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An Introduction to the Physics of the New Particles.

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PREFACE

The purpose of the present work is, as stated in the title, to provide an introduction to the physics of the new particles. It should serve as a guide to postgraduate students who intend to enter this field both as experimenters or theoreticians. For this reason the treatment has been kept at a level as elementary as possible.

Although only a few years have past since the discovery of the first new particle a large amount of information has been already accumulated; we felt that a review like the one presented here, covering all the aspects of this branch of physics might on one hand save much time to a rather large class of persons; and on the other help us in extending our knowledge and understanding on many points both of technical and of theoretical nature. We feel that we have, at least partially, reached our second goal; as far the first is concerned we hope that what follows may be of some use to somebody.

We are perfectly aware that in a few years much better statistics will be available and other new facts will be discovered; this review will then have only an historical value; however we believe that it will not be without in-

terest to retrace the rather fascinating history of the succession of experimental and theoretical discoveries which led to the classification of the particles in terms of strangeness; or to the discovery of the non conservation of parity; or to the observation of the longlived neutral heavy mesons etc.

We have divided this review into three parts. The first gives an account of the basic properties (decay modes, mass, mean life etc.) of the K mesons and hyperons, and of the experiments which lead to the determination of these properties. The discussion of the experiments has been carried out in some detail whenever we thought it was necessary to establish the confidence level to be associated with the reported results.

The second part gives an account of the attempts which have been made to interpret the behaviour of the new particles: the paradox strong production-slow decay, the introduction of the strangeness, the $K^+_{\pi 2}$, $K^+_{\pi 3}$ problem, the two neutral boson components and so on. We have only mentioned, without entering into details, the attempts to classify the new particles based on postulating invariance under rotations in a *four* dimensional isotopic spin space. We apologize for such an omission, both to the reader and to the respective authors, but our feeling is that these attempts are very rapidly evolving and it may be more profitable to review them when they will attain a less fluid state than at present.

The third part finally contains a survey of the interaction properties of the new particles with the old ones; that is: associated production in hydrogen and in complex nuclei, interaction of K^+ and K^-, hyperfragments etc.

This review was essentially completed by the end of December 1956; however we have decided to include also those results received after the above date and up to March 15, 1957 which have brought important additions in the knowledge previously accumulated; for instance we have included the results of the beautiful experiments of the Brookhaven and Berkeley bubble chamber groups; the results of the Brussels, Milan, Padua groups on the distributions in the $K^+_{\pi 3}$ decay; and a number of other important results.

Separate mention deserves the question of the non conservation of parity in the weak interactions; the experimental results which confirmed LEE and YANG's suggestion reached us after December 15, 1956. They have been briefly reported, together with some theoretical results, in Sect. **12·6**; the theoretical consequences for the neutral bosons have been summarized in Sect. **14·8**. Apart from these additions we usually have not modified the text as written previously to the experimental confirmations of the non conservation of parity; as a consequence in a few points some statements may need a modification; this is in the most part of the cases clear and straightforward.

The bibliography (at the end of each chapter) although not complete includes—we hope—all the important contributions.

We shall be grateful to anyone who points out to us mistakes contained in the text. We also thank in advance all those who will send us preprints or reprints which may facilitate the preparation of a book—on the same subject of the present article—to appear in the near future.

* * *

We would like to express our gratitude to the various groups doing research in this field and especially to those of Berkeley, Bologna, Bristol, Brookhaven, Columbia, École Polytechnique, Göttingen, Milan, M.I.T., Padua, Princeton, Sydney, for having regularly sent us preprints or reprints, without which this work could not have been accomplished; particular thanks we want to express to C. O'Ceallaigh, K. Crowe, G. and S. Goldhaber, N. Kroll, L. Lederman, T. D. Lee, A. H. Rosenfeld, J. Steinberger, C. N. Yang. C. Wu for preprints and/or discussions on particular points.

We are grateful to Prof. E. Amaldi for his continuous encouragement during the whole period in which this work has been done; we thank him also for having kindly communicated to us all the information which he received and which could be of interest to us.

Finally special thanks are due to Prof. G. Polvani, Editor of *Il Nuovo Cimento* and President of the Italian Physical Society, and Ing. Corbi, Secretary of the Editorial Board, for the interest that they have shown in our work; to Mr. Orlandi and the personnel of the Tipografia « Monograf » for their invaluable assistance during the correction of the proofs and to the Tipografia Compositori who printed this issue.

Note regarding references.

This article is divided into chapters (numbered consecutively throughout the whole work), into sections and in some cases also into subsections. The numeration of the formulae begins from one in each section, and contains the number of the formula, that of the chapter and that of the section; for example: eq. (3-16·9) refers to eq. 3 of Sect. **9** of Ch. **16**. A similar ordering has been used for the figures and for the tables. References to formulae, figures, and tables, in the current section are given by their number; in other cases the number is followed by the chapter section numbers. Bibliographic references have been always indicated by a number in square brackets [ ].

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PART I.

BASIC EXPERIMENTAL FACTS

Introduction.

The principal aim of Part I is to give a critical survey of the experiments and of the experimental results on which our present knowledge of some fundamental facts concerning the « new particles » is based. We shall deal here only with their masses, mean lives and modes of decay.

The amount of data accumulated in the last few years on this subject is indeed enormous. The construction of new powerful accelerators; of new devices for the focalization of the secondary charged particles produced by these machines; and finally the development of new instruments for the detection and the analysis of natural and artificial radiations, have all contributed to make possible a startling advance in our knowledge in this field.

Consequently little space could be devoted to some of the earlier experiments. Although interest in them still remains, it is chiefly confined to their technical aspect rather than to the result which they produced. The reader will find most of them described in detail in review articles published in the past on the *Progress in Cosmic Ray Physics* or in the *Annual Review of Nuclear Science* [1-4].

The classification of the experimental data has been made here on a phenomenological basis. « Particles » have been distinguished first according to their mass and charge: differences in their decay modes and in their mean life have provided reasons for further distinction.

The evidence reported refers to the following new particles:

i) K-*Mesons*. This name has been associated in the current literature with unstable particles whose mass lies between that of the proton and that of the π-mesons. Only K-mesons with a mass equal to 966 m_e—at least within a few electronic masses—have been proved to exist, associated with a charge $\pm e$ or neutral. The existence of different masses has been suggested by various authors [5, 6] on the basis of a few observations, but has not been confirmed by sufficient evidence and will not be considered.

ii) *Hyperons.* This name is used in connection with unstable particles heavier than protons. So far as we know, it includes the following:

1) the Λ^0 of mass $(2181 \pm 0.2)\ m_e$ and charge zero;

2) the Σ^+ Σ^- Σ^0 having slightly different masses, all within the interval 2327 to 2341 m_e;

3) the Ξ^- having a mass of $(2583 \pm 6)\ m_e$. A neutral partner is expected to exist on theoretical grounds, but so far has not been observed.

A resumé of the data pertinent to the various particles is given in the Tables on pag. 12, together with the nomenclature used in the text. It will be noted that for the K-mesons we have adopted a more uniform notation than has been done hitherto in the current literature.

REFERENCES

[1] H. S. Bridge: *Progress in Cosmic Ray Physics*, vol. III, ch. II.
[2] C. C. Dilworth, G. Occhialini and L. Scarsi: *Ann. Rev. of Nucl. Sci.*, **4**, 282.
[3] R. W. Thompson: *Progress in Cosmic Ray Physics*, vol. III, ch. III.
[4] J. G. Wilson: *Progress in Cosmic Ray Physics*, vol. II, ch. II.
[5] M. Danysz, W. O. Lock and G. Yekutieli: *Nature*, **169**, 364 (1952).
[6] R. B. Leighton, S. D. Wanlass and C. D. Anderson: *Phys. Rev.*, **89**, 148 (1953).

TABLE I. – *Properties of the established particles* (*).

Particle	Mass (m_e)	Mass (MeV)	Charge (e) (1)	Notations: used in the present article	Notations: alternative	Decay scheme (2)	Relative abundancies of different decay modes (percent)	Mean life (s)	Spin (2) ($\hbar$)
Photon	0	0	0	γ	—	stable	—	stable	1
Neutrino	0	0	0	ν	—	stable	—	stable	$\frac{1}{2}$
Electron	1	$0.510\,976 \pm 0.000\,007$ (3)	± 1	$e^{\pm}$	—	stable	—	stable	$\frac{1}{2}$
L-mesons	206.86 ± 0.11 (4)	105.70 ± 0.06	± 1	L $\mu^{\pm}$	—	$e^{\pm}+\nu+\nu$	—	$(2.22 \pm 0.02)\cdot 10^{-6}$ (4)	$\frac{1}{2}$
	264.37 ± 0.6 (4)	135.04 ± 0.16	0	L π^0	—	$\gamma+\gamma$ $e^{+}+e^{-}+\gamma$ $2e^{+}+2e^{-}$	98.8 1.2 0.004	$< 10^{-15}$ (6)	(0)
	273.27 ± 0.11 (4)	139.63 ± 0.06	± 1	L $\pi^{\pm}$	—	$\mu^{\pm}+\nu$ $e^{\pm}+\nu$	— $< 10^{-3}$ (5)	$(2.56 \pm 0.05)\cdot 10^{-8}$ (4)	(0)
	966.3 ± 1.4	493.73 ± 0.7	$+1$	$K^{+}_{\pi 3}$	τ^{+}	$\pi^{+}+\pi^{+}+\pi^{-}$	6.1 ± 0.3	$(1.12 \pm 0.03)\cdot 10^{-3}$	(0)
			$+1$	$K'^{+}_{\pi 3}$	τ'^{+}	$\pi^{+}+\pi^{0}+\pi^{0}$	2.2 ± 0.3		
			$+1$	$K^{+}_{\pi 2}$	χ^{+}, θ^{+}	$\pi^{+}+\pi^{0}$	27 ± 2		
			$+1$	$K^{+}_{\mu 3}$	$\varkappa^{+}$	$\mu^{+}+\pi^{0}+\nu$	1.9 ± 0.4		
			$+1$	$K^{+}_{\mu 2}$	K^{+}_{μ}	$\mu^{+}+\nu$	59 ± 2		
			$+1$	$K^{+}_{\beta 3}$	K^{+}_{e3}	$e^{+}+(\pi^{0})+(\nu)$	3.3 ± 1		
			-1	$K^{-}_{\pi 3}$	τ^{-}	$\pi^{+}+\pi^{-}+\pi^{-}$	?		
			-1	$K^{-}_{\pi 2}$	χ^{-}, θ^{-}	$\pi^{-}+\pi^{0}$	?		
K-mesons	965.6 ± 1.2	493.4 ± 0.6	-1	$K^{-}_{\mu 2}$	K^{-}_{μ}	$\mu^{-}+\nu$	?	$(1.4 \pm 0.2)\cdot 10^{-8}$	(0)

			0	—	—	(others)	(< 2) (?)		
			0	$K^0_{\pi 3}$ (K^0_2)	τ^0 (θ^0_2)	$\pi^+ + \pi^- + \pi^0$	?	$3\cdot 10^{-8} < \tau < 10^{-7}$	(0)
			0	$K^0_{\pi^0 3}$ (K^0_2)	τ^0 (θ^0_2)	$\pi^0 + \pi^0 + \pi^0$	?		
			0	$K^0_{\mu 3}$ (K^0_2)	— (θ^0_2)	$\mu^\pm + \pi^\mp + \nu$	?		
			0	$K^0_{\beta 3}$ (K^0_2)	K^0_{e3} (θ^0_2)	$e^\pm + \pi^\mp + \nu$	?		
Proton	$1\,836.12 \pm 0.04$	938.214 ± 0.024	± 1	N p	—	stable	—	stable	$\frac{1}{2}$
Neutron	$1\,838.65 \pm 0.04$	939.508 ± 0.024	0	N n	—	$p + e^- + \nu$	—	$(1.12 \pm 0.3)\cdot 10^3$ (4)	$\frac{1}{2}$
Hyperons	$2\,181 \pm 1$	$1\,114.7 \pm 0.6$	0	Y Λ^0	—	$p + \pi^-$; $n + \pi^0$	68 ± 5; 32 ± 5	$(3.03 \pm 0.18)\cdot 10^{-10}$	?
	$2\,323 \pm 7$	$1\,187 \pm 3.6$	0	Y Σ^0	—	$\Lambda^0 + \gamma$	—	$< 10^{-11}$	?
	$2\,327 \pm 1$	$1\,189 \pm 0.5$	$+1$	Y Σ^+	—	$p + \pi^0$; $n + \pi^+$	~ 50; ~ 50	$(0.903^{+0.17}_{-0.13})\cdot 10^{-10}$	?
	$2\,342 \pm 1.5$	$1\,196.7 \pm 0.8$	-1	Y Σ^-	—	$n + \pi^-$	—	$(1.74\ ^{+0.25}_{-0.20})\cdot 10^{-10}$	?
	$2\,583 \pm 5.5$	$1\,319.7 \pm 2.6$	-1	Y Ξ	—	$\Lambda^0 + \pi^-$	—	$\geqslant 1.8\cdot 10^{-10}$	?

(*) Most of the data given here are averages calculated from the published results, using the formula

$$\langle X \rangle = \frac{\Sigma X_i/\mu_i^2}{\Sigma 1/\mu_i^2} \pm 1\Big/\sqrt{\Sigma 1/\mu_i^2}$$

X_i being the i-th result and μ_i the error associated with it, as given by the respective authors. Whenever, in the writers' opinion, the calculation of an average was not justified, the most precise result has been reported. Reference to the source has been made only for those data not discussed in the text.

(1) $e = (4.80286 \pm 0.00009)\cdot 10^{-10}$ u.e.s. E. R. Cohen, J. W. M. DuMond, T. W. Layton and J. S. Rollet: *Rev. Mod. Phys.*, **27**, 363 (1955).

(2) Data given in brackets in this column are not to be considered as established.

(3) E. R. Cohen, J. W. M. DuMond, T W. Layton and J. S. Rollet: *Rev. Mod. Phys.*, **27**, 363 (1955).

(4) K. M. Crowe: *Nuovo Cimento*, **5**, 541 (1957).

(5) H. L. Anderson and G. M. G. Lattes: private communication.

(6) J. Orear, G. Harris and S. Taylor: *Bull. Am. Phys. Soc.*, Series II, vol. **2**, no. 1, p. 6.

TABLE II. – *Characteristic decay parameters regarding unstable particles.*

CHARGED PARTICLES												
Particle and decay mode	Q (MeV)	α^* [1]	ε^*	Characteristics of the charged decay products								
				p^* (MeV/c)	E^*_{kin} (MeV)	$(p\beta)^*$ (MeV/c)	β^*	Residual range				
								emuls. [2] (cm)	C [3] (g/cm²)	Al [3] (g/cm²)	Cu [3] (g/cm²)	Pb [3] (g/cm²)
$\mu^\pm \to e^\pm + \nu + \nu$	105.2	—	—	$\leqslant$ 52.8	$\leqslant$ 52.8	$\leqslant$ 52.8	$\lesssim 1$	—	—	—	—	—
$\pi^\pm \to \mu^\pm + \nu$	33.9	± 0.573	0.427	29.8	4.1	8.07	0.271	0.0592	0.148	0.180	0.22	0.33
$K^\pm_{\pi 3} \to \pi^\pm + \pi^\pm + \pi^\mp$	74.8	—	—	$\leqslant$ 125.2	$\leqslant$ 48.0	$\leqslant$ 83.7	$\leqslant$ 0.668	$\leqslant$ 3.3	$\leqslant$ 9.4	$\leqslant$ 10.8	$\leqslant$ 12.7	$\leqslant$ 17.4
$K'^\pm_{\pi 3} \to \pi^\pm + \pi^0 + \pi^0$	83.8	—	—	$\leqslant$ 132.6	$\leqslant$ 52.9	$\leqslant$ 91.3	$\leqslant$ 0.688	$\leqslant$ 3.9	$\leqslant$ 11.0	$\leqslant$ 12.7	$\leqslant$ 14.9	$\leqslant$ 20.4
$K^\pm_{\pi 2} \to \pi^\pm + \pi^0$	218.9	± 0.005	0.831	205.0	108.4	169.5	0.827	11.7	34.2	39	45.1	60.5
$K^\pm_{\mu 3} \to \mu^\pm + \pi^0 + \nu$	252.8	—	—	$\leqslant$ 215.0	$\leqslant$ 133.8	$\leqslant$ 193.0	$\leqslant$ 0.897	$\leqslant$ 15.2	$\leqslant$ 50.4	$\leqslant$ 57.6	$\leqslant$ 67.4	$\leqslant$ 89.4
$K^\pm_{\mu 2} \to \mu^\pm + \nu$	387.9	± 0.046	0.954	235.5	151.4	214.8	0.912	20.22	59.5	67.8	78.8	104.8
$K^\pm_{\beta 3} \to e^\pm + \pi^0 + \nu$	358.0	—	—	$\leqslant$ 228.3	$\leqslant$ 228.3	$\leqslant$ 228.3	$\lesssim 1$	—	—	—	—	—
$\Sigma^+ \to p + \pi^0$	115.7	$+0.609$	0.317	188.6	18.8	36.2	0.197	0.16	0.415	0.51	0.63	0.96
$\Sigma^+ \to n + \pi^+$	109.9	-0.611	0.311	184.7	91.7	147.5	0.798	9.0	26.2	29.8	34.9	47.1
$\Sigma^- \to n + \pi^-$	117.6	$+0.603$	0.322	192.4	98.1	155.7	0.809	11.5	29.1	33.1	38.7	52.6
$\Xi^- \to \Lambda^0 + \pi^-$	65.4	$+0.702$	0.209	138.2	56.9	97.3	0.704	3.53	12.5	14.3	16.8	23

(1) m_1 is identified with the positive product (in the decay of a positive primary) or with the neutral one (in the decay of a negative primary).

(2) Using BARONI *et al.* range energy curve [*Ric. Sci.*, **26**, 1718 (1956)]. The emulsion density was assumed to be 3.92 g/cm³.

Particle and decay mode	Q (MeV)	α^*	ε^*	p^* (MeV/c)	E^*_γ (MeV)	Particle and decay mode	Q (MeV)	p^* (MeV/c)	E^*_γ (MeV)
$\pi^0 \rightarrow \gamma+\gamma$	135.1	—	—	67.6	67.6	$\pi^0 \rightarrow e^+ + e^- + \gamma$	134.1	$p^*_\gamma < 67.1$	< 67.1
$K^0_{\pi 2} \rightarrow \pi^+ + \pi^-$	214.3	0	0.825	203.5	—	$K^0_{\pi 3} \rightarrow \pi^+ + \pi^- + \pi^0$	79.3	$p^*_{\pi^0} < 128.4$	$28.9 \div 157.3$
$K^0_{\pi^0 2} \rightarrow \pi^0 + \pi^0$	223.4	0	0.837	206.5	$21 \div 225$	$K^0_{\pi^0 3} \rightarrow \pi^0 + \pi^0 + \pi^0$	88.3	$p^*_{\pi^0} < 135.6$	$28.0 \div 163.5$
$\Lambda^0 \rightarrow p + \pi^-$	36.9	+0.693	0.179	99.6	—	$K^0_{\mu 3} \rightarrow \mu^\pm + \pi^\mp + \nu$	248.3	$p^*_\nu < 185.8$	—
$\Lambda^0 \rightarrow n + \pi^0$	40.1	0.695	0.193	107.8	$33 \div 134$	$K^0_{\beta 3} \rightarrow e^\pm + \pi^\mp + \nu$	353.5	$p^*_\nu < 226.9$	—
$\Sigma^0 \rightarrow \Lambda^0 + \gamma$	72.3	0.882	0.118	69.9	69.9	—	—	—	—

Mass values used in computing the data given above:

Particle:	μ	π^0	$\pi^\pm$	$K^{\pm,0}$	p	n	Λ^0	Σ^0	Σ^+	Σ^-	Ξ^-
Mass (MeV)	105.7	135.1	139.6	493.6	938.2	939.5	1114.7	1187.0	1189.0	1196.7	1319.7

SYMBOLS:

- M mass of the primary unstable particle.
- m_i mass of the i-th decay product.
- Q total kinetic energy associated with the decay products in the frame of reference in which the primary was at rest.
- α^* see Sect. **9·1**; $\alpha^* = (m_1^2 - m_2^2)/M^2$, m_1 and m_2 being the masses of the decay products emitted in 2-body decays.
- p^* decay product momentum, at emission, in the frame of reference in which the primary was at rest.
- ε^* see Sect. **9·1**; $\varepsilon^* = 2p^*/M$.
- E^*_{kin} decay product kinetic energy, at emission, in the frame of reference in which the primary was at rest.
- β^* decay product velocity (in c-units), at emission, in the frame of reference in which the primary was at rest.
- E^*_γ energy of γ-rays, directly emitted or emitted in the decay of secondary π^0's, in the frame of reference in which the primary was at rest.

CHAPTER 1.

Observations on Positively Charged K-Particles (K^+).

1·1. – The established modes of decay. Brief survey of the early work.

In this section we discuss briefly the evidence concerning the existence of the six decay modes listed in the Table on page 12 and the early experiments on which this evidence was based.

In the discussion it has been assumed that all K-mesons observed to decay from rest were positively charged. This assumption is certainly plausible after the recent results obtained from magnetically analyzed radiations, which indicate that negative K particles undergo nuclear capture when brought to rest.

The $K^+_{\pi 3} \to \pi^+ + \pi^+ + \pi^-$ *mode.* – The discovery of the first example of $K_{\pi 3}$ mode by the Bristol group [1] followed closely the application of electron sensitive nuclear emulsion to research in cosmic rays. The event, which is reproduced in Fig. 1 was in fact found in one of the first NT4 Kodak electron sensitive emulsions prepared by Dr. BERRIMAN [2]. A track (k) was associated

Fig. 1-1·1. – The first example of $K^+_{\pi 3}$ observed in nuclear emulsion [1].

Fig. 2-1·1. – The first example of a «complete» $K^+_{\pi 3}$ decay, reported by [6].

with a disintegration (A) from which two lightly ionizing particler (a, b) and a heavily ionizing (c) one were emitted. The latter produced a disintegration (B) leading to the emission of two slow particles (probably heavier than protons) and possibly of a slow recoil. It was assumed that the interaction B was a π^--star. Both particles a) and b) escaped from the emulsion after 2 000 μm and 116 μm respectively. Their ionization could be shown to be approximately 2.2 times minimum for both tracks; their scattering was small enough to exclude the possibility that they could be electrons. The precision attainable in mass measurements did not allow their identity to be established with certainty, although there were strong indications that they could only be π- or μ-mesons. Moreover, it could be shown that the three particles (a), (b) and (c) were coplanar at least within 4°, thus indicating that no neutral particle was involved in the disintegration (A).

Track (k) was 3 000 μm long and the gradual change of ionization clearly indicated that the particle producing it moved towards (A). Its mass, as measured by the grain counting method, appeared to be $(1\,080 \pm 160)\ m_e$.

Such an event could be interpreted as due to the interaction of an unstable particle with a nucleus and subsequent explosion of the latter. This hypothesis did not seem likely in view of the fact that the release of such a high energy, inside or even at the periphery of a nucleus, would be expected in general to lead to the evaporation of a considerable number of nucleons or nuclear fragments. The quoted Authors concluded that most probably it wsa the spontaneous disintegration of an instable particle.

Subsequent work proved them right. Two events very similar to the one just described were observed a year later by HARDING [3] and in another paper FOWLER *et al.* [4] were able to show, on the basis of conservation laws and grain density measurements, that the secondary particles were most probably due to π-mesons. This conclusion was supported by further work done in Rome [5] and finally proved by an observation made at Padua [6] where a $K^+_{\pi 3}$-meson with all its decay products stopping in emulsion was found (see Fig. 2).

The $K^+_{\mu 3} \to \mu^+ + \pi^0 + \nu$ *mode.* – The years between 1949 and 1951 witnessed a tremendous improvement both in the quality of high sensitivity emulsions and in the methods of analysis of the recorded radiations. The existence of new technical resources stimulated amongst other things reconsideration of old problems. It was at this stage of technical development that O'CEALLAIGH [7], studying the distribution in energy of electrons emitted in μ-e-decays, observed an event similar in appearance to a μ but associated with a secondary far too energetic to be of this type. Careful measurements on the primary track revealed that it was due to a particle more massive than a μ and less than a proton. A second example, which he found shortly after the first,

looked quite different: the primary particle (having a mass of $(1\,125 \pm 140)\, m_e$) decayed at rest into a slow secondary which could be identified with a μ, ejected at an energy of 5.9 MeV.

If the two cases were to be interpreted as due to the same particle their decay scheme had to involve at least two neutrals. The correctness of this interpretation was confirmed by MENON and O'CEALLAIGH [8] on the basis of eight further examples observed to decay into identified μ's whose energies were spread over a large interval from 0 to ~ 200 MeV. Already in his first paper [7] O'CEALLAIGH concluded that if the primary K was assumed to be a boson, the simplest possible decay mode, involving known particles only, was

$$K^+_{\mu 3} \to \mu^+ + \pi^0 + \nu\,.$$

For a long time, however, no proof for or against it was obtained. It must be pointed out that nuclear emulsions are most unsuitable for revealing π^0's in comparison with cloud chambers or bubble chambers. To prove or disprove the existence of γ-rays emitted either directly or through an intermediary π^0, a search should be made on a region extended over several cm² of emulsion. Even in the most favourable conditions, such a search is most difficult and in the majority of cases leads to questionable results.

The easiest way was to look for the rare mode of decay of the π^0 ($\to e^+ + e^- + \gamma$) which is known to occur with a relative frequency of ~ 1.2 percent. The resulting electron-positron pair, owing to the very short lifetime of the π^0, appears practically connected with the terminal part of the K and cannot be easily missed. Such was in fact the type of the experimental evidence which allowed recently the Rochester group to put on somewhat firmer ground O'Ceallaigh's original suggestion. YEKUTIELI *et al.* [9] and HOANG *et al.* [10] observed two disintegrations of K^+-mesons in emulsion, each associated with the creation of three ionizing particles. In both cases, two of them could be proved to be electrons and the third a μ-meson, the latter having energies of 18 and 110 MeV respectively (Fig. 3).

Fig. 3-1'1. – Reproduction in fac-simile of two $K^+_{\mu 3}$ decays associated with electron pairs, observed by the Rochester group [9, 10].

There was no obvious proof that the electron pairs were produced by a parent π^0: a scheme of the type $K^+_{\mu 3} \to \mu^+ + e^+ + e^- + n_1 + n_2 + \ldots$ (n_i indicating a neutral particle) could in fact be adequate to describe the events. These

authors showed, however, that if only one neutral n_i was postulated, the dynamics of the two cases were inconsistent with a unique value of its mass ($236 \leqslant m_{n_1} \leqslant 265$ MeV in one and $170 \leqslant m_{n_1} \leqslant 224$ MeV in the other case).

On the other hand, the scheme $K^+_{\mu 3} \to \mu^+ + \pi^0 + n_1$, was consistent with both events. An upper limit for the value of m_n could be calculated and shown to be 75 MeV. Restricting the possible choices to the known particles, n_1 could only be a γ-ray or a neutrino and assuming the K-particles to be bosons, the choice fell on the latter. Thus, though not definitely proved, this decay scheme seems highly probable.

The $K^+_{\pi 2} \to \pi^+ + \pi^0$ *mode.* – In the course of the investigation mentioned above [7], MENON and O'CEALLAIGH obtained also definite evidence for the existence of π's arising from a two body decay. At the Discussion Meeting of the Royal Society, held in London in 1953, they reported on the mass measurements of a number of secondaries from K's at rest and compared them to others obtained by DANIEL and PERKINS on identified π-particle tracks (see Fig. 4, *a*-*b*).

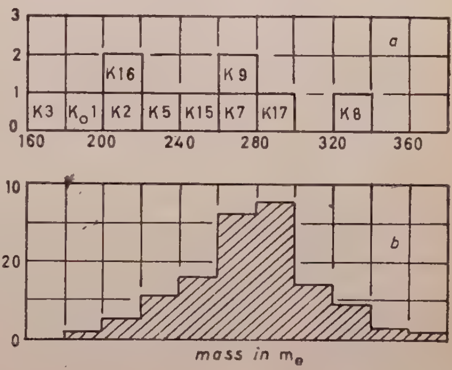

Fig. 4-1·1. - *a*) Mass measurements on secondary particles of K^+ decays; *b*) Mass spectrum obtained by DANIEL and PERKINS on 129 tracks produced by shower particles. (As reported in [8]).

Although the two diagrams are not directly comparable, having been obtained under different conditions, the Authors decided that the events in *a*) could not all be μ-mesons. Their judgement was further supported by the fact that, taking as μ all the particles associated with a measured mass smaller than 240 m_e and as π the others, the $p\beta$-distributions for the two groups had substantially different characteristics. As one can see from Fig. 5 the supposed π's are all between the values of $p\beta = 160$ and 200 MeV/c (diagram *a*) while the μ's appear to be spread from zero to about 320 MeV/c. Despite the poor statistics, it seemed to them

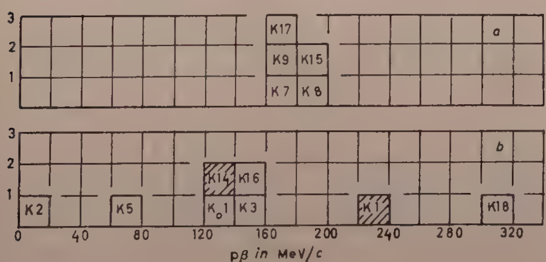

Fig. 5-1·1. – $p\beta$'s of K^+ secondary particles. Measurements related to the same event are marked with the same number in this and in the preceding diagram (Fig. 4-1·1).

very unlikely that this uniqueness of values for the π's could be due to chance only; they suggested therefore that a two-body decay of the type $K^+ \to \pi^+ + n$ existed. The nature of the neutral particle, (n), remained to be established. From momentum and energy conservation they could say that, if it had to be chosen among the known particles, (γ, neutrino, π^0, neutral K) the mass M of the parent K had to be

a) for $n \equiv \gamma$ or neutrino $M = 945\ m_e$

b) for $n \equiv \pi^0$ $M = 1019\ m_e$

c) for $n \equiv$ neutral K $(1000\ m_e)$ $M = 1610\ m_e$

Direct mass measurements on some of the primaries excluded the possibility *c*) but did not allow one to decide between *a*) and *b*). The answer came from cloud chamber experiments, far more apt to solve this type of problems. From the analysis of pictures taken operating a chamber at mountain altitudes, BRIDGE *et al.* [11] concluded in favour of decay scheme *b*). They argued as follows: the most frequent decay modes are those associated with the emission of two secondaries only, the $K_{\pi 2}$ and the $K_{\mu 2}$ (see Sect. 1·3) the others being present in negligible numbers. Those two are easily distinguished because the π emitted from the $K_{\pi 2}$ has a range of $\sim 60\ g/cm^2$ Pb and the μ from $K_{\mu 2}$ one of $\sim 105\ g/cm^2$ Pb. The observations show that:

a) of five events in which the secondary particle stopped after a range of $\sim 60\ g/cm^2$ Pb four are associated with soft showers;

b) in eight events in which the presence of associated photons was established and the secondary escaped from the chamber, the potential path of the secondaries was less than $60\ g/cm^2$ Pb;

c) in none of the cases in which the secondaries were seen to cover more than $60\ g/cm^2$ Pb associated photons could be detected.

The Authors concluded that indeed $K_{\pi 2}$ decays were associated with the emission of γ-rays. It remained to be established whether these were emitted directly or arose from the subsequent decay of a π^0. This could be done simply by observing whether all the γ's were moving in nearly the same directionand opposite sense as the charged secondary π's or not. The experiments showed that this was not the case [11, 12]. Further support for this conclusion is afforded by fourteen events observed in emulsion in which the π^0 is seen to decay into an electron pair [13-15].

The $K'^+_{\pi 3} \to \pi^+ + 2\pi^0$ *mode.* – The existence of a mode of decay involving one charged and two neutral π's was anticipated by DALITZ [16] in 1953. More than a year later his suggestion was substantiated by experimental findings. The first event to be attributed to this mode is due to CRUSSARD

et al. [17] who identified in nuclear emulsion a K particle decaying into a slow π^+-meson ($E_\pi \sim 14$ MeV). It could not be due to a $K^+_{\pi2}$ as $K^+_{\pi2}$-secondaries have an energy of ~ 108 MeV; it was on the other hand consistent with Dalitz' scheme which was expected to produce a continuous spectrum of π's from 0 to a maximum value of ~ 53 MeV. In view of the large statistics now available it can be inferred that the conclusions of CRUSSARD *et al.* were correct.

The $K^-_{\mu2} \to \mu^- + \nu$ *mode*. – The existence of the two body $K_{\mu2}$-decay was established, almost at the same time as the $K'^+_{\pi3}$, by the cloud chamber group of the École Polytechnique [18]. It is at first sight a surprising fact that several emulsion teams, all working on this subject, could fail to individuate this mode which is the most numerous amongst the K^+'s whilst they had detected the much rarer mode $K^+_{\mu3}$. A possible explanation may be found in Sect. **1·3** where experimental biasses are discussed. It will be seen that the high momentum of the secondary μ from $K_{\mu2}$ places these events in a region where ordinary methods of mass measurements in emulsion do not allow an easy discrimination between π's and μ's or between μ's and electrons. Only the recent introduction of very large stacks has made available to research workers a substantial number of long tracks on which accurate determinations are possible.

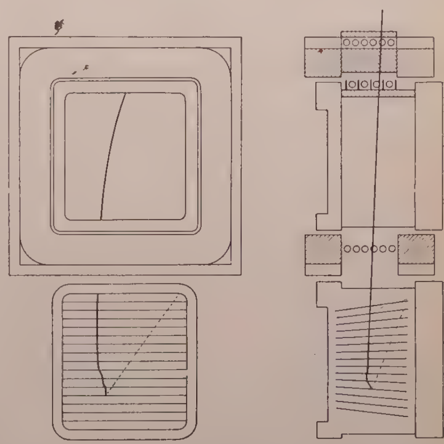

Fig. 6-1·1. – Schematic drawing of the arrangement used by the Paris group of l'École Polytechnique.

The Paris group, using large double cloud chambers (see Fig. 6) was in a better position with regard to this point. Take, for instance, what they consider « le cas idéal », i.e. a K-meson coming from above, crossing the upper magnet chamber and stopping and decaying in the multiplate below it. The combined knowledge of its momentum and residual range gives its mass with a precision often sufficient to individuate it with certainty as a K. Suppose, furthermore, that the secondary is emitted in a direction to allow it to cross a large number of plates: its residual range (or limits for it) can be determined and, from its behaviour in traversing heavy matter, information about its nature may be obtained.

Out of a large number of events observed with this apparatus, the quoted authors found one in which the primary mass was determined to be $(911 \pm 70)\ m_e$ and the secondary had a residual range larger than (84 ± 4) g/cm Pb. This could not be due to any K^+_π decay for which the maximum range is 60 g/cm Pb.

They also noted the presence of a large number of high momentum secondaries ($\gtrsim$ 100 MeV). Unless the momentum distribution of the K-secondaries was so shaped as to be significantly different from zero only above that value (which appeared unlikely), the result strongly suggested the existence of a two body decay. In the latter case the neutral decay product could be shown to have a negligible mass. The absence of associated soft radiation excluded the γ-ray, leaving the neutrino as the only choice [19]. These conclusions were confirmed by the work of the M.I.T. group [11].

The $K^+_{\beta 3} \to e^+ + ? + ?$ *mode.* – Experimental evidence for this mode was found by FRIEDLANDER *et al.* [20], in a photographic emulsion exposed to cosmic rays. They observed a K particle, decaying at rest into a fast secondary having a $p\beta$ of about 90 MeV/c and grain density equal to the plateau value within 2%. They noted that a π or a μ meson of this specific ionization would have had too high a $p\beta$ to be consistent with the other independent measurement.

Careful inspection of the secondary track showed that the particle producing it suffered a sudden change of direction at a distance of 2.3 cm from the point at which it originated. After that deflection the track could be identified with certainty as due to an electron of ~ 5 MeV.

The observed deviation of the secondary track could be due to the decay in flight of a secondary μ. This explanation however seemed rather unlikely in view of the long lifetime of μ mesons: in fact the Authors calculated that the probability of observing such a process was only $4 \cdot 10^{-4}$, considering the total number of μ-mesons observed at rest in their emulsions.

It seemed much more reasonable to assume that the secondary track was due to an electron right from the point of decay. The observed angular deviation could then be explained as a collision involving the emission of bremsstrahlung, an interpretation which was consistent with the fact that the corresponding mean free path of an electron of 90 MeV for this type of process is 2.9 cm of emulsion and the observed collision occurred after 2.3 cm of its path.

Further evidence was reported by DAHANAYAKE *et al.* [21] and by JOHNSTON and O'CEALLAIGH [22]. The energy of the secondaries were found to be respectively ~ 50 and ~ 80 MeV thus suggesting a continuous spectrum and therefore a three body decay. On the nature of the neutral particles nothing can be said at present.

1·2. – The masses and the Q-values of the K$^+$'s.

Such a large number of experiments was performed in the last few years to determine the mass of K-particles that it would be neither useful nor interesting to discuss all of them in detail in the present article. The continuous

progress of the techniques involved in this type of experiments has considerably reduced the interest associated with some of the early ones (*).

Precise mass measurements on K^+'s have been made practically only using emulsions. In this type of measurements the emulsion is definitely superior to other techniques for it provides us with a detailed knowledge of the path of a particle precise within the limits imposed by the size of a AgBr grain in a medium which is dense enough to produce a measurable average scattering even for high energy radiation.

The problem has been attacked in two different ways: one of *direct mass determination* based on the study of the K-particles behaviour in traversing matter and magnetic fields; another (which we shall denote as *Q-method*) based on the determination of the energy released in the K-particle decays.

In what follows a brief description will be given of the experimental methods. The results are summarized in Table I.

i) *Direct mass determination.* – Of the methods which fall in this group, the most accurate one is based on the determination of the magnetic rigidity of individual particles and of their residual range. By this method, the mass of K^+ particles has been measured with an accuracy of $\sim 2\ m_e$. Chiefly, this remarkable result has been achieved as a consequence of the construction of new strong focussing devices, by a group of physicists working at the Radiation Laboratory [26], and of their application to the study of radiations produced by the Bevatron.

Their apparatus (see Fig. 1), focussed inside a stack of emulsions particles of a given momentum ejected from the target at 90° with respect to the primary proton beam. An analyzing magnet, placed on the path of the secondary particles after the strong-focussing lenses, introduced a horizontal momentum-dispersion. By the combined effects of this magnet and of the lenses, particles having the same mass and charge were focussed on a surface whose position and shape depended on those two parameters. Large numbers of particles of a given mass could then be brought to rest in a narrow region of the emulsions thus enormously facilitating their search which is very laborious in stacks exposed to cosmic rays.

Due to the presence of the analyzing magnet, the point of entry of a particle in the stack depends on its momentum. The latter was determined by measuring the residual range of protons (+) entering the stack at the same point. The mass value so obtained is not strongly dependent on the density

(*) For earlier references consult [23], [24] and [25].

(+) Protons are largely represented in the beam from the Bevatron. Smith *et al.* [27] find, for a primary proton energy of 4.8 GeV impinging on a Ta-target, at 90° to the primary beam at ~ 3 m from the target, the following composition of the 375 MeV/c

of the emulsion which is known to vary considerably from a batch to another and caused one of the largest sources of uncertainty in cosmic ray work.

Experiments of this type have been made by a number of different authors [28-30], who used very nearly the same procedure outlined above.

A different way to determine the mass of K particles was pointed out by CHUPP *et al.* [31]. In an emulsion exposed to a focussed K^+ beam they observed the track of a particle which had undergone a large scattering by a nucleus. Both scattered primary and recoiling target nucleus stopped in the emulsion thus allowing to be recognized as a K and a proton respectively. The tracks were observed to be coplanar within less than a degree and no other track could be seen in association with the point of interaction. They assumed this to be the elastic scattering of a K-particle on a free proton of the emulsion and calculated the mass from the dynamics of the event. From the range of the proton the transverse momentum of either particle was deduced and from it the momentum of the K; its residual range being known, the mass could be estimated as shown above. They obtained $M_K = (973 \pm 12)\, m_e$, in good agreement with other determinations. Unfortunately this type of collision is too rare to make the method practical on large scale measurements.

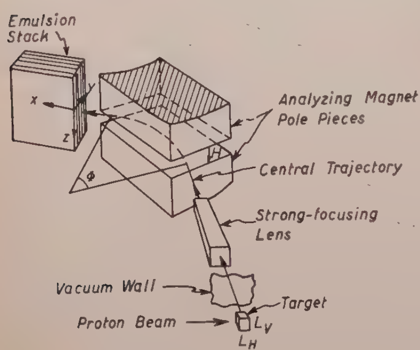

Fig. 1-1·2 – Apparatus employed by the Berkeley physicists to focus K^+-beams on emulsion stacks [30].

Methods based on the measurements of Coulomb scattering and specific ionization cannot reach such a precision level. The error involved in individual mass determinations very often exceeds 10%.

ii) *The Q-method.* – Whenever the magnetic analysis of the observed particles is not possible, this method is by far the best. Its application is restricted to the decays in which *a*) the scheme is known or can be established

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positive secondary flux

e^+	0.75 ± 0.25
π^+	3.84 ± 0.18
K^+	0.036 ± 0.002
^1H	87.31 ± 2.1
^2H	5.13 ± 1.12
^3H	1.22 ± 0.17
^3He	0.97 ± 0.49
^4He	0.75 ± 0.25

without previous knowledge of the primary mass, *b*) not more than one neutral secondary is emitted and *c*) the mass of the latter is known. Of the established modes, the $K_{\pi3}$, $K_{\pi2}$ and $K_{\mu2}$ satisfy these requirements, while the others are known to decay into more than one neutral.

The $K_{\pi3}$ is the most suitable to this type of mass determination, all the secondary particles being charged. When all can be brought to rest inside an emulsion stack, the total energy released is immediately deduced from their ranges. The experimental error is only that imposed by *a*) errors in the determination of the ranges of the pions, *b*) range straggling of the pions, *c*) uncertainty in the pion masses, *d*) uncertainty in the range-energy relation and in the density of the emulsion (*).

a) Tends to shorten the range and so to reduce systematically the measured Q-value. In general the sum of the chords which are used to approximate a pion track is taken as the true range. HADDOCK [32] estimates that the reduction is $\sim 0.5\%$ in range, which corresponds $\sim 0.25\%$ in the energy of the π's.

b) Is about 0.5% and statistical in nature.

c) Will contribute for less than 1%.

d) May be a dangerous source of errors if no calibration is made on the same emulsions which are used for the measurements. This is comparatively simple in emulsions exposed to momentum analyzed beams which contain a large number of different particles of known masses (and in such a case, the error can be reduced to a negligible size) but it is not so easy otherwise.

Most interesting in this connection is the method due to BACCHELLA *et al.* [33]. It consists in determining the energy relation in their emulsions by using the same $K_{\pi3}$ decays which are the object of their research.

Direct application of the momentum conservation theorem imposes that

$$\frac{p_i}{m_\pi c}\,\frac{1}{\sin\alpha_i} = \text{const} = \lambda\,, \qquad i = 1, 2, 3,$$

the p_i being the momentum of the i-th π-meson, α_i the angle between $\boldsymbol{p}_{i+1}$ and $\boldsymbol{p}_{i-1}$, and m_π the pion mass. In terms of λ, the Q-value is expressed by

$$Q + 3m_\pi c^2 = m_\pi c^2[(1+\lambda^2\sin^2\alpha_1)^{\frac{1}{2}} + (1+\lambda^2\sin^2\alpha_2)^{\frac{1}{2}} + (1+\lambda^2\sin^2\alpha_3)^{\frac{1}{2}}]\,.$$

For each value of Q, λ can be deduced and, from λ, the energy of each π. Out of a large number of decays an experimental range energy relation for

(*) For a detailed discussion on this question see [32].

π's can be obtained. Since Q is unknown, an independent point is required. This was provided from the π-μ-decay. The energy of the μ from decays at rest is known today with great precision and if one measures the residual range of a number of μ's, the range of a π having the same velocity can be calculated from the well known relation

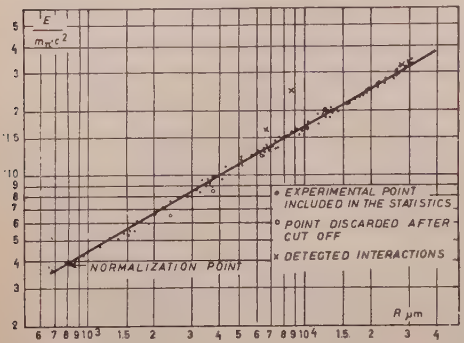

Fig. 2.1'2. – Range energy relation for pions as deduced from $K_{\pi 3}$ decays (after BACCHELLA *et al.* [33]).

$$R_\pi(\beta) = \frac{m_\pi}{m_\mu} R_\mu(\beta) \, .$$

The range energy relation thus obtained is shown in Fig. 2. The Q-value which was deduced is (see Table II) one of the most precise ($\sim 0.13\%$) so far published.

Errors in mass determination of two body decays ($K_{\pi 2}$ and $K_{\mu 2}$) as obtained by the Q-method are considerably higher ($\sim 2\%$). This is mainly due to the fact that the energy release is, in the latter cases, higher than in the $K_{\pi 3}$. Thus, while an error in the $Q(K_{\pi 3})$ of 1% corresponded to an error of less than 0.14% in the primary mass, here in the $K_{\pi 2}$ it would correspond to about 0.4%, i.e. three times as big.

iii) *Results.* – The data reported in both tables clearly indicate that, if the masses associated with the various modes of decay differ, the difference is within the experimental errors and, most probably, not larger than $\sim 3\, m_e$.

If the view is taken that all the K^+'s have indeed the same mass, the question arises as to which is the best method to determine it. An inspection of Tables I and II indicates that—at the present moment—the Q-method applied to the $K_{\pi 3}$ yields the most precise results.

In calculating the weighted average of the data listed in Table II, regarding the $K_{\pi 3}$ mode, it was not clear to us what weight to give to the different results since it appeared that the errors had been estimated in different ways. Owing to the high degree of precision which has been claimed by the respective authors the question of the criterion to be used in calculating the errors has become fundamental.

The value given by AMALDI [24] is an average of 54 events reported by a number of different laboratories and the error was calculated by treating as statistical those associated with the individual data. He pointed out that, owing to the uncertainty in the knowledge of the range energy relation and to the low degree of precision often involved in calibrating the stopping

Ref.	Source of radiation	Time of flight (s)	Masses (m_e) $K^+_{\pi 3}$	$K'^+_{\pi 3}$	$K^+_{\pi 2}$	$K^+_{\mu 3}$	$K^+_{\mu 2}$	$K^+_{\beta 3}$	All K^+'s
[28, 30]	Bevatron	$1.4\cdot10^{-8}$	966.3 ± 2.7	967.7 ± 4.3	966.9 ± 2.7	$969\ \pm 5$	967.2 ± 2.8	$967\pm$	967.0 ± 2.1
[35]	»	$1.2\cdot10^{-8}$	961.8 ± 2.6	951.5 ± 10.9	972.2 ± 4.7	951.2 ± 10.6	962.5 ± 3.2	—	964.2 ± 1.7
			Mass differences (m_e) $\Delta M = M_{K_i^+} - M_{K_{\pi 3}}$						
[28]	Bevatron	$1.4\cdot10^{-8}$	—	$-\ 2.4\pm 9.8$	$-9\ \pm13$	$12\ \pm16.2$	-1.8 ± 10.3	—	—
[36]	»	$\sim 10^{-8}$	—	—	—	—	—	—	$4\ \pm4$ (°)
[29]	»	$1.2\cdot10^{-8}$	—	-10.3 ± 10.9	10.4 ± 4.7	-10.6 ± 10.6	0.7 ± 3.2	—	2.4 ± 1.7 (°)
[30]	»	$1.4\cdot10^{-8}$	—	$+\ 1.4\pm 4.5$	0.6 ± 2.9	1.3 ± 5	0.9 ± 3	-2.7 ± 8	—
[37]	»	$1.5\cdot10^{-8}$	—	$+\ 2\ \pm 8$	$-3\ \pm9$	$-10\ \pm 9$	$-5\ \pm 7$	$+18\pm25$	—

(°) Not inclusive of $K^+_{\pi 3}$.

TABLE II-1.2. – *Mass determinations and Q-values (Q-method).*

Ref.	Source of radiation	Time of flight (s)	Range-energy relation (*)	Masses (m_e) $K^+_{\pi 3}$	$K^+_{\pi 2}$	$K^+_{\mu 2}$	Q (MeV) $K^+_{\pi 3}$	$K^+_{\pi 2}$	$K^+_{\mu 2}$
[24]	Cosm. Rays	$\sim 10^{-10}$	—	966.4 ± 0.7	—	—	$75\ \pm0.2$	—	—
[33]	Bevatron	10^{-8}	Bc	965.6 ± 1.3	—	—	$74.9\ \pm0.4$	—	—
[28]	»	$1.4\cdot10^{-8}$	B-Y	—	962.8 ± 1.8	965.8 ± 2.4	—	217.4 ± 1.3	387.8 ± 2.3
[38]	»	—	Ba	965.6 ± 0.7	—	—	$74.9\ \pm0.2$	—	—
[39]	»	$5\ \cdot10^{-10}$	Ba	—	$971\ \pm4$	$958\ \pm3$	—	222.2 ± 2	385.2 ± 2.3
—	—	—	B-Y	—	$969\ \pm4$	$952\ \pm3$	—	221.0 ± 2	380.3 ± 2.3
[40]	Cosm. Rays	$3\ \cdot10^{-10}$	Ba	—	$969\ \pm3$	$977\ \pm6$	—	220.3 ± 1.7	389.8 ± 3
[32]	Bevatron	10^{-8}	B-Y	966.5 ± 0.74	—	—	75.13 ± 0.2	—	—
[35]	»	$1.2\cdot10^{-8}$	Bk	966.3 ± 0.7	—	—	75.08 ± 0.2	—	—
[37]	»	$1.5\cdot10^{-8}$	B-Y	—	$964\ \pm4$	$964\ \pm6$	—	217.6 ± 2	386.6 ± 3

(*) Bc = BACCHELLA *et al.* range energy relation [33]. B-Y = BARKAS and YOUNG range energy relation [34].
Ba = BARONI *et al.* range energy relation [42]. Bk = BARKAS *et al.* range energy relation [41].

power of the emulsion did not allow one to claim a precision of better than 2% in the Q, i.e. 4 m_e in the mass of the $K_{\pi 3}$.

The Copenhagen group [38] applied the curve of BARONI *et al.* without previous calibration of the stopping power of their track: the error reported is only statistical.

HADDOCK [32] used the range energy relation of BARKAS and YOUNG [34], calculated from Vigneron's parameters and extrapolated to high energies using a mean ionization potential of 322 eV. The error given takes into account the uncertainty of the stopping power due to possible differences in the emulsion density. HECKMAN *et al.* [35] use a range energy relation experimentally verified on a number of points. Safest of all is—in our opinion—the result of BACCHELLA *et al.* [33] based on the method discussed above. The error given by them includes *all* possible sources of uncertainty.

In view of these considerations we are inclined to consider some of the errors given in Table II rather below their real value. At the present moment, the average Q cannot be claimed to be better known than to ~ 0.2 MeV. We find accordingly

$$Q_{K_{\pi 3}} = (75.0 \pm 0.2)\ \text{MeV}\ , \qquad M_{K_{\pi 3}} = (966.3 \pm 0.7)\ m_e\ .$$

1·3. – Relative frequencies of the various modes of decay.

The relative frequencies of the various modes of decay have been measured both in experiments on artificial and cosmic ray particles.

We shall discuss here only those results which are based on a large number of events. Those based on small numbers are not significant if taken separately and, on the other hand, cannot be compared with each other since it is difficult to estimate to what extent and in what way they might have been affected by experimental bias.

An idea of the difficulties involved in this type of experiment and of how they can be overcome is probably better given by reviewing briefly the procedures which have been followed. We shall consider only observations on K-mesons decaying at rest in nuclear emulsion, since the majority of the results have been obtained using this technique.

i) *Some remarks on the methods used in the identification of secondaries from* K's *in emulsion.* – With the exception of the $K_{\pi 3}$, all the modes are associated with the emission of only one charged particle. The identification of the secondary is straightforward if the latter is brought to rest in the emulsion. Then by simple inspection of the end of its track its nature can be established and therefore its mass. The range also tells us the energy at the point of

the spontaneous disintegration of its parent K. This information is sufficient to establish the type of decay.

The task is less easy when the secondary escapes from the stack before being brought to rest or for some reasons cannot be followed to the end of its range; its mass is then established by measuring the scattering and the grain density of the track. In Fig. 1 we have reported a typical « grain density versus scattering » plot. The grain density (g) is usually defined as the ratio between the measured value g and the « plateau » value g_0 (*).

The average angle of scattering is practically proportional to the inverse of $p\beta$ (momentum $\times$ velocity) which is the parametre reported as abscissa in the graph. Points corresponding to individual measurements will be placed on curves the position of which depends on the mass and charge of the particle concerned. Three typical curves, corresponding to π and μ mesons, and to electrons are shown. The position and the shape of these curves depend on the sensitivity of the emulsions and on the intensity of the development. Both may vary from stack to stack and in general it is necessary to « calibrate » each stack on particles of known mass.

The curves shown in Fig. 1 do not refer to any particular paper published on this subject but may be considered a fair example, certainly adequate for the use which we are going to make of them in the discussion which follows.

Let us consider a number of points obtained from measurements of K^+ secondaries. Both π's and μ's arising from $K^+_{\pi 2}$ and $K^+_{\mu 2}$ modes have single $p\beta$ values, respectively

$$\pi \text{ from } K_{\pi 2} \qquad p\beta = 170 \text{ MeV/c}$$

$$\mu \text{ from } K_{\mu 2} \qquad p\beta = 215 \text{ MeV/c}$$

apart from the experimental errors which still affect our knowledge of these values. Furthermore let us consider only tracks of secondaries which make an angle with the plane of the emulsion of not more than 15° and are at least 5 cm long. Both the g^* and the $p\beta$ will be known with a certain error. The horizontal and vertical bars which we have associated with each point give an idea of the size of the « standard errors » associated with the results.

We shall consider two tracks as « distinguishable » through g^*-$p\beta$ measurements when their points are expected to be separated by at least three

(*) In emulsion, the observed grain density — for velocities larger than that corresponding to the « minimum of ionization » — increases with the increasing velocity of the particles, but not as predicted by theory. It reaches in fact a « plateau value » g_0. This is *not* due to the Fermi effect of polarization of the medium but is a consequence of the mechanism which leads to the formation of a track by an ionizing particle in emulsion.

« standard errors ». Following DILWORTH *et al.* ([43], p. 301. See also [44]) we shall divide the g^*-$p\beta$ plan into three regions *a*), *b*), *c*). In region *a*) $p\beta \leqslant 140$ MeV/c and all possible secondaries from K decays (i.e. π, μ and e) can be distinguished (*). In region *b*) ($140 < p\beta < 190$ MeV/c) π's can still be distinguished from μ's and electrons but not μ's from electrons. In region *c*) (190 MeV/c $< p\beta$) π's cannot be distinguished from μ's. Electrons can be individuated only if their $p\beta$ is 400 to 600 MeV/c, but this region does not include any decay product from K's.

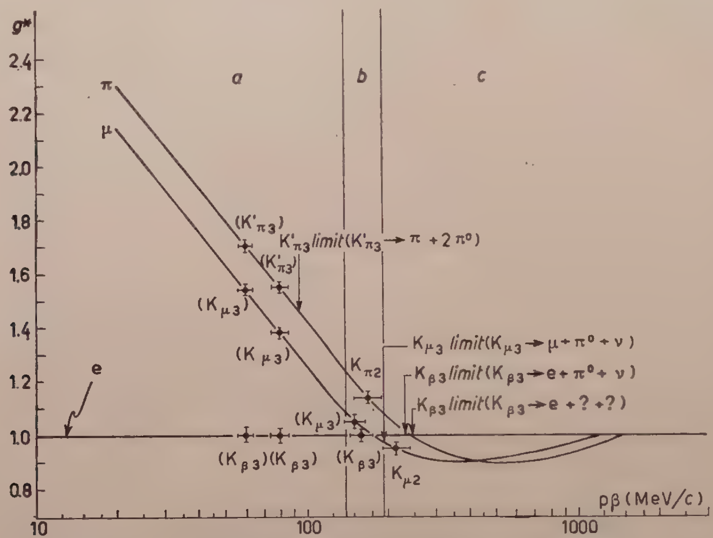

Fig. 1-1·3. – Relation between g^* and $\log p\beta$ in emulsion for electrons, μ and π mesons g^* is the number of grains per unit length of track normalized to the plateau value (see footnote page 31) and $p\beta$ is the momentum times velocity of the detected particles, as deduced from its average scattering angle $\langle\alpha\rangle = C/p\beta$, C being a constant. The « points » marked on the graph indicate where the secondaries from two body K decays ($K_{\mu 2}$, $K_{\pi 2}$) are expected to fall and the arrows mark the upper limits for the secondaries from the 3-body decays. A number of other « points » has been reported to give an idea of the size of the errors involved in (g^*-$p\beta$)-measurements at various energies. (See text).

Thus, if we select events in which the charged secondary produces a track satisfying the geometrical requirement mentioned above, the $K_{\pi 2}$ can always be recognized. Its secondary emitted at an energy of about 108 MeV, i.e. $p\beta = 170$ MeV/c, falls in region *b*) where it cannot be mistaken for

(*) The values given here for the limits of the three regions are different from those given by DILWORTH *et al.*, due to the different selection of events which we consider ($l > 5$ cm, dip angle $< 15°$).

a μ or an electron. The μ from $K_{\mu 2}$ falls into region c). Knowing the maximum $p\beta$ which is permissible to each type of decay, we can rule out the π when observing a $p\beta > 190$ MeV/c, but we cannot disregard the possibility of an electron from a $K_{\beta 3}$ unless we can decide this issue on the basis of a different type of observation. In fact, after 5 cm of emulsion, such an electron would on the average have lost 80% of its energy and it can be recognized on the basis of a ($dp\beta/dR$ — versus — R)-plot (see the example reported in Fig. 2). Although the radiative loss of energy is subject to large fluctuations, the chances that more than 1 or 2% of the electrons may be mistaken for μ's after 5 cm appear to be remote.

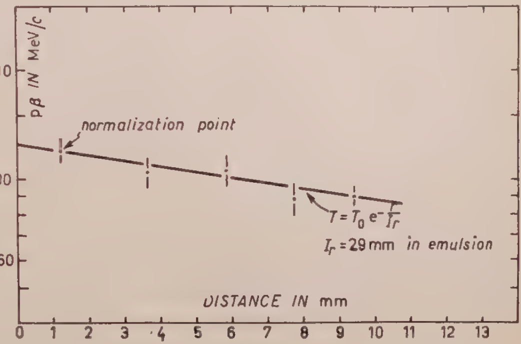

Fig. 2-1·3. – Loss of energy due to bremsstrahlung for electrons in emulsion as observed by HECKMAN [44] on a secondary track emitted in a $K^+_{\beta 3}$ decay.

Of the three body decays the $K'^+_{\pi 3} \to \pi^+ + 2\pi^0$ produces charged π's having a $p\beta \leqslant 94$ MeV/c. All its secondaries then fall into region a) where there is no possibility for ambiguity. For the $K_{\mu 3}$ the upper limit of $p\beta$ permissible to the μ is 193 MeV/c, i.e. the μ would fall invariably either in a) or in b).

In a) it cannot be mistaken for either a π or an electron, but in b) it could be mistaken for an electron. The same considerations discussed above in relation to the discrimination between $K_{\beta 3}$ and $K_{\mu 2}$ apply here, with the additional difference that there is no evidence that the $K_{\mu 3}$-decay spectrum is very rich in the high energy region. On the contrary, from considerations on statistical factors only one can see that the spectrum has most probably a peak in an intermediate region ($p\beta \sim 130$ MeV/c) and is comparatively poor in the high energy tail. So far, this has not been found in contradiction with the experimental data (see Sect. 1·4).

A partial discrimination between the various modes may also be obtained from measurements of grain counting. Only secondaries from $K_{\pi 2}$'s have, at the decay point, a $g^* \sim 1.16$ (see Fig. 1) whilst the μ's from the $K_{\mu 2}$ have $g^* \sim 1$. If g^* can be measured more exactly than $\sim 2 \div 3\%$, the discrimination between the two modes is possible by this method. They will not be separated however, from $K_{\mu 3}$ or $K_{\beta 3}$ associated with fast secondaries unless the other methods mentioned above are invoked. At intermediate velocities ($1.2 < g^* < 1.4$) the only possible assignment is the $K_{\mu 3}$. For lower velocities ($g^* > 1.4$) additional information has to be obtained from scattering measurements. Using

large stacks, this last question is, however, of secondary importance since the residual range of a μ having such a grain density is less than 5 cm.

In conclusions we can say that if the relative abundance of the various modes of decay is measured by selecting events producing secondaries of at least 5 cm of visible track and making an angle of less than 15° with respect to the plan of the emulsion, only the relative proportion of $K_{\beta3}$, is likely to be altered and even that only slightly.

ii) *Experiments on focussed beams of artificially produced* K^+*'s.* – Experiments on K^+ produced by the Berkeley Bevatron have been mentioned in Sect. 1'2 and details on the experimental arrangements were given there. K^+-particles were brought to rest in a definite region of an emulsion stack and their secondaries analysed using the methods mentioned above [28, 45, 47, 37, 27]. The results are reported on Table I. Of those listed, that of BIRGE *et al.* has the largest statistical weight. They used big stacks (90 pellicles, each 600 μ thick and 9×17.5 m² of area) in order that some of the secondaries would be brought to rest in the emulsion, thus ensuring unambiguous identification. To have an unbiassed selection of events, the relative abundance was computed by taking only those whose secondaries were associated with a « potential range » (*) of 21 cm. This is the residual range of a μ from $K_{\mu2}$ decay and is the longest of those of K secondaries other than $K_{\beta3}$. In this way all geometrical bias due to the different ranges of secondaries was avoided.

Additional data were obtained by « blob counting » tracks of particles which escaped from the stack. Fig. 3 shows to what extent a mass separation is possible for fast mesons. The criterion of selection was less restrictive than that considered above (angle of dip $\leqslant 22°$ instead of $\leqslant 15°$); on the other hand

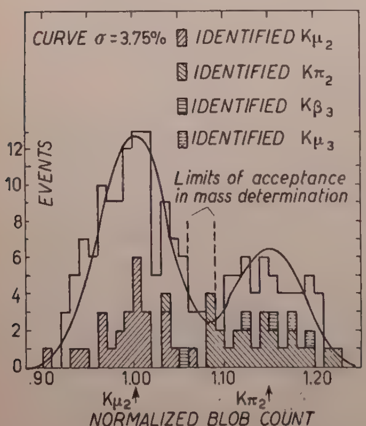

Fig. 3-1'3. – Distribution of the blob densities (here normalized to the density of the $K^+_{\mu2}$ secondary which is below the plateau value - see Fig. 1) as measured on the secondaries from K^+'s. The cross-hatched area indicates events identified by other methods. The two peaks have been interpreted as due to $K_{\pi2}$ and $K_{\mu2}$ decays. (After BIRGE *et al.* [28]).

(*) By potential range is meant the distance which a particle would cover inside the recording medium proceeding on a straight line in the same direction which it had at emission.

the development was found to be very even throughout the whole stack and the fluctuation of g^* was remarkably small.

A number of measurements performed on previously identified events have been reported for comparison and confirm the validity of the method. Interference from fast electrons was eliminated by identifying the latter on the basis of their characteristic increase of multiple scattering. $K_{\mu3}$ remained in the sample. This negligence is justified *a posteriori* by the final result which shows that the spectrum of μ's from $K_{\mu3}$ decays is extremely poor in the region $g^* \leqslant 1.2$.

iii) *Experiments on K^+'s produced inside the emulsions.* – In the experiments described above, the emulsions were exposed at a distance from the target, where K-particles were produced, corresponding to a time of flight of $\sim 10^{-8}$ s. Short lived particles would not have been detected. When they are created inside the emulsion stack, either by cosmic rays or by beams of artificially accelerated primaries, lifetimes of 10^{-11} s are still tolerable.

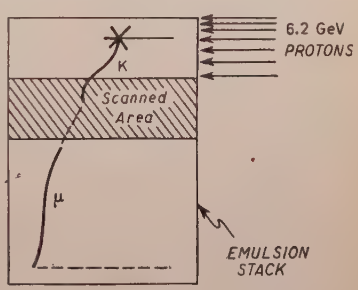

Fig. 4-1·3.

Experiments of this type have been performed by exposing emulsions to high altitude cosmic rays and also directly in the beam of 6.2 GeV protons produced at Berkeley. The results are reported in the first two rows of Table I.

The arrangement used by CRUSSARD *et al.* [39] to this effect is sketched in Fig. 4. A stack of 600 μm thick emulsions was placed on the edge of the stack 6.2 GeV protons produced by the Bevatron. A mechanical system allowed the stack to be « shot » into the beam in time to cross it just before the end of a pulse thus insuring that the energy of the protons was 6.2 GeV and not lower. The parallel lines on the upper part of the drawing indicate the direction of the beam of primary protons.

Scanning was carried out only in the shaded region. According to the Authors most of the K-particles observed there came from the part of the highest primary density i.e. covered a distance of 5 to 20 cm before stopping. This corresponds to a time of flight of 3 to $9 \cdot 10^{-10}$ s. The remaining part of the emulsions was used to follow the secondaries.

The relative abundances obtained from this type of experiments agree well with those from focussed beams of K mentioned in ii) (see Table I).

TABLE I-1·3. – *Data on the relative abundances of the* K^+*-decay m*

Ref.	Source of rad.	Primary beam energy (GeV)	Target	Secondary K energy (MeV)	Angle of production	Time fligh (10^{-8}
[39]	Bevatron	6.2 (p)	emulsion	100—240	90° favoured	3—9·1
[40]	Cosm. rad.	—	»	—	0—180°	5·1
					Average values	
[28]	Bevatron	6.2 (p)	Cu	114	90°	1.35
[46]	»	2.9 (p)	Cu	63	60°	2.16
[47]	»	6.2 (p)	Cu	114	90°	1.35
[37]	»	6.2 (p)	Ta	100	90°	1.5
[27]	»	4.8 (p)	Ta	126	90°	1.2
					Average values	

1·4. – Energy spectra of secondaries emitted in K^+_{L3}-decays.

Of the energy spectra of charged secondaries emitted in 3-body K^+-decays, only that of the $K_{\pi3}$-meson has been studied in detail. Of the others (the $K'_{\pi3}$, $K_{\mu3}$ and $K_{\beta3}$) little is known at present. This is partly due to the relative scarcity of the latter modes which account for less than 10% of all the K^+ decays, but chiefly to the experimental difficulties and biasses discussed in the previous section. These have much more serious consequences when not only the identification of the secondaries is required but also their frequency of occurrence in a specific energy interval.

For this reason a theoretical interpretation of the decay spectrum of $K_{\mu3}$ and $K_{\beta3}$ is not worth doing as yet and also the conclusions drawn from the measured $K_{\pi3}$-decay spectra need be qualified. In the following we shall comment separately on the value of the experimental results regarding the different modes.

The $K^+_{\pi3}$ *decay spectrum.* – Several hundred examples of $K^+_{\pi3}$-decays have been observed in nuclear emulsion during the last few years. The relevant results of the analysis and the theoretical implications will be discussed in Chapter **13**.

It will be seen that the analysis is based on the energy distribution of the « unlike » meson and on the θ-distribution, θ being the angle between the momentum $\boldsymbol{p}_3$ of the « unlike » meson and the vector $\boldsymbol{p}'=\boldsymbol{p}_1-\boldsymbol{p}_2$, $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$

ion data (compiled by BIRGE *et al.* [28]).

	Relative abundances					Remarks
3	$K^+_{\pi'3}$	$K^+_{\pi2}$	$K^+_{\mu3}$	$K^+_{\mu2}$	$K^+_{\beta3}$	
)	0.8 $\pm$0.4	25 $\pm$ 5	3.8 $\pm$2.3	57 $\pm$ 6	7.9 $\pm$3.5	Experiments on K⁺'s produced inside the emulsion stack [see (iii), p. 35]. The data in these rows have been corrected for scanning efficiencies [28] assuming 1 .0 % efficiency for $K_{\pi3}$ and a $K_{\pi3}$ abundance of 5.6 %.
)	1.1 $\pm$0.6	19 $\pm$ 5	2.5 $\pm$1.2	63 $\pm$ 8	9.1 $\pm$4.1	
)	0.9 $\pm$0.3	22.3$\pm$ 3.5	3.5 $\pm$0.5	59 $\pm$ 5	8.5 $\pm$2.7	
0.41	2.15$\pm$0.47	27.7$\pm$ 2.7	2.83$\pm$0.95	58.5$\pm$ 3.0	3.23$\pm$1.3	Experiments on focussed beams of artificially produced K⁺ [see (ii), p. 34]. Data marked by the asterisk are based on a fraction of the complete spectrum.
1.6	3.5 $\pm$1.4	21 $\pm$ 6	6 $\pm$? (*)	59 $\pm$ 8	5 $\pm$3.5(*)	
0.6	3.9 $\pm$1 6	27.5$\pm$ 4.2	7.2 $\pm$2.1	51 $\pm$ 5.7	3.9 $\pm$1.6	
)	1.6 $\pm$0.8	20 $\pm$10	1.3 $\pm$0.7	60 $\pm$10	2 $\pm$2	
1.4	1.9 $\pm$0.8	1.4$\pm$ 5.6	1.1 $\pm$0.7 (*)	65.2$\pm$10.3	2.6 $^{+2.2}_{-1.7}$ (*)	
0.3	2.2 $\pm$0.3	227 $\pm$ 2	1.9 $\pm$0.4	59 $\pm$ 2	3.3 $\pm$1	

being the momenta of the remaining decay products — all being measured in the center of mass system of the $K_{\pi3}$. If we restrict ourselves to consider K-mesons decaying at rest, we can safely assume that they are all positively charged, i.e. the unlike meson is the negative one.

A very convenient method for plotting the experimental results was suggested by DALITZ (Sect. **13·5**). In an equilateral triangle the sum of the perpendicular to the sides drawn from a point P, inside the triangle, is independent of P and equal to the height of the triangle. If the latter is taken equal to the total release of energy $Q_{K_{\pi3}}$ (= 75 MeV), then the perpendiculars from P—measured in the same units—uniquely define a set of values E_1, E_2 and E_3 which satisfy the equation $Q = E_1 + E_2 + E_3$ and can be interpreted as the kinetic energies of the secondary π's. Momentum conservation in a non-relativistic approximation restricts the region allowed for P to the inscribed circle (see Fig. 1). Obviously half of the triangle is sufficient to describe any $K^+_{\pi3}$

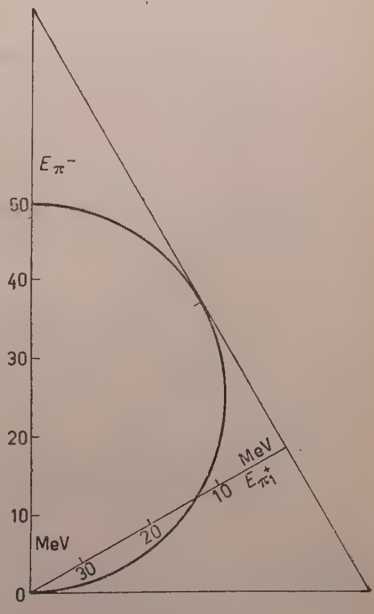

Fig. 1-1·4.

decay, as the exchange of one π^+ meson with the other π^+ would merely change a configuration into another which is indistinguishable from it.

Since the magnetic field is of no use in studying events taking place in emulsion, the methods of analysis mentioned above can be applied to the experimental observations only selecting those cases in which

a) the π^- meson, or

b) two of the three secondary π's

end in the emulsion.

The first selection method *a*) is likely to introduce serious distortions in the shape of the E_{π^-}-spectrum. This has been pointed out by BHOWMIK *et al.* [48] who have calculated the relative probability observing a π^- (from $K_{\pi 3}$'s) of a given energy stopping in a stack of 30 emulsions, each 600 μm thick.

The method *b*) leads to more precise results and BRENE *et al.* [38] have shown that in most cases the biasses introduced with it are not serious. The trend of the possible alterations, introduced in the experimental distributions, is indicated in Fig. 2 and 4. Assuming the decay planes to be distributed at random, these Authors have calculated for a number of points, on a Dalitz diagram, the probability that the corresponding events have at least two secondaries stopping in a stack of 20 (600 μm thick)-emulsions. Lines connecting points of equal probability are shown in Fig. 2 separately for events in a pellicle in the middle of the stack (4*a*) and in one close to the limiting surface (4*b*).

Which fraction of the events observed in a stack of *n* 600 μm emulsions can be selected to form an unbiassed sample is shown in Fig. 3. Curves *a*, *b*, *c* refer respectively to events located in the central emulsion of the stack, in an emulsion close to the surface and (case *c*) to a popul-

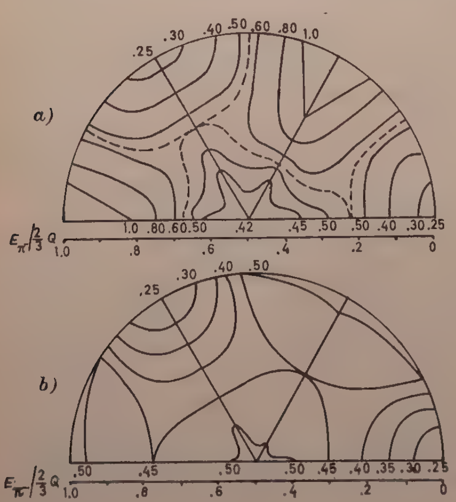

Fig. 2-1·4. – « Iso-bias » lines in Dalitz' diagram. The lines connect points associated with the same probability *P* that a $K_{\pi 3}$ decay has at least two secondaries brought to rest inside the emulsion stack. The value of *P* associated with each line is marked on the contour of the diagrams. Diagram *a*) refers to events observed in the middle of a stack of 20 pellicles-(600 μm thick); *b*) to events observed very close to the surface of the same stack. (After BRENE *et al.* [38]).

TABLE I-1·4. – *Data on the stacks used in investigation on $K^+_{\pi 3}$-decays.*

Reference	No. of emuls. per stacks	Thickness of the emuls. (μm)	Total No. of events
[49]	{120, 300}	600	400
[48]	30	600	76
[50]	120	600	87
[38]	{43, 38}	600	{57, 18}
[51]	100	400	—
[52]	{40, 108}	600	16

lation uniformly distributed through the whole stack. The number of « unbiassed » events is small for $n \leqslant 20$ but it grows quickly for higher n's and in practical cases ($n \geqslant 30$; see Table I) represents more than 50% of the total number.

Brene's results enable ut so have an idea of the weight which we can give to the observations which have been reported. Most of the published data have been found in stacks of more than 40 emulsions, (see Table I), and therefore energy spectra are not seriously distorted. The angular distributions are not appreciably altered.

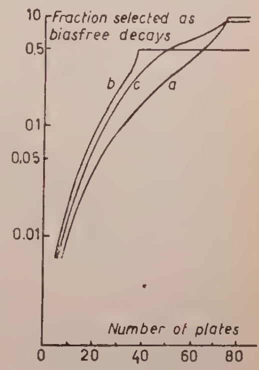

Fig. 3-1·4.

The $K'^+_{\pi 3}$ *decay spectrum.* – The secondary π^+'s can only have an energy lower than 53 MeV. If a selection is made according to the criterion discussed

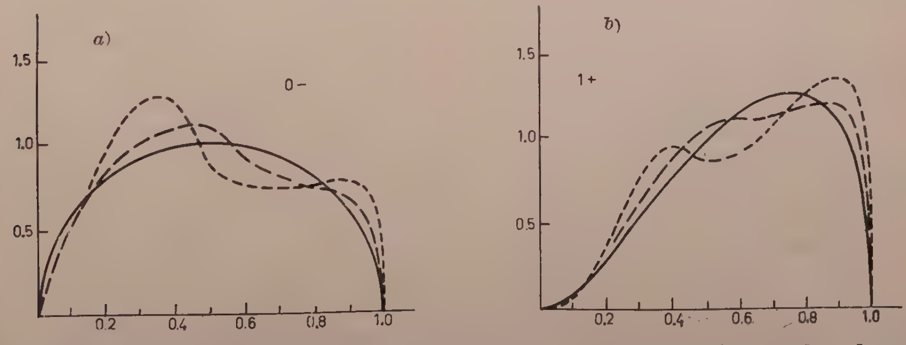

Fig. 4-1·4. – Alterations on the π^- energy spectrum (from $K_{\pi 3}$ decays) based on indiscriminate selection of events in a 20-(600 μm thick)-pellicle stack (according to BRENE *et al.* [38]) ——— expected distribution for *a*) 0^- spin parity assignment, *b*) 1^+ spin parity assignment; ------ as it would appear selecting events in the middle of the stack or — — — at the edge of the stack.

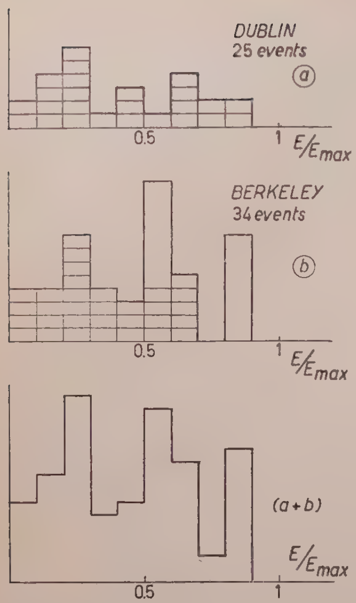

Fig. 5-1·4. – π^+ energy distribution for $K'^+_{\pi 3}$ decays.

previously, they would fall invariably within the region a (Fig. 1-1·2) and once found they would be distinguished from other modes.

In Fig. 5 we have reported the experimental data available at the moment. Dublin results (a) have been selected from secondaries above minimum at an angle of dip $\leqslant 12°$ [54]. Berkeley events (b) were partly identified by systematic following, partly by blob counting on a selected portion of K-meson secondaries [53]. It was reckoned that a bias against fast $K_{\pi 3}$ secondaries existed in their selection and a correction was introduced accordingly giving different weights to their different observations, as shown in Fig. 5.

Both spectra, as well as their sum, are not inconsistent with a distribution symmetrical with respect to the median value.

The $K^+_{\mu 3}$ *decay spectrum.* – The μ energy can extend up to 134 MeV. The presence of some events having very nearly this energy excludes other decay schemes involving a larger number of heavy neutrals.

The graphs reported have been compiled at the Rochester Conference 1956, the first, and by the Dublin group [54], the second. The difference between them is not significant as yet, although the second seems richer of high energy events. A possible uncontrolled contribution from $K_{\pi 2}$ secondaries—which have a g^* just corresponding to that of a 90 MeV μ-meson—was considered by O'CEAL-

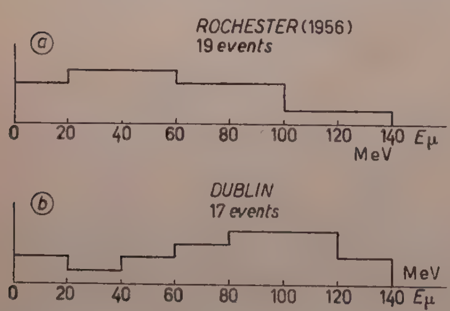

Fig. 6-1·4. – μ energy distribution for $K^+_{\mu 3}$ decays.

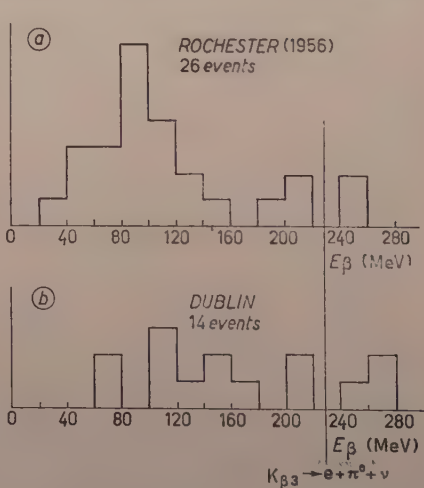

Fig. 7-1·4. – Electron energy distribution for $K^+_{\beta 3}$ decays.

LAIGH and found to be negligible. The region below 80 MeV should be bias free.

The two graphs are not to be summed because some events are contained in both.

The $K^+_{\beta 3}$*-decay spectrum.* – The nature of the neutrals, has not been established. From the existence of the neutral mode $K^0_{\beta 3} \rightarrow e^{\pm} + \pi^{\mp} + \nu$, one may infer that the right scheme is the following one:

$$K^+_{\beta 3} \rightarrow e^+ + \pi^0 + \nu \, .$$

If so, the electron maximum energy is 228 MeV.

The shape of the experimental decay spectrum (Fig. 7) is most probably a considerably distorted version of the true one. We have seen that the determination of the energy—at emission—of the observed secondaries is in general based on several cm of range. Electrons loose a considerable amount of energy in 1 cm of emulsion, due to bremsstrahlung and do so in a way subject to large statistical fluctuations. The energy at emission is then determined by plotting the loss of energy as a function of the distance from the parent K-decay and extrapolating to zero a theoretical curve fitted on the experimental points. The errors involved are in general very large.

It is noteworthy that among the data available at present, 5 events seem to be above the upper limit allowed to the $(e^+ + \pi^0 + \nu)$-mode, but permissible to a $K^+_{\beta 2} \rightarrow e^+ + \nu$ mode. Should this mode be proved to exist (and this is not yet the case) it may well be responsible for an appreciable fraction of the events so far attributed to the high energy tail of the $K_{\beta 3}$ spectrum.

1.5. – The mean life of the K^+'s.

The order of magnitude of the mean life of K^+-particles has been known for several years, though only recently precise determinations have been made, most of them on K^+-beams produced artificially. A variety of methods have been used many of them of great interest also from the technical point of view. They can be divided into three groups.

i) *Emulsion experiments.* – The attenuation of the $K^+_{\pi 3}$ flux in a beam of magnetically analysed secondary particles, produced by the Bevatron has been determined by ALVAREZ and GOLDHABER [55] by measuring the $K^+_{\pi 3}$/proton ratio at two different distances from the target. The selected momentum channel corresponded to particles of 350 MeV/c, which were detected by placing stacks of nuclear emulsions on their path. The time of flight plus moderation time

for particles of mass 966 m_e corresponding to the two distances were $1.3\cdot10^{-8}$ and $1.8\cdot10^{-8}$ s respectively.

For $K^+_{\pi3}$-meson their measurements yielded the value

$$\tau(K^+_{\pi3}) = (1.0^{+0.7}_{-0.3})\cdot10^{-8}\ \text{s}\ .$$

The samə method was used also by OREAR *et al.* [56] who found

$$\tau(K^+_{\pi3}) = (1.04^{+0.43}_{-0.23})\cdot10^{-8}\ \text{s}\ ,$$

$$\tau(K^+_{L}) = (1.11^{+0.18}_{-0.11})\cdot10^{-8}\ \text{s}\ .$$

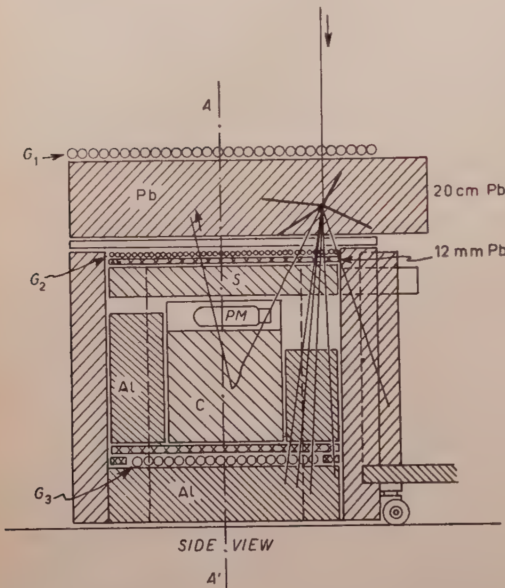

Fig. 1-1·5. – The apparatus used by MEZZETTI and KEUFFEL [58] for determining the lifetime of cosmic ray K particles. A typical event registered by the apparatus has been sketched on the original drawing.

ILOFF *et al.* [57] searched emulsions exposed to the K^+ beam produced by the Bevatron for decays in flight of K's by scanning « along the track ». In addition to a number of interactions, they observed 19 events which could be only interpreted as decays in flight of K^+ particles into a lighter particle. The number of the decays did not allow a separation of the various K-modes. Altogether, the mean lifetime was found to be

$$\tau(K^+) = (1.01^{+0.33}_{-0.21})\cdot10^{-8}\ \text{s}\ .$$

In this, as in the majority of the experiments performed, the particles were allowed to enter the stack of emulsion only after having travelled several metres from the point of origin. In all these cases the time of flight was $\sim 10^{-8}$ s, so that a particle having a lifetime of 10^{-9} s or less would have not been observed.

ii) *Experiments using counters.* – MEZZETTI and KEUFFEL [58] measured the lifetimes of cosmic ray K's using both scintillation and Čerenkov counters. A sketch of their experimental arrangement is shown in Fig. 1. *S* is a scintillation counter and *C* two Čerenkov counters placed, one behind the other (only one visible in the sketch). Photomultipliers collecting light from the Čerenkov's were placed in such a way as to record only particles travelling upwards. The arrangement was prepared to detect particles crossing the scintillator, stopping in the Čerenkov and emitting a fast secondary in the upper direction within a certain solid angle.

π-μ decays could not interfere with the measurements since the event could be recorded only when the secondary had an energy exceeding $T = 0.52\ mc^2$; nor for the same reason could $K'_{\pi 3}$. The majority of the events which could trigger the apparatus was thus $K^+_{\mu 2}$, $K^+_{\pi 2}$ and $K^+_{\beta 3}$. The results obtained by the Authors are shown in Fig. 2. The prominent peak centered near zero of the time scale was due to spurious counts. From about $1.5 \cdot 10^{-8}$ s the experimental curve departs from the nearly gaussian distribution of the spurious counts (dotted curve) and continues on an exponential which was interpreted as due to the decay of K^+-particles. The corresponding mean life was found to be

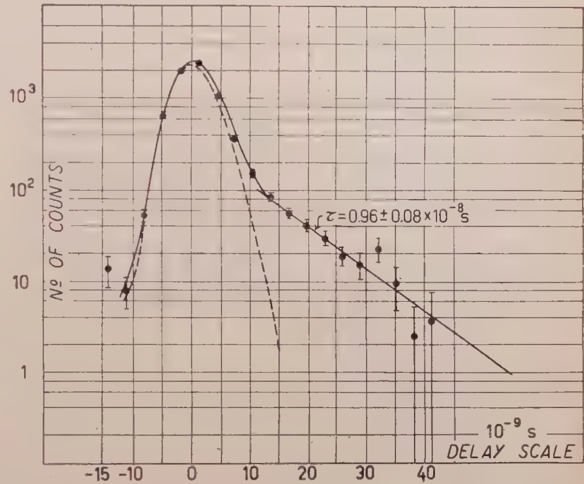

Fig. 2 1.5. – Time distribution of delayed counts obtained by MEZZETTI and KEUFFEL [58]

$$\tau(K^+) = (0.96 \pm 0.08) \cdot 10^{-8}\ \text{s}$$

in good agreement with the other results.

An experiment in principle similar to this just described, was performed by ROBINSON [59] who also analyzed decays of cosmic ray particles.

He obtained

$$\tau(K^+) = (0.805 \pm 0.066 \cdot) 10^{-8}\ \text{s}\ .$$

An arrangement of scintillator and Čerenkov counters was employed by BARKER *et al.* [60] in conjunction with a cloud chamber (see Fig. 3), thus combining the high temporal resolution of the former with the precise spatial definition of the latter.

A Čerenkov counter C_2 was placed inside a cloud chamber containing six lead plates. A layer of 1.5 cm lead was also placed between the S_2 and C counters. The Čerenkov counters were surrounded by reflecting surfaces to enable the photomultiplier to respond to the total amount of light emitted by the particle, irrespective of its direction. The selection of cosmic ray K particles was obtained by taking $S_1 + S_2 + G - C_1$ coincidences. Such a combination indicated that a particle having a velocity below the critical value V_c associated with the Čerenkov counter C_1 was able to traverse 1.5 cm of lead. The value

of V_e and of the lead thickness were so chosen that only particles having a mass larger than 300 m_e would reach the cloud chamber, and K-particles would probably stop in one of the plates inside it.

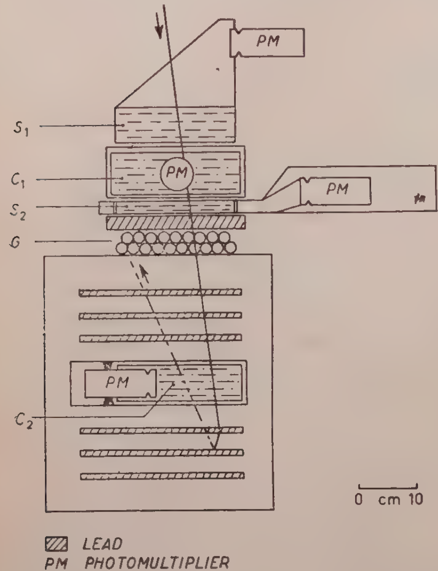

Fig. 3-1·5. – The apparatus of BARKER *et al.* [60] for measuring the mean life of cosmic ray K^+.

If a fast decay product was emitted by the triggering particle, in such a direction to cross the Čerenkov C_2, the delay between the S_1+S_2 and C_2 signals could be observed on a cathode ray oscillograph and measured. A cloud chamber picture was taken at the same time for each coincidence recorded and this enabled the Authors to distinguish genuine K-decays from other spurious events. Thirteen events were observed which gave a value of the mean lifetime of K's, irrespective of their sign, of

$$\tau(K) = (1.10^{+0.41}_{-0.24})\cdot 10^{-8}\ \text{s}\ .$$

Most probably all K observed were K^+ since slow K^- would have interacted rather than decay.

FITCH and MOTLEY [61, 62] measured the lifetimes of $K_{\mu 2}$ and $K_{\pi 2}$ mesons produced by the Brookhaven Cosmotron. They selected magnetically K^+ particles having a momentum of 465 MeV/c and focussed them at a distance of 15 ft from the target. This corresponded to a time of flight $\sim 2\cdot 10^{-8}$ s for particles having a mass of 966 m_e.

The selected particles were allowed to cross a system of Čerenkov and scintillation counters schematically shown in Fig. 4. C_1 is a counter only sen-

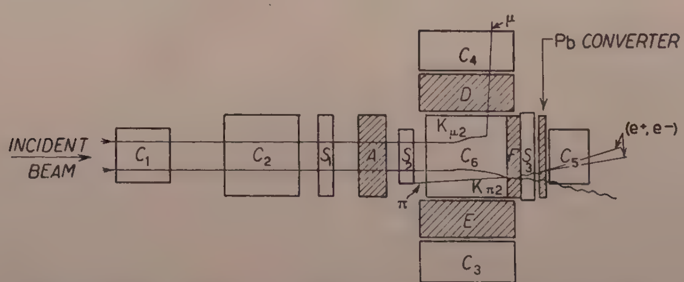

Fig. 4-1·5. – The « counter telescope » used by FITCH and MOTLEY [61] for measuring the mean life of $K^+_{\mu 2}$ and $K^+_{\pi 2}$.

sitive to particles having a velocity between 0.62 and 0.78c. To eliminate π-mesons which had been recorded by C_1, a second Čerenkov counter C_2 was used in anticoincidence with C_1 with a threshold above the value of the K particle velocity.

K particles of 465 MeV/c would trigger a $C_1 - C_2 + S_1 + S_2 - S_3$ coincidence. Those which stopped in C_6 and decayed emitting a particle fast enough to trigger C_6 (i.e. chiefly $K_{\pi 2}$, $K_{\mu 2}$) could be recorded and the delay between C_1 and C_6 measured. An absorber A was introduced to stop the selected K^+ inside C_6.

The identification of $K_{\mu 2}$ events was based on the response of either of the two additional Čerenkov (C_3 or C_4) to the fast μ's associated with this mode. The absorbers D and E where so chosen that no secondary emitted from $K_{\pi 2}$ could traverse them. Most of the other modes were also eliminated; moreover their relative frequency is known to be low and certainly could not interfere seriously with the measurements.

$K_{\pi 2}$ were selected by stopping them in the absorber F and by requiring a coincidence between C_6 and C_5 not accompanied by a signal from S_3. The total sequence was then $C_1 - C_2 + S_1 + S_2 + C_6 - S_3 + C_5$. In this way one could recognize π^0's coming from the $K_{\pi 2}$'s decaying inside F by the materialization of γ's which traversed S_3 and produced electrons inside the Pb-converter.

Ionizing particles arising from the decay of K's in F could not reach C_5 without triggering S_3 also and would not be counted. Here again $K_{\mu 3}$ and possibly $K_{\beta 3}$ could simulate a $K_{\pi 2}$ if their decay scheme involves the emission of π^0's or γ's. As stated above, however, their relative frequencies are low.

The distribution of the delays obtained with this apparatus is shown in Fig. 5. The slopes of the exponentials correspond to

$$\tau(K_{\mu 2}) = (1.17^{+0.08}_{-0.07}) \cdot 10^{-8}\ \text{s}\ ,$$

$$\tau(K_{\pi 2}) = (1.21^{+0.11}_{-0.10}) \cdot 10^{-8}\ \text{s}\ .$$

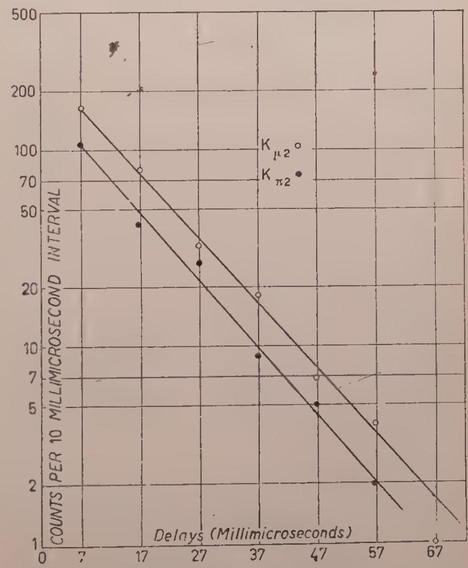

Fig. 5-1·5. – Distribution of the delays in $K^+_{\pi 2}$ and $K^+_{\mu 2}$ decays. (After Fitch and Motley [61]).

A slightly different apparatus was used by the same Authors to measure the lifetime of the $K_{\pi 3}$ meson. The $K_{\pi 3}$'s were brought to rest in a scintillator and identified on the basis of the large amount of energy lost by their secon-

daries in it. By selecting delayed pulses of 35 MeV, or more in energy, they felt confident that negligible contributions could come from other modes. They found

$$\tau(K_{\pi3}) = (1.17^{+0.08}_{-0.07})\cdot 10^{-8}\ s\ .$$

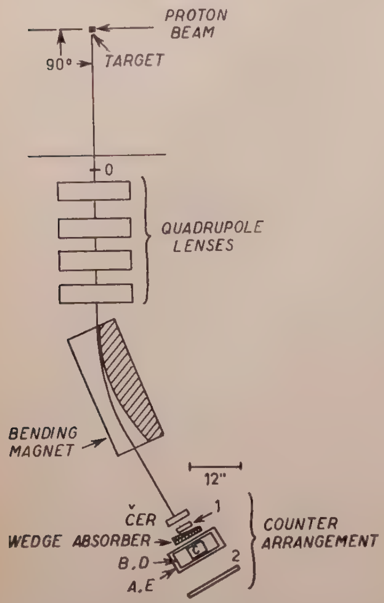

Fig. 6-1·5. – Experimental arrangement used by ALVAREZ *et al.* to measure the mean life of K^+ particles [63].

A subsequent experiment made by ALVAREZ *et al.* (see Fig. 6) at Berkeley [63] was based on very similar principles. K^+ were brought to rest in a scintillator C surrounded by a system of other counters schematically shown in Fig. 7. μ's from $K_{\mu2}$ decays were the only secondaries capable of triggering simultaneously the scintillators D and E, owing to the 2 cm Cu absorber which was sufficient to slow down secondaries from $K^+_{\pi2}$ or $K^+_{\mu3}$, apart from the other modes which were ignored, being rare. $K^+_{\pi2}$ were detected by observing the simultaneous signals from D and A unaccompanied by any other from B and E. Such a combination was what one had to expect if the charged π crossed D—subsequently being brought to rest in the Cu-absorber—and at least one of the γ's, originating from the decay of the π^0, materialized in the Pb below. Simultaneous signals from B and D, non-associated with any other from E or A, were interpreted as due to $K_{\pi3}$ mesons. The delay interposed between the « trigger » pulse, due to the K^+ stopping in C, and any of the additional coincidences due to the lateral counters, was measured by recording the response of individual counters on an oscilloscope. The values thus obtained are

$$\tau(K_{\mu2}) = (1.4 \pm 0.2)\cdot 10^{-8}\ s\ ,$$

$$\tau(K_{\pi2}) = (1.3 \pm 0.2)\cdot 10^{-8}\ s\ .$$

Too few $K_{\pi3}$'s were recorded to obtain a precise determination of their mean life.

iii) *Experiments using cloud chambers.* – Cloud chamber experiments cannot in general yield results as precise as those obtainable from the other techniques des-

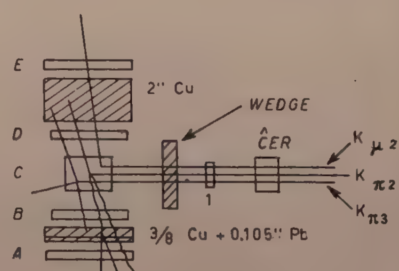

Fig. 7-1·5. – Details of the counter arrangement used by ALVAREZ *et al.* [63].

TABLE I-1·5. – *Mean lives of K⁺-particles.*

Ref.	Source of rad.	Secondary momentum channel (MeV/c)	Time of (*) flight (10^{-8} s)	Detector	Measured mean life (10^{-8} s)				
					$K^+_{\pi 3}$	$K^+_{\pi 2}$	$K^+_{\mu 2}$	K^+_L (+)	All K^+
[63]	Bevatron	370 ± 30	(2.02)	counters	—	1.3 ± 0.2	1.4 ± 0.2	—	—
[55]	»	350 ± 15	$1.3 \div 1.8$	emulsion	$1^{+0.7}_{-.3}$	—	—	—	—
[64]	Cosm. Rad.	—	($\sim 10^{-2}$)	cl. chamber	—	—	—	—	$0.052^{+0.33}_{-2.5}$ (×)
[60]	»	—	($\sim 10^{-2}$)	counters	—	—	—	—	$1.08^{+0.36}_{-0.22}$
[61]	Cosmotron	465	1.6	»	—	$1.21^{+0.11}_{-0.10}$	$1.17^{+0.08}_{-0.07}$	—	—
[62]	»	480	(~ 1.6)	»	$1.17^{+0.08}_{-0.07}$	—	—	—	—
[56]	»	356	1.3	emulsion	$1.04^{+0.42}_{-0.23}$	—	—	$1.11^{+0.18}_{-0.11}$	—
[57]	Bevatron	$390 \div 450$ $335 \div 360$	1.06	»	—	—	—	—	$1.01^{+0.33}_{-0.21}$
[58]	Cosm. Rad.	—	—	counters	—	—	—	—	0.96 ± 0.08
[59]	»	—	—	»	—	—	—	—	0.8 ± 0.07
				Average values	$1.16^{+0.07}_{-0.06}$	$1.22^{+0.10}_{-0.08}$	$1.21^{+0.7}_{-0.6}$	$1.11^{+0.18}_{-0.11}$	$0.91^{+0.07}_{-0.05}$

(*) Values reported in brackets were not explicitly stated in the original papers and have been estimated from other data given by the authors.
(+) Not inclusive of $K^+_{\pi 3}$.
(×) Not included in the average.

cribed above, mainly because of the large errors involved in momentum measurements which make a precise determination of the velocity of the observed particles, difficult.

We shall describe briefly one of the most recent experiments performed with this technique on cosmic ray K-particles, by ARNOLD *et al.* [64]. It involved two large cloud chambers placed one above the other; the upper one, 16 in. × 16 in. × 16 in. in size was placed inside a magnetic field of about 5 500 G; the lower one, 20 in. × 20 in. × 7 in., contained 7 lead plates 0.5 in. thick which were replaced by 7 copper plates of the same thickness during the course of the experiment. The expansion of the chambers was controlled by a counter system which could be triggered only by a penetrating shower. Since no electronic device was used to measure any time delay between the recorded particles, lifetime determinations could only be made on those which were observed to decay in flight.

Out of 46 charged unstable particles of both signs which these Authors observed, only 9 of the positive ones could be individuated as K-particles decaying into two secondaries. The distribution of these events in the chamber, analyzed by means of Bartlett's (see Sect. **9**˙3) method gave a value of the mean life of $(5.3^{+3.3}_{-2.5}) \cdot 10^{-10}$ s, well below that found using different techniques. So far, this result has not been confirmed by other findings.

iv) *Remarks on the results.* – All the results obtained in the investigations described above, are summarized in Table I. Average values, calculated giving to each individual result a weight equal to the inverse square of its error, have been given in the last row. It appears clearly that those available are all consistent with a unique mean life: the average of all values is $\tau_{K^+} = (1.12 \pm 0.03) \cdot 10^{-8}$ s.

Should small differences exist, it would be very difficult to detect them

TABLE II-1˙5. – *Relative abundancies and mean lives of the* K^+'s (*after* HOANG *et al.* [65]).

Decay mode	Abundancies: Stack *a* 3.52 m from the target	Abundancies: Stack *b* 4.27 m from the target	$\tau(K^+) - \tau(K^+_{\mu 2})$ (*) (10^{-8} s)
$K^+_{\pi 3} + K'^+_{\pi 3}$	7.9 ± 1.7	8.1 ± 1.8	0 ± 0.17
$K^+_{\pi 2}$	23.5 ± 5.8	20.6 ± 5.5	-0.37 ± 0.08
$K^+_{\mu 2}$	60.1 ± 12	61.5 ± 14.8	—
$K^+_{\mu 3}$	4.2 ± 1.3	4.0 ± 1.8	-0.21 ± 0.13
$K^+_{\beta 3}$	4.7 ± 3.5	5.5 ± 3.2	$+0.75 \pm 0.56$

(*) Calculated assuming $\tau_{K^+_{\mu 2}} = 1.2 \cdot 10^{-8}$ s. The errors are only statistical.

through direct measurements as those described in this section unless the experimental errors were reduced to a small fraction of their present value.

An alternative method to be used on artificial particles is based on the comparison of the relative frequencies of the various modes of decay measured at different distances from the target. Small differences in the mean lives could then be appreciated.

Of the data reported in Table I-**1**.3 those of the G-stack [40] and of the Ecole Polytechnique group referred to particles which needed to survive only for a time of $\sim 3\cdot 10^{-10}$ s to be selected. All the others refer to particles which had to travel for a considerable distance from the target to the detector ($\gtrsim 10^{-8}$ s). Comparison of the two sets of data does not indicate a substantial difference in the mean lives of the various modes. The same was found by the Rochester group [45] who analyzed K decays in two stacks of emulsions exposed at 3.52 and 4.27 m from the target respectively. Mean life differences as obtained by these authors are reported in Table II.

REFERENCES (Ch. 1)

[1] R. M. Brown, U. Camerini, P. H. Fowler, H. Muirhead, C. F. Powell and D. M. Ritson: *Nature*, **163**, 47 (1949).

[2] R. W. Berriman: *Nature*, **162**, 992 (1948).

[3] J. B. Harding: *Phil. Mag.*, **41**, 405 (1950).

[4] P. H. Fowler, M. G. K. Menon, C. F. Powell and O. Rochat: *Phil. Mag.*, **42**, 1040 (1951).

[5] G. Baroni, C. Castagnoli, G. Cortini, C. Franzinetti and A. Manfredini: *Proc. Roy. Soc.*, A **221**, Discussion Meeting (1954).

[6] G. Belliboni, B. Sechi and B. Vitale: *Suppl. Nuovo Cimento*, **12**, 195 (1954).

[7] C. O'Ceallaigh: *Phil. Mag.*, **42**, 1032 (1951).

[8] M. G. K. Menon and C. O'Ceallaigh: *Proc. Roc. Soc.*, A **221**, 294 (1954).

[9] G. Yekutieli, M. F. Kaplon and T. F. Hoang: *Phys. Rev.*, **101**, 506 (1956).

[10] T. F. Hoang, M. F. Kaplon and G. Yekutieli: *Phis. Rev.*, **101**, 1834 (1956).

[11] H. S. Bridge, H. De Staebler, B. Rossi and B. V. Sreekantan: *Nuovo Cimento*, **1**, 874 (1955).

[12] H. Courant: *Phys. Rev.*, **99**, 282 (1955).

[13] R. Cester, T. F. Hoang, M. F. Kaplon and G. Yekutieli: to be publ. on the *Phys. Rev.*

[14] D. Keefe: *Nuovo Cimento*, **4**, 412 (1956).

[15] J. Orear, G. Harris and S. Taylor: *Bull. Am. Phys. Soc.*, **2**, 6 (1957).

[16] R. H. Dalitz: *Proc. Roy. Soc.*, **66**, 710 (1953).

[17] J. Crussard, M. F. Kaplon, J. Klarman and J. H. Noon: *Phys. Rev.*, **93**, 253 (1954).

[18] B. Gregory, A. Lagarrigue, L. Leprince-Ringuet, F. Muller and Ch. Peyrou: *Nuovo Cimento*, **2**, 292 (1956).

[19] R. ARMENTEROS, B. GREGORY, A. HENDEL, A. LAGARRIGUE, L. LEPRINCE-RINGUET, F. MULLER and CH. PEYROU: *Nuovo Cimento*, **1**, 915 (1955).
[20] M. W. FRIEDLANDER, D. KEEFE, M. G. K. MENON and L. VAN ROSSUM: *Phil. Mag.*, **45**, 1043 (1954).
[21] C. DAHANAYAKE, P. E. FRANCOIS, Y. FUJIMOTO, P. IREDALE, C. J. WADDINGTON and M. YASIN: *Phil. Mag.*, **45**, 1219 (1954).
[22] R. W. H. JOHNSTON and C. O'CEALLAIGH: *Phil. Mag.*, **46**, 393 (1955).
[23] K. M. CROWE: *Nuovo Cimento*, **5**, 541 (1957).
[24] E. AMALDI: *Suppl. Nuovo Cimento*, **4**, 179 (1956).
[25] C. C. DILWORTH, A. MANFREDINI, G. D. ROCHESTER, J. WADDINGTON and and G. T. ZORN: *Suppl. Nuovo Cimento*, **12**, 433 (1954).
[26] L. T. KERTH, D. H. STORK, R. W. BIRGE, R. P. HADDOCK and M. N. WHITEHEAD: *Bull. Am. Phys. Soc.*, **30**, no. 3.
[27] F. M. SMITH, H. H. HECKMAN and W. H. BARKAS: UCRL 3289.
[28] R. W. BIRGE, D. H. PERKINS, J. R. PETERSON, D. H. STORK and M. N. WHITEHEAD: *Nuovo Cimento*, **4**, 834 (1956); R. W. BIRGE, R. P. HADDOCK, L. T. KERTH, J. R. PETERSON, J. SANDWEISS, D. H. STORK and M. N. WHITEHEAD: *Suppl. Nuovo Cimento*, **4**, 351 (1956).
[29] H. H. HECKMAN, F. M. SMITH and W. H. BARKAS: *Nuovo Cimento*, **3**, 85 (1956).
[30] J. R. PETERSON: *Phys. Rev.*, **105**, 693 (1957).
[31] W. W. CHUPP, G. GOLDHABER, S. GOLDHABER, W. R. JOHNSON and J. E. LANNUTTI: *Phys. Rev.*, **99**, 1042 (1955).
[32] R. P. HADDOCK: *Nuovo Cimento*, **4**, 241 (1956).
[33] G. L. BACCHELLA, A. BERTHELOT, M. DI CORATO, O. GOUSSU, R. LEVI SETTI, M. RENÉ, D. REVEL, L. SCARSI, G. TOMASINI and G. VANDERHAEGHE: *Nuovo Cimento*, **4**, 1529 (1956).
[34] W. H. BARKAS and D. M. YOUNG: UCRL 2549.
[35] H. H. HECKMAN, F. M. SMITH and W. H. BARKAS: *Nuovo Cimento*, **4**, 51 (1956).
[36] S. FUNG, A. PEVSNER, D. M. RITSON and N. MOHLER: *Phys. Rev.*, **101**, 493 (1956).
[37] D. M. RITSON, A. PEVSNER, S. FUNG, M. WIDGOFF, G. T. ZORN, G. GOLDHABER and S. GOLDHABER: *Phys. Rev.*, **101**, 1085 (1956)
[38] N. BRENE, K. H. HANSEN, J. E. HOOPER and M. SCHARFF: *Nuovo Cimento*, **4**, 1059 (1956).
[39] J. CRUSSARD, V. FOUCHÉ, J. HENNESSY, G. KAYAS, L. LEPRINCE-RINGUET, D. MORELLET and F. RENARD: *Nuovo Cimento*, **3**, 731 (1956).
[40] G-STACK COLLABORATION: *Nuovo Cimento*, **2**, 1063 (1955).
[41] W. H. BARKAS, P. H. BARRET, P. CÜER, H. H. HECKMAN, F. M. SMITH and H. K. TICHO: *Phys. Rev.*, **102**, 583 (1956).
[42] G. BARONI, C. CASTAGNOLI, G. CORTINI, C. FRANZINETTI and A. MANFREDINI: *Ric. Sci.*, **26**, 1718 (1956).
[43] C. C. DILWORTH, G. OCCHIALINI and L. SCARSI: *Ann. Rev. of Nucl. Sci.*, vol. **4**.
[44] H. H. HECKMANN: UCRL 3003.
[45] M. F. KAPLON, J. KLARMAN and G. YEKUTIELI: *Phys. Rev.*, **99**, 1528 (1955).
[46] T. F. HOANG, M. F. KAPLON and G. YEKUTIELI: *Phys. Rev.*, **102**, 1185 (1956).
[47] C. O'CEALLAIGH, G. ALEXANDER and R. H. W. JOHNSTON: *Proc. of the 6th Rochester Conference* (1956).
[48] B. B. BHOWMIK, D. EVANS, I. J. VAN HEERDEN and D. J. PROWSE: *Nuovo Cimento*, **3**, 574 (1956).
[49] M. BALDO-CEOLIN, A. BONETTI, W. D. B. GREENING, S. LIMENTANI, M. MERLIN and G. VANDERHAEGHE: to be published.

[50] N. B. Biswas, L. Ceccarelli-Fabbrichesi, M. Ceccarelli, K. Gottstein, N. C. Varshneya and P. Waloschek: *Nuovo Cimento*, **3**, 825 (1956).
[51] B. P. Feld, A. C. Odian, D. M. Ritson and A. Wattenberg: *Phys. Rev.*, **100**, 1535 (1955).
[52] H. Winzeler, M. Teucher and E. Lohrmann: *Helv. Phys. Acta*, **29**, 75 (1956).
[53] M. N. Whitehead, D. H. Stork, J. R. Peterson, D. H. Perkins and R. W. Birge: UCRL 3295.
[54] G. Alexander, R. H. W. Johnston and C. O'Ceallaigh: private communication, to be published.
[55] L. W. Alvarez and S. Goldhaber: *Nuovo Cimento*, **2**, 344 (1955).
[56] J. Orear, G. Harris, and S. Taylor: *Phys. Rev.*, **104**, 1463 (1956).
[57] F. L. Iloff, W. W. Chupp, G. Goldhaber, S. Goldhaber, J. L. Lannutti, A. Pevsner and D. M. Ritson: *Phys. Rev.*, **99**, 1617 (1955).
[58] L. Mezzetti and W. Keuffel: *Nuovo Cimento*, **4**, 1096 (1956).
[59] K. W. Robinson: *Phys. Rev.*, **99**, 1606 (1955).
[60] P. R. Barker, D. M. Binnie, B. D. Hyams, R. J. Rout and J. Shepherd: *Phil. Mag.*, **46** 307 (1955); *Suppl. Nuovo Cimento*, **4**, 597 (1956).
[61] V. Fitch and R. Motley: *Phys. Rev.*, **101**, 496 (1956).
[62] V. Fitch and R. Motley: circulated preprint, see also *Rep. 6th Rochester Conference* (1956).
[63] L. W. Alvarez, F. S. Crawford, M. L. Good and M. L. Stevenson: *Phys. Rev.*, **101**, 303 (1956).
[64] W. H. Arnold, J. Ballam and G. T. Reynolds: *Phys. Rev.*, **100**, 295 (1955).
[65] T. F. Hoang, M. F. Kaplon and G. Yekutieli: *Phys. Rev.*, **105**, 278 (1957).

Chapter 2.

Observations on negatively charged K-particles (K^-).

K^- basic properties are not as well known as those of their positive counterparts. We especially lack information with regard to their modes of decay: this is because K^- interact strongly with matter and when brought to rest in a recording medium are usually captured before they have time to disintegrate (*) [1]. Only a few decays in flight have been observed and of those observed only 14 identified (see below).

Within the experimental errors, both the mass and the lifetime seem to be very close if not coincident with the corresponding values determined for the K^+ particles.

2·1. – Identified modes of decay.

To our knowledge the following events have been identified:

i) *The* $K^-_{\pi 3} \to \pi^+ + \pi^- + \pi^-$ *mode.* – Eight cases have been observed ([2] 1 case; [3] 1 case; [4] 2 cases; [5] 4 cases) in cloud chambers exposed to cosmic rays; and four cases in a hydrogen bubble chamber exposed to the K^--flux from the Bevatron [6]. In all cases, mass and Q-values were found consistent with those observed in $K^+_{\pi 3}$ decays.

ii) *The* $K^-_{\pi 2} \to \pi^- + \pi^0$ *mode.* – One case has been observed in a stack exposed at the Berkeley Bevatron [7]. A K^--meson (identified by direct mass measurements on its track) was seen to decay into a lighter particle having a mass of $(265 \pm 30)\,m_e$ and giving rise to a « σ-star ». Thus, the secondary was

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(*) Of course the atomic capture is supposed to take place by the usual falling down mechanism.

interpreted as a π^- and its momentum at the point of creation was found to be (170 ± 9) MeV/c. This value gave for the corresponding momentum in the decay center of mass system (202 ± 12) MeV/c, in good agreement with that obtained for $K^+_{\pi 2}$.

iii) *The* $K^-_{\beta 3} \rightarrow e^- + ? + ?$ *mode.* – One case to be attributed to this scheme was observed in the same investigation which yielded the event described in ii) [7]. The secondary particle was identified as an electron through ionization and scattering measurements on various intervals along its track. Its energy, in the center of mass system, was found to be 30 MeV.

Several cases have been reported of events observed in cloud chambers which are most probably due to K^--decay and yield a value for the mass of the primary close to 966 m_e when interpreted, some in terms of the $K^-_{\pi 2}$ and others of $K^-_{\mu 2}$ scheme. Details are given in Table I. The first three cases listed here favour a $K^-_{\pi 2}$ scheme; the others a $K^-_{\mu 2}$ scheme.

TABLE I-2·1. – *Probable two-body decays.*

Reference	$M(\pi^-, \pi^0)$ (m_e)	$M(\mu^-, \nu)$ (m_e)	Probable decay mode
[4]	(932 ± 105)	(871 ± 115)	$K^-_{\pi 2}$
[4]	(935 ± 58)	(833 ± 65)	$K^-_{\pi 2}$
[8, 9]	954 ± 30	790 ± 25	$K^-_{\pi 2}$
[4]	(1080 ± 204)	(966 ± 210)	$K^-_{\mu 2}$
[4]	(1062 ± 110)	(995 ± 120)	$K^-_{\mu 2}$
[9]	(1065 ± 55)	986 ± 60	$K^-_{\mu 2}$
[9]	1045 ± 45	941 ± 50	$K^-_{\mu 2}$
[9]	1049 ± 40	936 ± 30	$K^-_{\mu 2}$

Values in brackets have been deduced from the values of the momentum p^* of the secondary particles in the frame of reference of the decaying K^- — as given in the original papers.

Indirect evidence in support of the existence of modes of decay other than i), ii) was obtained by the Ecole Polytechnique group [10]. Examining a large number of V^- they concluded that it was impossible to interpret them if all the secondaries were assumed to be π-mesons. They suggested therefore that some of their cases were necessarily due to $K^-_{\mu 2}$ or $K^-_{\mu 3}$.

Noteworthy one event which was found associated with two electron showers. The dynamics of the event was consistent with its being a $K^-_{\pi 2}$ and with the assumption that both γ-rays had been produced by the disintegrating π^0 secondary. Unfortunately the primary was so fast that the sign of its charge could not be established with certainty.

2·2. – The mass of the K⁻'s.

Experimental methods to determine the mass of K^- particles are not substantially different from those employed for positive K's. Most of the work has been done using nuclear emulsions and the results have been reported in Table I.

WEBB *et al.* [11] determined the mass of 42 selected events from the simultaneous measurement of their residual range and their momentum on entering the emulsion stack. The measurement of the momentum was more elaborate in this case that it was for K^+. (See Sect. 1·2). There it could be immediately deduced from the residual ranges of protons deflected through the same angle: here it had to be deduced from the range of positive K's channelled and focussed in the same way as the negative ones by reversing the magnetic field of the focussing spectrometre. What was thus obtained was the ratio $M(K^-)/M(K^+)$ which was found to be 0.998 ± 0.013.

Determinations have also been based on the study of the dynamics of the interactions of K^- with free protons of the emulsion (See Table I). The most precise ($\Delta m/m \lesssim 0.5\%$) are those based on the estimate of the energy released in the capture of K^- by free protons leading to the ejection of a positive Σ hyperon and a negative pion. Those associated with the emission of a negative Σ are not as useful since the mass of the latter is not known as precisely as that of the Σ^+ (see Sect. 3·2).

$K^-_{\pi 3}$ decays observed in bubble and cloud chambers allow a precision 1% in favourable cases.

It is apparent from Table I that the masses obtained through the different methods are all consistent with a single value, very close to that of the K^+'s. An average on the two best estimates [14] gives

$$m_{K^-} = (965.6 \pm 1.2)\ \mathrm{m_e}\,.$$

2·3. – The mean life of the K⁻'s.

The mean life of K^- has been measured in emulsion and cloud chamber experiments and recently with counters. The emulsion work is due to ILOFF *et al.* [34]. Stacks of stripped emulsions were exposed to a magnetically analyzed beam of K^-, and particles having momenta 285 to 425 MeV/c were allowed to enter the stacks, parallel to the emulsion layers. The K^- tracks, recognized from the background of π^- of the same momentum on the basis of their grain density on entering the emulsion, were followed until they decayed or interacted, or were brought to rest. For each event the time of flight t_i was

TABLE I-2·2. – *Summary of* K^-*-mass determinations.*

i) *Direct determinations*, based on the characteristics of the K^--particle tracks and/or their magnetic rigidity (emulsion work only).

Ref.	Source of radiation	No. of observed events	Time of flight (s)	Mass (m_e)	Remarks
(*)	Cosmic rays	21	$\geqslant 10^{-11}$	987 ± 24	—
[1]	Bevatron	51	$\sim 10^{-8}$	970 ± 30	Range versus scattering (constant sagitta) method.
[11]	»	42	$1.4 \cdot 10^{-8}$	963 ± 12	Range — $H\varrho$ method
(**)	Bevatron and Cosmotron	13	$\geqslant 10^{-9}$	939 ± 20	—
[12]	Bevatron	3	$\sim 10^{-3}$	989 ± 10	From elastic scatterings on free protons. (Individual values: 980 ± 220; $1\,008 \pm 26$; 993 ± 11).
[13]	»	1	—	979 ± 11	From elastic scattering on free protons,
[14]	»	6	—	978 ± 57	»

(*) [15-24]. (**) [25-31].

ii) *Indirect determinations*, based on the analysis of the decays or of the disintegrations following capture by nuclei.

Ref.	Source of radiation	Detector	No. of events	Time of flight	Mass (mW)	Remarks
a) $K^-_{\pi 3}$ *decays*						
[2]	Cosm. rays	Cloud Ch.	1	—	973 ± 18	$Q = 77 \pm 9$ Average mass (960 ± 5) m_e
[4]	»	»	1	—	$1\,006 \pm 26$	$Q = 95 \pm 13$
[4]	»	»	1	—	954^{+32}_{-22}	$Q = 69^{+16}_{-11}$
[3]	»	»	1	—	956 ± 6	$Q = 70 \pm 3$
[6]	Bevatron	Bubble Ch.	4	10^{-8}	963 ± 10	—
b) $K^-_{\pi 2}$ *decays*						
[7]	Bevatron	Emulsion	1	10^{-8}	957 ± 40	$Q = (214 \pm 20)$ MeV; $p^* = (202 \pm 12)$ MeV/c
c) K^- *capture by nuclei*						
[14]	Bevatron	Emulsion	1	—	965 ± 1.5	from $K^- + {}^1H \to \Sigma^+ + \pi^-$ at rest
[32]	»	»	1	—	966 ± 6	»
[33]	»	»	1	—	966.7 ± 2	»
[12]	»	»	1	—	966 ± 5	from $K^- + {}^{12}C \to \pi^- + {}^7_\Lambda Li + {}^4He + Q$; ${}^7_\Lambda Li \to {}^7Be + \pi^- + Q$

measured and hence the lifetime calculated. Out of 13 decays in flight they obtained $\tau_{K^-} = (0.95^{+0.36}_{-0.31})\cdot 10^{-8}$ s. Preliminary results obtained by BARKAS *et al.* [14] from a similar experiment indicate a slightly higher value ($(1.46^{+0.38}_{-0.31})\cdot 10^{-8}$ s).

CORK *et al.* [35] estimated the mean life by measuring the decreasing intensity of the K^- flux between two counters placed in the beam of negative particles produced by the Berkeley Bevatron. The momentum of the K^- was 900 MeV/c: the distance between the two counters was 13.4 feet and that of the first counter from the target was approximately 100 feet. Their experiment gave $\tau_{K^-} = (1.49^{+0.22}_{-0.24})\cdot 10^{-8}$ s in good agreement with BARKAS *et al.* The average calculated on the results listed above is

$$\tau_{K^-} = (1.32 \pm 0.17)\cdot 10^{-8}\text{ s},$$

which is very close to the mean life of K^+ (See Sect. **1**·5).

Different results have been obtained by the Princeton and by the Pasadena group. ARNOLD *et al.* [9, 37] studied penetrating showers by means of two cloud chambers, one immersed in magnetic field and another equipped with lead plates. Out of 44 V events, 6 negative were found which could not be due to hyperons and were assumed to be K^-. The mean life was measured using Bartlett's method (see Sect. **9**·3) and it was obtained

$$3 \leqslant \tau_{K^-} \leqslant 8\cdot 10^{-10}\text{ s},$$

with 50% probability. The probability that τ_{K^-} were greater than $5\cdot 10^{-9}$ s was found to be 11%.

TRILLING and LEIGHTON ([36]; see also [38]) also in a cloud chamber investigation on charged V-particles found some evidence for a short lived negative component. Their events, however, can be interpreted as Σ^- or K^-, the individual errors in mass and moment estimates being rather large, and Σ^- particles are known to have a lifetime of $\sim 10^{-10}$ s.

TABLE I-2·3. – *Summary of recent results on* K^-*-mean life.*

Reference	Source of radiation	Detector	Time of flight (s)	Lifetime (10^{-8} s)
[14]	Bevatron	emulsion	—	$1.46^{+0.38}_{-0.31}$
[35]	Bevatron	counters	$6\cdot 10^{-8}$	$1.49^{+2.2}_{-2.4}$
[34]	Bevatron	emulsion	$1.2\cdot 10^{-8}$	$0.95^{+0.36}_{-0.32}$
[37, 9]	Cosmic Rays	Cl. Ch.	—	$3 < - < 8\cdot 10^{-2}$
[36]	Cosmic Rays	Cl. Ch.	—	$< 10^{-1}$

Thus we are led to conclude that:

i) The existence of K^- particles having a lifetime $\sim 10^{-8}$ is proved.

ii) The possibility of the existence of a short lived component seems very doubtful because presumably it would have been observed in emulsion in the same way as the Σ^+ emitted from energetic disintegrations are observed.

iii) No evidence for a long lived component ($\tau \geqslant 10^{-7}$ s) has ever been reported.

REFERENCES (Ch. 2)

[1] E. P. George, A. J. Herz, J. H. Noon and N. Solntseff: *Nuovo Cimento*, **3**, 94 (1956).

[2] T. L. Aggson, W. B. Fretter, E. W. Friesen, L. F. Hansen, R. G. Kepler and A. Lagarrigue: *Phys. Rev.*, **102**, 243 (1956).

[3] V. A. J. Van Lint and G. H. Trilling: *Phys. Rev.*, **92**, 1089 (1953).

[4] G. H. Trilling and G. Neugebauer: circulated pre-print, in press on the *Phys. Rev.*

[5] R. Armenteros, B. Gregory, A. Hendel, A. Lagarrigue, L. Leprince-Ringuet, F. Muller and Ch. Peyrou: *Suppl. Nuovo Cimento*, **4**, 520 (1957).

[6] B. Waldman: UCRL 3507.

[7] A. G. Ekspong and G. Goldhaber: *Phys. Rev.*, **102**, 1187 (1956).

[8] W. H. Arnold, J. Ballam, A. L. Hodson, G. K. Lindeberg, R. R. Rau, G. T. Reynolds and S. B. Treiman: *Suppl. Nuovo Cimento*, **4**, 559 (1956).

[9] W. H. Arnold, J. Ballam and G. T. Reynolds: *Phys. Rev.*, **100**, 295 (1955).

[10] R. Armenteros, A. Aster, C. D'Andlau, B. Gregory, A. Hendel, J. Hennessy, A. Lagarrigue, L. Leprince-Ringuet, F. Muller, Ch. Peyrou and R. R. Rau: *Suppl. Nuovo Cimento*, **4**, 541 (1956).

[11] F. B. Webb, W. W. Chupp, G. Goldhaber and S. Goldhaber: *Phys. Rev.*, **101**, 1212 (1956).

[12] G. G. Gilbert, C. E. Violet and R. S. White: *Phys. Rev.*, **103**, 1825 (1956).

[13] Bern-group: as quoted in the *Proc. 6th Rochester Conference* (1956); V-26.

[14] W. H. Barkas, W. F. Dudziak, P. C. Giles, H. H. Heckman, F. W. Inman, C. J. Mason, N. A. Nichols and F. M. Smith: UCRL 3627.

[15] E. Amaldi, G. Baroni, C. Castagnoli, G. Cortini, C. Franzinetti and A. Manfredini: *Nuovo Cimento*, **11**, 207 (1954).

[16] L. Bacchella, M. Di Corato, M. Ladu, R. Levi Setti, L. Scarsi and G. Tomasini: *Suppl. Nuovo Cimento*, **4**, 465 (1956).

[17] Tsai-Chü: *Suppl. Nuovo Cimento*, **4**, 479 (1956).

[18] J. Crussard, V. Fouché, J. Hennessy, G. Kayas, L. Leprince-Ringuet, D. Morellet and F. Renard: *Nuovo Cimento*, **3**, 731 (1956).

[19] M. Di Corato, C. Dilworth, D. Hirschberg, L. Hirschberg, L. Scarsi, P. Solari and G. Tomasini: *Suppl. Nuovo Cimento*, **4**, 490 (1956).

[20] C. C. Dilworth, A. Manfredini, G. D. Rochester, J. Waddington and G. T. Zorn: *Suppl. Nuovo Cimento*, **12**, 433 (1954).

[21] M. W. Friedlander, D. Keefe and M. G. K. Menon: *Nuovo Cimento*, **2**, 666 (1955).

[22] K. Gottstein, B. Roederer, J. Roederer, N. Varshneya and P. Waloschek: *Suppl. Nuovo Cimento*, **4**, 440 (1956).
[23] D. Hirschberg and L. Hirschberg: *Nuovo Cimento*, **12**, 296 (1954).
[24] M. Teucher: *Suppl. Nuovo Cimento*, **4**, 482 (1956).
[25] M. W. Friedlander, D. Keefe, M. G. K. Menon, R. H. W. Johnston, C. O'Ceallaigh and A. Kenan: *Phil. Mag.*, **46**, 144 (1955).
[26] F. W. Fry, J. Schneps, G. A. Snow and M. S. Swami: *Phys. Rev.*, **100**, 1448 (1955).
[27] R. D. Hill, E. O. Salant and M. Widgoff: *Phys. Rev.*, **94**, 1794 (1954).
[28] J. Hornbostel and E. O. Salant: *Phys. Rev.*, **93**, 902 (1954).
[29] J. Hornbostel and E. O. Salant: *Phys. Rev.*, **99**, 338 (1955).
[30] J. E. Naugle, E. P. Ney, P. S. Freier and W. B. Cheston: *Phys. Rev.*, **96**, 1383 (1954).
[31] S. Rosendorf, R. Stehl and G. Yekutieli: *Phys. Rev.*, **93**, 901 (1954).
[32] W. W. Chupp, G. Goldhaber, S. Goldhaber, E. L. Iloff and F. H. Webb: as reported by G. Goldhaber at the *6th Rochester Conference* (1956), p. V-27 of the mimeogr. rep.
[33] W. F. Fry, J. Schneps, G. A. Snow, M. S. Swami and D. C. Wold: *Phys. Rev.*, **104**, 270 (1956).
[34] I. L. Iloff, G. Goldhaber, S. Goldhaber, J. E. Lannutti, F. C. Gilbert, C. E. Violet, R. S. White, D. M. Fournet, A. Pevsner, D. M. Ritson, and M. Widgoff: *Phys. Rev.*, **102**, 927 (1956).
[35] B. Cork, G. R. Lambertson, O. Piccioni and W. Wenzel: UCRL 3654.
[36] G. H. Trilling and R. B. Leighton: *Phys. Rev.*, **100**, 1468 (1955).
[37] W. H. Arnold: *Thesis* (Princeton, 1955).
[38] C. M. York, R. B. Leighton and E. K. Bjørnerud: *Phys. Rev.*, **95**, 159 (1954).

CHAPTER 3.

Observations on Neutral K-particles (K^0).

3·1. – Early experiments on V^0's.

In the course of an investigation on cosmic ray penetrating particles, by means of a cloud chamber, ROCHESTER and BUTLER [1] observed two forked tracks « of a very striking character ». In either case the apex of the fork was in the gas; and in neither case could they see any other ionizing particle associated with the apex. They noted that if the two events were due to some collision process, then they should have observed several hundreds of them in the lead plates of the chamber; besides, the momentum transferred to the target nucleus would have been large enough to produce a visible recoil. It was also proved that neither event could be interpreted as due to an electron pair. On the basis of momentum measurements they concluded that the two events represented the spontaneous decay of new particles presumably into light mesons: in one case of a neutral particle and in the other of a charged one, both having masses around 1 000 m_e.

Confirmation of their conclusion was obtained by SERIFF *et al.* [2] who used a similar experimental arrangement. They observed 34 V-shaped events among cosmic ray particles and assumed them to be of the same type as those observed by ROCHESTER and BUTLER. The name « V-particle » was then suggested by C. ANDERSON and P. M. S. BLACKETT (see [3]) to indicate this type of events.

Of the 34 events, 30 were interpreted as due to the decay of neutral V (or V^0) (*). The nature of their secondary fragments was in most of these cases obscure, but it could be proved that light mesons (π or μ) were present among them, and that presumably there were no electrons since no radiative

(*) Although their composition and nature is now much better known than it was then, and different names have been introduced for the various components, we shall use the symbol V^0 to indicate indifferently any event due to a neutral particle which decays into two charged secondaries.

collision was observed associated with those crossing the lead plate inside the chamber.

Moreover in twelve cases the plane of the V^0 also contained the centre of an interaction observed simultaneously in the chamber and most probably responsible for the creation of the V^0 itself. This suggested that the decay scheme of the V^0's did not involve more than the two visible fragments.

The first evidence that V^0's consisted of at least two different types of particles was obtained by the Manchester group in 1951 [3]. Among the positive secondaries of 36 V's, 4 protons were identified with certainty and 3 particles observed which were definitely lighter than protons. Among the negatives, 3 were identified as light mesons, 18 were found to be lighter than protons and none was identified as a particle of protonic mass. Unfortunately in no case the mass of both secondaries could be determined. However in the four decays associated with a positive fragment of protonic mass, the negative fragment was found unlikely to be anything but a light meson.

The presence of positive fragments both of protonic and of mesonic mass made it difficult to explain all the observed events in terms of alternative modes of decay of the same particles (*).

The quoted Authors suggested the existence of two different particles which decayed in the following ways

$$V_1^0 \to p + \pi^- ,$$

$$V_2^0 \to \pi^+ + \pi^- .$$

On these assumptions, the mass of the two particles, as obtained from momentum and/or ionization measurements, turned out to be:

$$m(V_1^0) \sim 2250 \ m_e ,$$

$$m(V_2^0) \sim \ 950 \ m_e .$$

The existence of at least two completely different kinds of V's was confirmed by subsequent work which also confirmed the order of magnitude of the masses. In this chapter we shall confine our attention to the lighter V^0's defined as those decaying into secondaries both lighter than protons. They will be referred to as K^0, without implying anything about their nature or that of their decay products. The purpose of the following section will be in fact that of answering such questions.

The V^0 protonic decays will be considered in the next chapter.

(*) To do so it would have been necessary, either to assume the existence of negative protons (not known to exist at that time) or that the decay scheme of V^0's involved more than two particles (and this was not favoured by the observations of SERIFF *et al.*).

3·2. – Nature of the charged secondaries in K^0 decays.

Following the pioneer work of [3], cases of K^0 were reported by various people [4-6]. Further information on the nature of the secondaries was obtained from the study of their interaction properties. A number of interactions undergone by secondaries from K^0 decays was reported [5, 7-9]. The one found at the Ecole Polytechnique [9] in which both interact is particularly interesting. The Manchester group [4] observed two interactions by secondaries of either sign, crossing a total of 20 g cm^{-2} Pb which is a distance about 1.3 times longer than the geometric mean free path. REYNOLDS [8] observed two nuclear interactions over a range of 132 g cm^{-2} Pb.

Three 2-branch stars observed in photographic emulsions and identified with a high degree of probability as K^0-decays [10-12] furnished independent evidence that most of the secondaries from K^0 interact strongly with matter. In all the three cases the negative fragment, stopped in the emulsion producing a disintegration, and consequently was assumed to be a π^-. The positive ones could be π's or μ's; they were too fast to decide between the two alternatives. One example of π-μ decay of a meson emitted from the same two-branch-star in emulsion was reported by LAL *et al.* [13]. This case will be discussed again in Sect. 3·7 since it was found to be associated with a low Q-value.

Recent cloud and diffusion chamber observations have shown the existence of V^0's which disintegrate into (π, e) and (π, μ) pairs [14-19].

3·3. – The plurality of K^0-events.

The evidence surveyed in Sect. 3·2 indicates that there must be more than one mode of decay to be associated with K^0-events. In the following sections, leaving the historical accuracy aside, we shall analyze the ensemble of the experimental data accumulated in the last few years and proceed to see in some detail how many K^0-modes can be proved to exist.

The line of approach will be the reverse of that used in connection with charged K's. There we could select the particles on the basis of their mass as deduced directly from their observed behaviour in interacting with matter or external fields before they decayed; subsequently their various properties such as decay mode, lifetime etc., were considered.

For neutral particles the knowledge of the decay mode is in general the starting point. Once the nature of the secondaries and the energy released in the disintegration have been established, the other relevant dynamical parameters can be calculated.

Precise mass and energy determinations on both secondaries are seldom possible in cloud and diffusion chambers—which were responsible for most of the early work. In most cases the information obtainable in individual events is only « partial » and not sufficient to allow an unambiguous identification. It was soon realised, however that definite conclusions could be reached even out of a comparatively small number of « partial data » and very elegant methods of analysis were developed to this purpose [20, 21] (see Ch. 9). Even if the production of intense beams of unstable heavy particles by the big machines has made them less essential than they were a few years ago, they are still of great interest. The discussion of the data given in the following pages is largely based on their application.

3·4. – The $K^0_{\pi 2} \to \pi^+ + \pi^-$ mode of decay.

In this section we shall describe the evidence for the existence of a neutral heavy meson decaying according to the scheme

$$K^0_{\pi 2} \to \pi^+ + \pi^- + (214 \pm 2.5)\ \text{MeV}\,.$$

We shall begin by considering two experimental facts:

3·4.1. *Existence of co-planar events among* K⁰*-decays.* – Whenever the origin of a V⁰-event can be individuated, it is possible to check whether its line of motion and those of its charged secondaries lie or not in the same plane. Any appreciable departure from coplanarity indicates the existence of at least one neutral decay product. It must be pointed out that the reverse is not necessarily true since for appropriate values of the masses and velocities of the secondaries the « uncoplanarity » may well be, on the average, of the same order of magnitude of the experimental errors involved in angular measurements and therefore practically undetectable.

Coplanarity in K⁰-events—within the experimental errors—has been observed by BARKER [23] in 7 cases, by FRETTER *et al.* [24] in 11 cases, by BRIDGE *et al.* [7] in 4 cases, by GAYTHER [25] in 8 events.

3·4.2. *Transverse momentum distribution.* – In a 2 body decay, if the emission of the products is isotropic in the center of mass system, the distribution of the transverse momentum (*) follows a simple law

$$\omega(p_T)\,\mathrm{d}p_T = \frac{p_T\,\mathrm{d}p_T}{p^*(p^{*2} - p_T^2)^{\frac{1}{2}}}\,, \tag{1-3·4}$$

(*) I.e. the component of the momentum of the secondary perpendicular to the line of motion of the primary (see Fig. 1).

where p^* is the momentum of the secondaries in the system in which the primary is at rest.

Due to errors in the determination of p_T the experimental curve is to be expected to differ from (1). BARKER [23], assuming a Gaussian distribution for the errors, predicted a p_T-distribution of the type

$$\omega(\bar{p}_T)\,\mathrm{d}\bar{p}_T = \mathrm{d}\bar{p}_T \int_0^{p^*} \frac{p_T}{p^*(p^{*2}-p_T^2)^{\frac{1}{2}}} \exp\left[-\frac{(p_T-\bar{p}_T)^2}{2\Delta p^2}\right] \mathrm{d}p_T\,, \tag{2-3·4}$$

where $\bar{p}_T$ indicates a value of the transverse momentum as experimentally measured and Δp the average error in p_T determinations. A histogram based on 13 K^0-cases was fitted with the $\bar{\omega}(\bar{p}_T)$-curve calculated for $p^*=200$ MeV/c and $\Delta p=35$ MeV (Fig. 2). The agreement with the experimental points strongly suggests that both assumptions (2-body decay and isotropy) are correct in so far as the events considered are concerned.

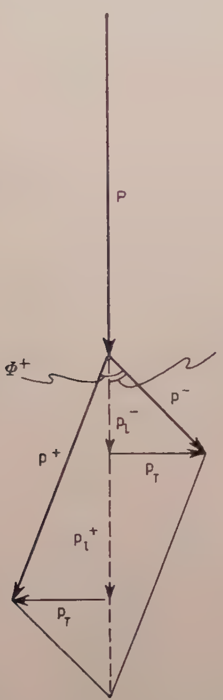

Fig. 1-3·4. – Momentum diagram of neutral 2-body decays.

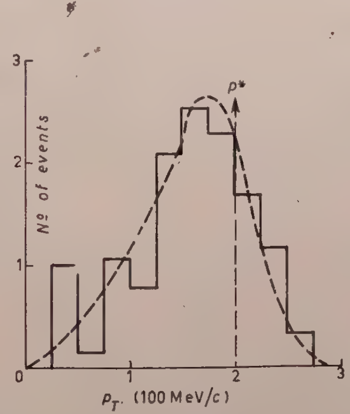

Fig. 2-3·4. – Transverse momentum distribution in $K^0_{\pi 2}$ decays. (After BARKER [23]).

3·4.3. *Dynamical analysis of* K^0 *decays* (*). – It was shown by THOMPSON [4, 21] and by PODOLANSKI and ARMENTEROS [20] that, for two body decays, momentum and energy conservation laws, expressed as a function

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(*) For details on the methods of analysis discussed here and in the following paragraphs see Sect. 9·1.

of the quantities p_T and $\alpha = (p_l^+ - p_l^-)/(p_l^+ + p_l^-)$ (p_l^+ and p_l^- being the longitudinal components of the momenta of the secondary particles (see Fig. 1)) result into the equation

$$p_T^2 + \frac{(\alpha - \alpha^*)^2}{4((1/P^2) + (1/M^2))} = p^{*2}\,, \tag{3-3·4}$$

where M is the primary mass, P the primary momentum, $\alpha^* = (m_+^2 - m_-^2)/M^2$, m_+ and m_- the masses of the charged secondaries. For a given 2-body decay scheme, the values of M, α^* and p^* are uniquely determined. Viceversa if one determines experimentally the values of M, α^* and p^*, the decay scheme (i.e. also m_+ and m_-) can be deduced.

For each observed decay the quantities $1/P$, α, p_T can be determined experimentally *without any previous knowledge* of the decay scheme. In a $(1/P, \alpha, p_T)$-space each triplet of values is represented by a point and equation (3) tells us that all points associated with the same « decay scheme » fall on the same « decay surface ».

One could in principle individuate the surface given a sufficient number of experimental points. This is practically impossible: but it is possible to check an assumed decay scheme against the experimental observations.

Before we see how this can be done, let us remark that if we are observing decays of fast K⁰'s and the condition $P^2 \gg M^2$ is satisfied, the projection of the points in the $\alpha - p_T$ plane ($1/P = 0$) will be very close to the curve

$$p_T^2 + \frac{(\alpha - \alpha^*)^2}{4/M^2} = p^{*2}\,. \tag{4-3·4}$$

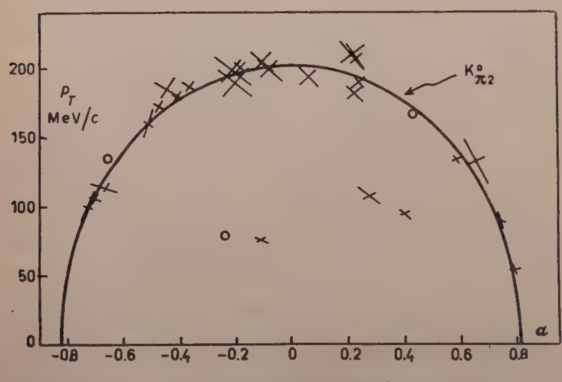


Fig. 3-3·4. – α-p_T diagrams of $K^0_{\pi 2}$ data. (After THOMPSON *et al.* [26]).

We can then make use of the far more manageable two-dimensional α—p_T plot instead of the three dimensional one discussed above. Slow decays will be projected on different « iso-P » curves. It can be easily shown, however, that the different iso-P all converge on to the « limiting ellipse » (4) (i.e. the decay surfaces degenerate into a cylinder) if instead of α the quantity α' is used

$$\alpha' = \alpha^* + \beta(\alpha - \alpha^*)\,,$$

β being the velocity of the neutral primary. In what follows we shall use the symbol α for α' assuming non the less this transformation as being applied.

The indications summarized in Sect. 3·2 and 3·4 give to a dynamical analysis in terms of α and p_T a more precise meaning. In fact they suggest a check of the experimental data against the scheme $K^0_{2\pi} \to \pi^+ + \pi^-$ associated with a value of $p^* \sim 200$ MeV/c. Of the large amount of data accumulated in the last few years on K^0 particles, certainly few are more convincing than those of THOMPSON *et al.* [26] (Fig. 3). Out of 28 events 24 crowd a narrow region close to the ellipse corresponding to $p^* = 203$ MeV/c and to a $Q = 214$ MeV. One can hardly escape the conclusion that the above decay mode exists although it does not explain all the observations. The « anomalous » cases will be discussed in Sect. 3·7.

3·4.4. *The mass and the Q-value of the* $K^0_{\pi 2}$*'s*. – A list of results concerning mass measurements on identified $K^0_{\pi 2}$ particles observed in cloud chamber experiments is given in Table I. The corresponding Q-values, as defined by the relation $Q = M - m_+ - m_-$, are also given.

An average calculated on numbers obtained from different experiments, probably affected to a different extent by systematic errors, seems to us meaningless. In view of the high degree of technical precision reached by the Indiana group, we are inclined, to consider their results as the closest approach to the true value so far achieved, i.e.

$$Q(K^0_{\pi 2}) = (214 \pm 2.5)\ \text{MeV}\ ; \qquad M(K^0_{\pi 2}) = (965 \pm 5)\ m_e\ (*)\ .$$

Few cases of $K^0_{\pi 2}$ have been observed in nuclear emulsion (YASIN [12], DI CORATO *et al.* [10], HIRSCHBERG [11]). The estimated Q's are in good agreement with those of Table I ((202 ± 11) MeV, (210 ± 35) MeV, (216 ± 45) MeV).

TABLE I-3·4. – *Mass and Q-values regarding* $K^0_{\pi 2}$*-decays.*

Ref.	Source of radiation	Detector	No. of events	Q (MeV)	M (m_e)
[28]	Cosmic Rays	Magn. Cloud Ch.	31	212 ± 4	961 ± 8
[29]	Cosmotron	»	29	200 ± 10	936 ± 20
[31]	Cosmic Rays	»	12	222 ± 4	974 ± 8
[26]	»	»	24	214 ± 2.5	965 ± 5

3·4.5. *The lifetime of the* $K^0_{\pi 2}$*'s*. – The experimental procedures followed in lifetime measurements on K^0's are very similar.

Measurements on cosmic ray particles, all recorded in cloud chambers, have

(*) With regard to the value of the error given here see K. M. CROWE [27], footnote p. 552.

been analyzed using Bartlett's classical work (see Sect. **9**·3). They differ, however in the method of selection. Particularly in the early ones, the selection of the events was largely based on the assumption that only two types of V⁰-particles existed, the Λ^0 and the $K^0_{\pi 2}$ and so long as an event was *consistent* with the $K^0_{\pi 2}$ mode and inconsistent with the other, it was automatically assumed that it was a $K^0_{\pi 2}$. Presently we are unable to estimate the relative proportion of the different modes (Sect. **3**·7) which are likely to have interfered with this type of experiments and we cannot decide on the exact significance of the interpretation given by the respective Authors of their results. Presumably however they are not seriously in error because the other modes of decay presently known to exist are mostly associated with a mean life probably 100 times longer than that of the $K^0_{\pi 2}$. Unless the long lived ones are produced in a much larger proportion than $K^0_{\pi 2}$ (which does not seem the be case) only a small fraction of them would be present in the samples selected in most of the cosmic ray experiments which in general recorded particles decaying very close to the generating material.

In the experiments in which a magnetic analysis of the momenta was possible [31, 32] the selection of the events was based on the Q-value determination. In others [25, 33, 34] the analysis was carried out following the method of Podolanski and Armenteros (see Sect. **9**·1) which requires only the knowledge of the angles Φ_1 and Φ_2 (as defined in Fig. 1) (*) combined with an estimate of the ionization. Their procedure was the following: in the cases in which the line of flight of the V^0 and those of either secondaries was known, the parameters

$$\alpha = \frac{\sin(\Phi_2 - \Phi_1)}{\sin(\Phi_2 + \Phi_1)} \qquad \text{and} \qquad \varepsilon = \frac{2\sin\Phi_1 \sin\Phi_2}{\sin(\Phi_1 + \Phi_2)},$$

were calculated. In the α, ε plane, the point having these co-ordinates must lie on an ellipse which is determined by the decay scheme and by the momentum P of the primary. As noted in Sect. **9**·1.3, the distances f_1 and f_2 from the point $R(\alpha, \varepsilon)$ to the foci of the decay ellipse (see Fig. 4-**9**·1) determined the angle Φ^* of emission in the center of mass system and the velocity β of the primary. The V^0 were only assumed to be either a $K^0_{\pi 2}$ or a Λ^0. For each choice one can draw only one ellipse going through R, i.e. only one set of values of Φ^* and β is possible. Assuming alternatively the values of p^*, m_1 and m_2 related to the $K^0_{\pi 2}$ and Λ^0 modes, the momenta p_1 and p_2 can be deduced from the relation

$$p_1 \sin\Phi_1 = p_2 \sin\Phi_2 = p^* \sin\Phi^*$$

(*) When, in the absence of a magnetic field, the sign of the secondary charges is not known, the symbols Φ_1 and Φ_2 will be used to indicate the same angles instead of Φ_+ and Φ_-.

and the ionization of the tracks produced by the secondary predicted. If these particles are not fast, a comparison between the predicted and the observed ionization may allow a choice between the two modes. For example, in 29 out of 44 examined $K^0_{\pi 2}$ Gayther found consistency with only one of the two schemes.

Measurements on artificially produced K^0's have been made at Brookhaven using cloud and bubble chambers. BLUMENFELD *et al.* [29] analyzed 29 K^0's observed in a 36 in diameter magnet cloud chamber, produced by 1.9 GeV π^- in lead and carbon. On the basis of the measured Q-values they identified 25 $K^0_{\pi 2}$.

SCHWARTZ *et al.* [35] studied K^0 particles produced and decaying in a 12 in. diameter propane bubble chamber exposed to the flux of 0.85 to 1.3 GeV/c π^- beam produced by the cosmotron and immersed in a magnetic field. The origin of the K^0 could be located very precisely whenever the parent π^- interaction was seen in the chamber. Such origin is in fact marked by the point of disappearance of the primary π^- such as that shown in Fig. 6-3·5 due to the reaction (compare also Sect. **17·3**)

$$\pi^- + p \to \Lambda^0 + K^0 .$$

Thus, combining the knowledge of the dynamics of the production events with the estimates of the momenta of the secondaries (as obtained from magnetic analysis of the Λ^0 and K^0 decays) the time of flight of the unstable particles could be measured more precisely than previously done.

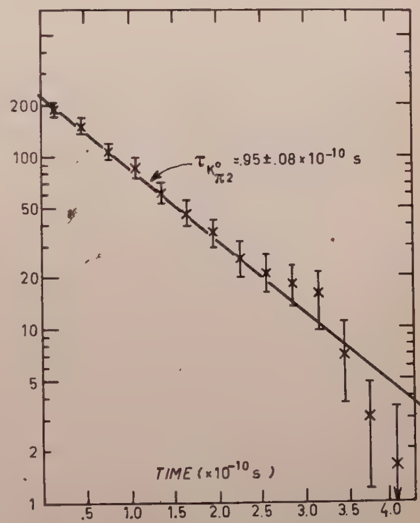


Fig. 4-3·4. – Time distribution of $K^0_{\pi 2}$ decay. (After SCHWARZ *et al.* [35]).

Moreover these Authors were able to analyze a large number of events, almost twice as many as those observed in several years of work on cosmic rays (see Table II). Their result

$$\tau(K^0_{\pi 2}) = (0.95 \pm 0.08)\cdot 10^{-10}\ \text{s} ,$$

has thus more weight than all the others listed in Table II.

The distribution of the individual times of flight (Fig. 4) is beautifully fitted by an experimental line corresponding to this value of τ.

A weighted average of the cosmic ray results alone gives $\tau = (0.65 \pm 0.08)\cdot 10^{-10}$ s, which is appreciably lower than that given by the machine work. Despite

TABLE II-3·4. – $K^2_{\pi 2}$ *mean life measurements.*

Ref.	Source of Radiation	Detector	No. of obs. events	Mean life 10^{-10} s
[32]	Cosmic rays	Magnet Cloud Ch.	11	$1.6^{+2.2}_{-0.6}$
[34]	»	Multiplate Cloud Ch.	6	$0.92^{+0.3}_{-0.18}$
[7]	»	»	6	$0.9^{+1.6}_{-0.3}$
[36]	»	»	9	$2.3^{+2.1}_{-0.7}$
[25]	»	»	8	$0.6^{+0.4}_{-0.2}$
[33, 37]	»	»	52	0.6 ± 0.1
[31]	»	Magnet Cloud Ch.	14	$0.7^{+0.3}_{-2.0}$
[29]	Cosmotron	»	25	$0.8^{+0.3}_{-0.2}$
[35]	»	Magnet propane bub. ch.	277	0.95 ± 0.08

the small numerical value of the statistical errors which have been estimated by the authors of the various cosmic ray experiments, there are too many possible sources of uncertainty, to leave ground for a claim of substantial difference.

3·5. – The $K^0_{\pi^0 2} \to \pi^0 + \pi^0$ mode.

When a beam of accelerated particles of sufficiently high energy strikes a target it will produce all sort of charged and uncharged radiations. In particular it may produce γ-rays either directly or by an intermediate unstable particle which decays with the emission of one or more γ's. Neutral π's are an example of such processes since they are known to decay in the majority of cases according to the scheme $\pi^0 \to \gamma + \gamma$. Other examples have been quoted in this article when the following decays were mentioned: $K^{\pm}_{\pi 2} \to \pi^{\pm} + \pi^0 \to \pi^{\pm} + \gamma + \gamma$ and $K^{\pm}_{\mu 3} \to \mu^{\pm} + \pi^0 + \nu \to \mu^{\pm} + \gamma + \gamma + \nu$; others will be mentioned in the next chapter.

γ-rays produced in the decay of unstable particles can be distinguished from those directly produced in the target if the lifetime of the unstable particles is long enough to bring the emission of γ's at an appreciable distance from the target itself. This will have the effect of producing an « extended » source of γ-rays, on either side of the target if the dynamics of the individual processes allow their emission both in the forward and backward direction.

If an experimental arrangement is used, which is capable of detecting γ-rays coming from a well defined direction such as that schematically drawn in Fig. 1 it may be possible to measure the « extension » of the γ-ray source and hence the mean life of the unstable particles responsible for it.

The knowledge of the lifetime and of the « excitation curve » may allow the identification of the individual processes which lead to the γ-emission. Of the known processes, the π^0 decay, being associated with a mean life $\leqslant 10^{-15}$ s will produce γ's practically all inside the target. If a $K^0_{\pi^0 2} \to \pi^0 + \pi^0$ mode occurs with appreciable frequency, and has a lifetime comparable with that of the $K^0_{\pi 2}$ mode, its contribution may be detected from the study of the γ-intensity as a function of the distance d from the target (see Fig. 1) at various energies of the primary beam. Analogous considerations may be applied to hyperon decays and these will be discussed in the next chapter. The contribution from $K^{\pm}$ would not interfere with that from K^0's since the former are known to have a lifetime about hundred times longer than the latter.

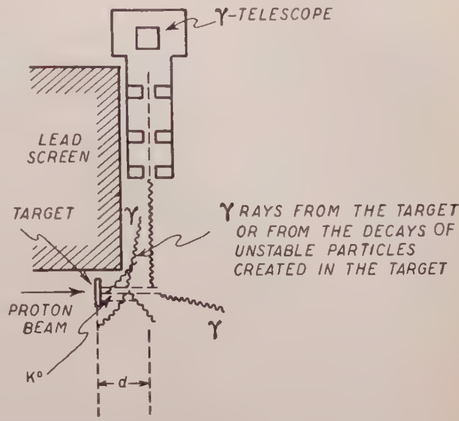


Fig. 1-3·5. – Disposition of apparatus employed by COLLINS [38], RIDGWAY *et al.* [39], OSHER *et al.* [40] for the study of γ-rays ejected in the decay of unstable particles produced by the impact of energetic protons in matter.

A search by this method was carried out by COLLINS [38] and by RIDGWAY *et al.* [39] who had at their disposal a proton beam of energies up to 3 GeV. Details of their apparatus are given in Fig. 2.

Only γ-rays emitted at right angles with respect to the primary beam were detected by a γ-telescope, all radiations, not coming from a well defined « viewed region », being prevented from reaching it by a heavy lead shielding.

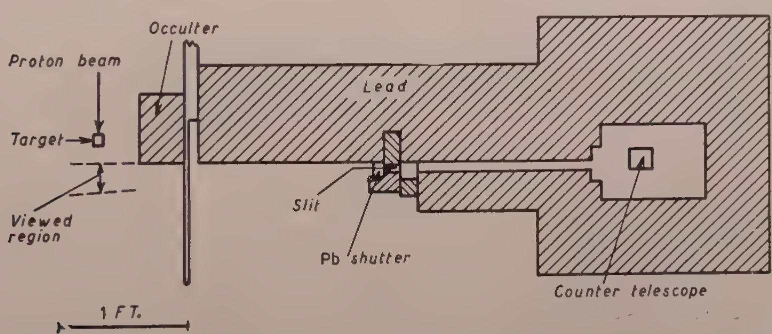


Fig. 2-3·5. – Details of the apparatus used by RIDGWAY *et al.* [39].

The γ-telescope consisted of four scintillation counters and of a Čerenkov counter disposed as indicated in Fig. 3. Gammas converted into electron pairs

in a $1\frac{1}{4}$ inch thick lead plate could produce a 4-fold coincidence in the S_2-S_3-S_4-C counters. The anticoincidence counter S_1 placed before the converter ensured that the particle triggering the telescope was neutral, while the Čerenkov C selected only relativistic particles from a background of other charged pro-

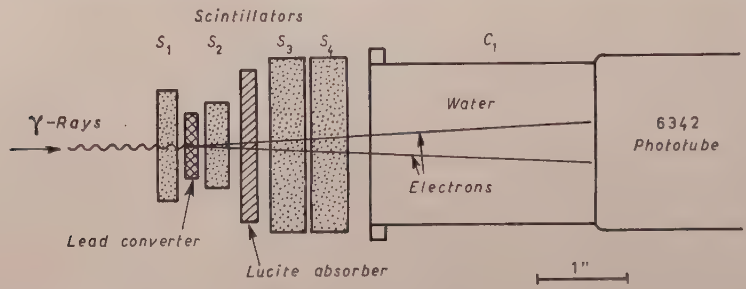


Fig. 3-3'5. – The counter telescope placed inside the apparatus in Fig. 2.

ducts of disintegrations produced in the converter or in the first layers of S_2 by neutrons. The amount of matter to be traversed by the electrons created in the converter fixed a minimum energy for detectable γ-rays at 10 MeV.

In these conditions the Authors felt confident that most of the unwanted background was eliminated.

The relevant point of their result is that the intensity of the γ-rays which their apparatus recorded when the « viewed » region was at a distance 1.3 cm from the target, begins to rise above the negligible value of the background at a primary energy of ~ 1 GeV (see Fig. 1-**18**'1). This (see Sect. **17**'1) agrees very well with the expected threshold energy for associated production of heavy mesons and hyperons induced by nucleons. In fact, of all the reactions induced by nucleons, predicted by the theory, the one with the lowest threshold is

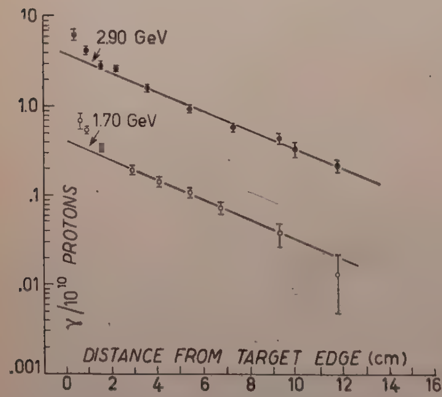
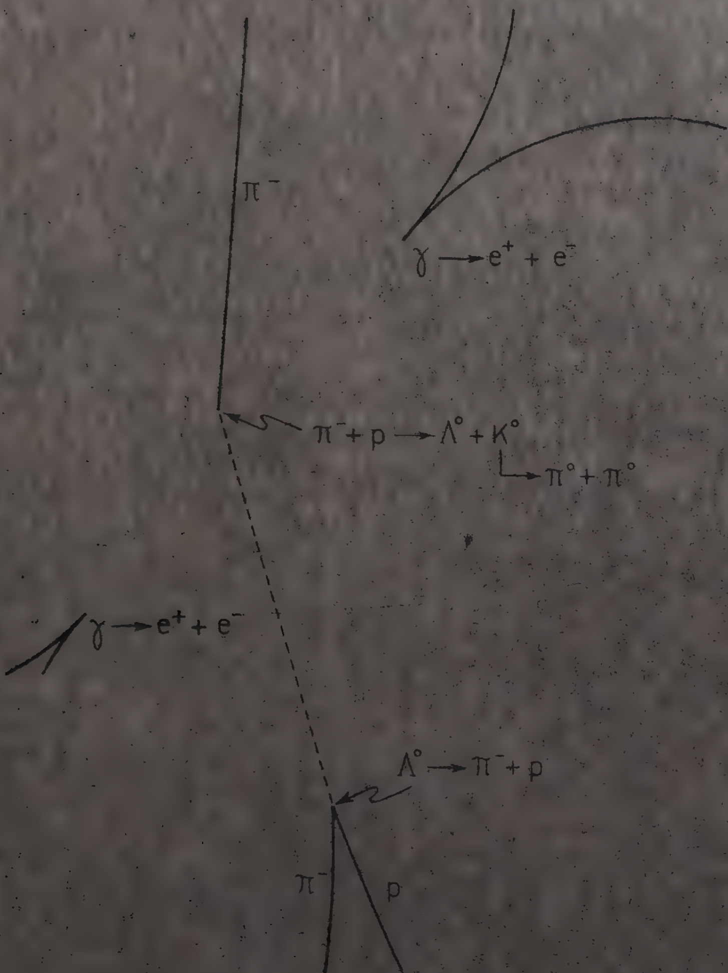


Fig. 4-3'5. – Spatial dependence of the γ-ray intensity as observed by RIDGWAY *et al.* (See text).

$$N + N \to K + \Lambda^0 + N ,$$

which requires a primary energy of about 1.1 GeV.

They also measured the γ-yield as a function of the distance d for two different primary energies (1.7 and 2.9 GeV). Disregarding the points for $d \leqslant 2$ cm, which are certainly affected by directly produced radiations, the



π^-
$\gamma \rightarrow e^+ + e^-$
$\pi^- + p \rightarrow \Lambda^0 + K^0$
$\rightarrow \pi^0 + \pi^0$
$\gamma \rightarrow e^+ + e^-$
$\Lambda^0 \rightarrow \pi^- + p$
π^-
p

Fig. 6-3·5. – Evidence favouring the existence of the neutral decay mode $K^0_{\pi^0 2}$. The event has been interpreted as due to associated production of a Λ^0 and a K^0 in a $\pi^- + p$ collision. The K^0 subsequently has decayed into π^0's from which four γ's must have originated. Two of them have materialized into electron pairs in the chamber. (Courtesy of Prof. J. STEINBERGER).

distributions indicates a decay constant of ~ 0.24 and ~ 0.263 cm^{-1} respectively (Fig. 4). The interpretation of these curves relies however on the *assumed* energy spectrum of the unstable particles which are believed to be responsible for the γ-rays. Consequently, no definite proof can be given that the observed intensity is necessarily due to the $K^0_{\pi^0 2}$ mode of decay.

A similar experiment has been performed recently by OSHER *et al.* [40] at Berkeley over a wider range of energies ($0.8 \div 6.2$ GeV). They measured the γ-intensity (see Fig. 5) from both sides of the target, i.e. both upstream and downstream with respect to the direction of the proton beam. In the diagram, the position of the target is centred at $d = 0$. The prominent peak is due to γ's directly produced or emitted by directly produced π's. The two tails have been interpreted as due to $K^0_{\pi^0 2}$ and $\Lambda^0_{n\pi^0}$ decays (see Sect. **4**·2). For primary protons having energies within the quoted range, Λ^0's or heavier products could not be emitted in the upstream direction in the laboratory system. It is significant that the downstream intensity indicates two components of different lifetime whilst upstream only one is visible.

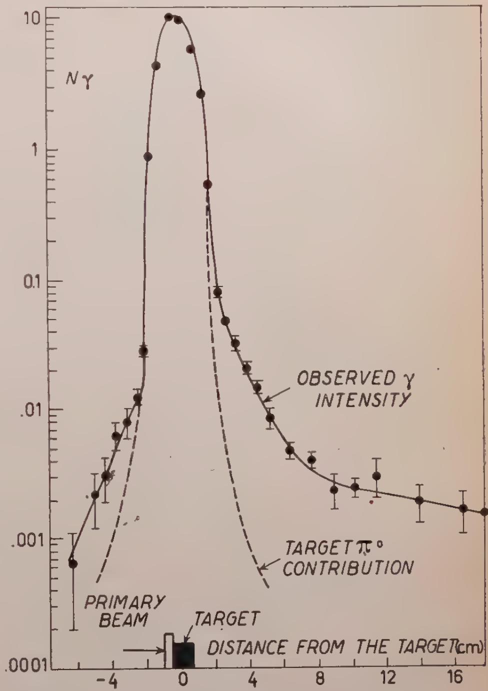


Fig. 5-3·5. – γ-ray intensity produced on both sides of the target by 6.5 GeV protons, as observed by OSHER *et al.* (See text).

The analysis of this curve indicates that the $K^0_{\pi^0 2}$ particle may be the source of radiation responsible for the component with the shorter life, while the longer lived one is consistent with the Λ^0 particle (compare Sect. **4**·1.4).

The above evidence, however strong, is circumstantial. Decisive and independent evidence has been recently obtained by SCHWARTZ *et al.* [35] in the experiment to which reference has already been made in connection with the determination of the $K^0_{\pi 2}$ mean life (see Sect. **3**·4.5). They start from the assumption that production of K^0 particles takes place only in association with another heavy unstable particle as predicted by theory and confirmed by experience. Whenever an unaccompanied Λ^0 is observed they *assume* that a K^0 has also been produced in the same parent interaction. If the momentum of the Λ^0 can be determined and the production can be pro-

ved to have originated from a reaction

$$\pi^- + p \rightarrow \Lambda^0 + K^0 ,$$

then the momentum of the K^0 is univocally determined and the K^0 can be searched for. In 6 of the cases in which an unaccompanied Λ^0 was observed, an electron pair was seen in the picture whose line of flight intersected that of the unseen K^0 (determined as indicated above): in another case two γ's are seen, both satisfying this rule. Two of these cases are reproduced in Fig. 6 and 7. The probability of a chance coincidence is estimated by SCHWARZ *et al.*: they found that not more than one of the 8 pairs could be accidentally in the right direction with the right energy.

This is a strong evidence in favour of a neutral mode of decay of the K^0 but it does not tell us yet either the decay mode or its relative frequency of occurrence. To this purpose the same authors examined the energy distribution of the observed γ's (Fig. 8). Decays such as $K^0 \rightarrow 2\gamma$ or $K^0 \rightarrow \pi^0 + \gamma$ could be rejected immediately since the first would produce γ's of a unique energy (246 MeV) and the second a peak at 229 MeV containing $\frac{1}{3}$ of the events, superimposed on a flat distribution ranging from 17 to 246 MeV.

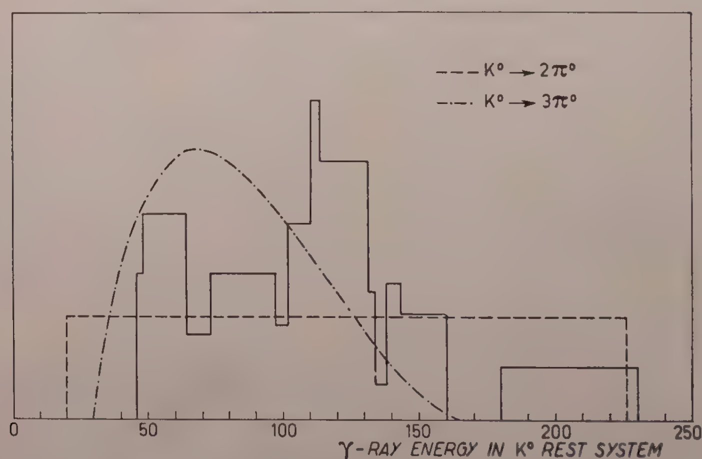
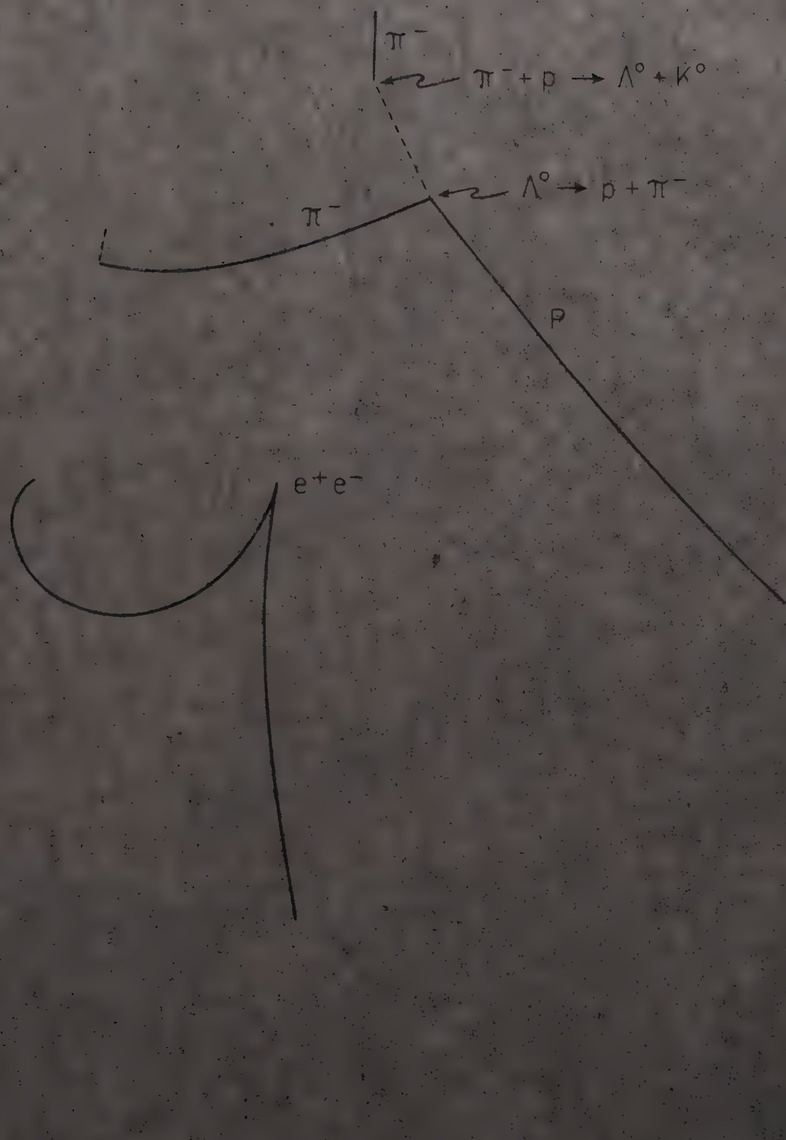


Fig. 8-3·5. – Distribution in energy of 8 γ-rays from neutral K^0 decays. (After SCHWARTZ *et al.* [35]).

The expected distributions for $K^0_{\pi^0 2}$ and $K^0_{\pi^0 3}$ are shown in Fig. 8. It was concluded that probably the $K^0_{\pi^0 2}$ is the right choice in view of the fact that if a $K^0_{\pi^0 3}$-mode existed, also the corresponding mode $K^0_{3\pi} \rightarrow \pi^+ + \pi^- + \pi^0$ having comparable frequency of occurrence should exist. This was not observed.

On the basis of these conclusions, the frequency of occurrence of the neutral



π⁻
π⁻ + p → Λ° + K°
Λ° → p + π⁻
π⁻
p
e⁺e⁻

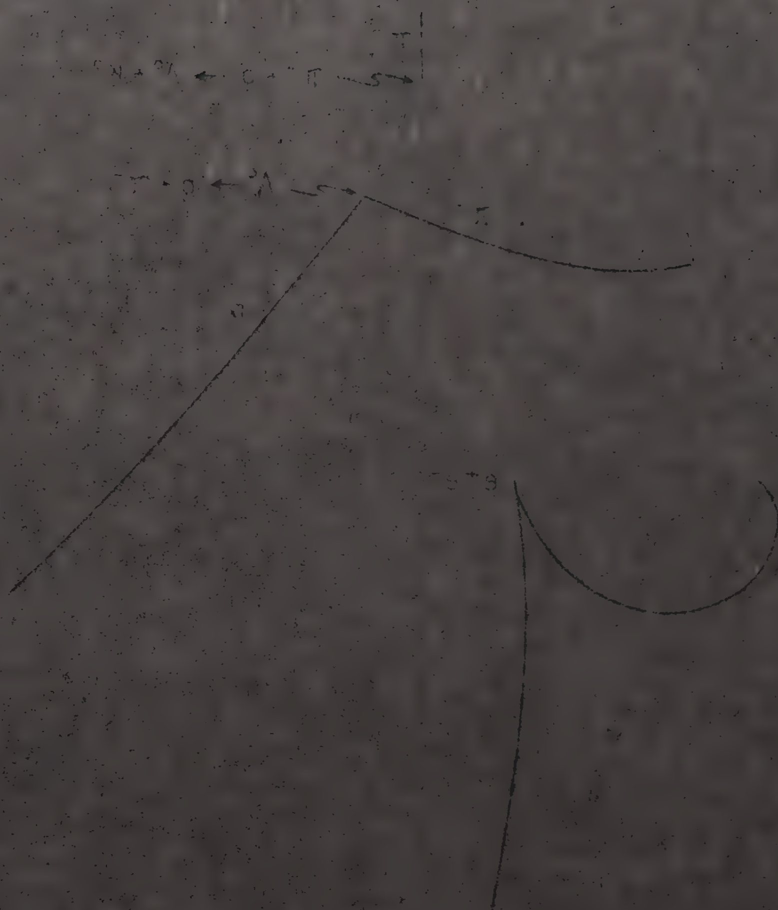


Fig. 7-3·5. – Additional example interpreted as an evidence for the neutral decay of a K^0 into π^0's. See caption under Fig. 6. (Courtesy of Prof. J. Steinberger).

mode $K^0_{\pi^0 2}$ could be calculated from the observed number of γ's. It was found

$$f = \frac{R(K^0_{\pi^0 2})}{R(K^0_{\pi 2})} = 0.14 \pm 0.06\,,$$

$R(K^0_{\pi 2})$ indicating here the *total* number of both the $(\to \pi^+ + \pi^-)$ and $(\to \pi^0 + \pi^0)$ modes.

3·6. – Three body decays.

In Sect. 3·2 we have already mentioned the possible existence of K^0 decay different from the $K^0_{\pi 2}$ or $K^0_{\pi^0 2}$. We may add here that a large number of K^0 events *not interpretable in terms of the* $K^0_{\pi 2}$-mode (henceforth called « anomalous ») were observed in various laboratories; in fact about 10% of the cosmic ray K^0-decays was « anomalous » (Sect. 3·7).

Interest in such « anomalous » cases increased in view of a theory by Gell-Mann and Pais (Chapter 14) which predicted in fact the existence of two K^0 components having different lifetime: the longer-lived K^0's (called in the following K^0_2) should decay, according to the theory, exclusively by anomalous modes; the short lived component (K^0_1) should decay predominantly by the $K^0_{\pi 2}$-$K^0_{\pi^0 2}$ modes and, only in a very small fraction, by « anomalous » modes.

3·6.1. *Existence of a long lived component.* – To decide this issue LANDÉ *et al.* [18] exposed their 36 in. magnet cloud chamber to the neutral radiation produced by the 3 GeV proton beam of the Brookhaven accelerator at a convenient distance from the target to avoid $K^0_{\pi 2}$ or Λ^0. The experimental arrangement is indicated in Fig. 1. Secondary particles created in a copper target and emitted inside a narrow cone at 68° to the beam direction were selected by means of a thick Pb collimator. A $4 \cdot 10^5$ gauss magnet deflected the charged particles through a sufficient angle to prevent them from reaching the cloud chamber which was placed on the path of the neutral ones 6 m from the target.

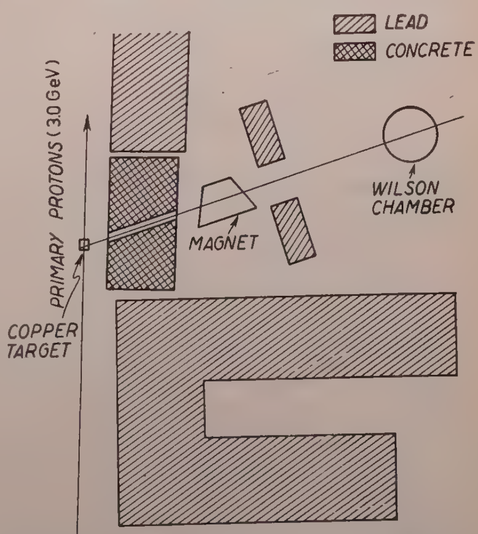


Fig. 1-3·6. – Experimental arrangement employed by LANDÉ *et al.* [18] for revealing K^0_2 decays.

This distance was sufficient to ensure that practically no particle

with a mean life around 10^{-10} s—produced by the Cosmotron—would reach the chamber.

The results of the observations was in line with predictions. In 1200 pictures 23 V^0 events were observed, none of which could be interpreted as a $K^0_{\pi 2}$ (or a Λ^0). None of the secondaries, in fact, could be a proton and all but three were certainly lighter than K-particles. *All but one were « non-coplanar »* with the line of flight of the neutral beam: moreover, a decay mode of the type $\Lambda^0 \to \binom{\mu\pm}{e\pm} + e^{\mp} + \text{neutron}$ could be excluded.

The lifetime of these events could only be estimated within distant limits: $3\cdot 10^{-9} < \tau < 10^{-6}$ s.

The identification of the various decay modes will be discussed in the next paragraph. Here we would like to mention another result pertinent to the question of the lifetime only.

Reference has been already made in Sect. **3**'4.5 to the bubble chamber experiment of SCHWARTZ *et al.* [35]. We add here that only in 40% of the cases the production of a Λ^0 by a π^- colliding in the propane bubble chamber was seen to be accompanied by a K^0 decaying into charged particles inside the chamber. This 40% becomes (49 ± 7.5)% when one corrects for losses of events in examining the photograms and for the estimated number $K^0_{\pi^0 2}$ decays (Sect. **3**'5). This figure shows that if associated production is always true, (51 ± 7.5)% of the K^0 produced in association with the Λ^0 are missing. It is plausible to regard this fact as a strong evidence of long lived K^0_2 (compare also Sect. **17**'3). Combining the results of [35] with those of [19] it is now possible to bracket the mean life of the K^0 between the values $3\cdot 10^{-8} \div 10^{-7}$ s.

3'6.2. *The modes of decay of the long lived component.*

i) *The* $K^0_{\beta 3} \to \pi^{\pm} + e^{\mp} + \nu$ *mode.* – Prior to the work of LANDE *et al.* [18, 19] the existence of this mode had been proved by a number of observations due to COWAN [15], HARMON [17], BLOCK *et al.* [14], D'ANDLAU *et al.* [16]. Their evidence is based on six events, listed in Table I. In 100 K^0_2 decays LANDE *et al.* [19] identified four cases associated with a π^+e^- and two with a π^-e^+

TABLE I-3'6. – *Details on the early* K^0 *events.*

Ref.	Source of radiation	Detector	Nature of the charged secondaries		Q^* (MeV)
[14]	Cosmotron	Diff. Ch.	?	e^-	18 ± 4
[14]	»	»	e^+	$< 500\ m_e$	48 ± 5
[14]	»	»	?	e^-	$\geqslant 62$
[15]	Cosm. Rays	Mag. Cl. Ch.	π^+	p^- or e^-	225 ± 20
[16]	»	»	π^+	e^-	$83 \sim 110$
[17]	»	Mult. Cl. Ch.	$\pi^{\pm}$ or $\mu^{\pm}$	$e^{\mp}$	$350 \sim 223$

pair. As for the nature of the neutral particle, evidence that it probably has a negligible mass came from the transverse momentum distribution (see Fig. 2). All the 300 secondaries are plotted in the same graph—the momenta of the neutral products being determined by the knowledge of those of the charged ones and of the primary K^0 line of motion. The distribution is extended up to about 230 MeV. If one restricts the analysis to consider only those modes of decay which are known to have a corresponding counterpart among the charged K's, the momenta above 190 MeV/c can only be due to the mode indicated above. The agreement between the expected theoretical distribution—as given by statistical factors only—and the experimental data is better seen in the inset graph of Fig. 2. Confirming evidence for this mode came from a more detailed dynamical analysis of some events selected among those having secondaries above 190 MeV/c.

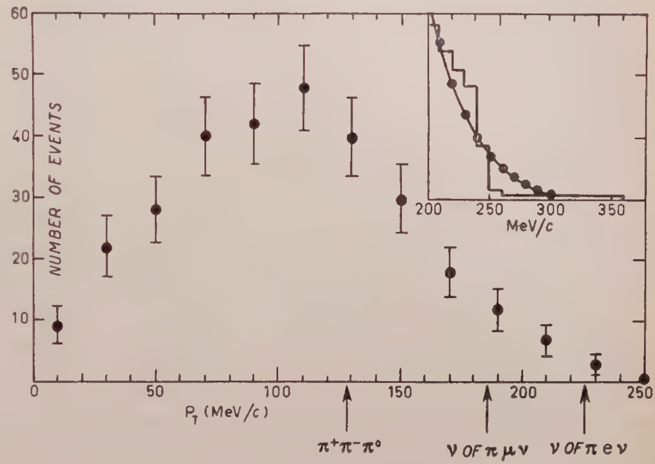


Fig. 2-3·6. – Transverse momentum distribution of charged and uncharged secondaries from 100 K-three-body-decays, measured by LANDÉ *et al.* [19]. Arrows indicate the maximum momenta for secondaries involved in the various decay modes.

ii) *The* $K^0_{\pi 3} \rightarrow \mu^\pm + \pi^\mp + \nu$ *mode.* – Evidence also for this mode has been reported by LANDÉ *et al.* [19], who identified one $(\pi^-\mu^+)$ pair among the decays seen in their chamber. Indirect support was received from the shape of the transverse momentum distribution (Fig. 2).

iii) *The* $K^0_{\pi 3} \rightarrow \pi^+ + \pi^- + \pi^0$ *mode.* – Of 19 cases consistent with this mode one was found for which both charged secondaries could be identified as π-mesons. For the remaining 18, LANDÉ *et al.* were able to show that their distribution of the $Q^*(\pi, \pi)$ (see Sect. 9·2) was in good agreement with that deduced from statistical factors only. They concluded that if the other modes were responsible for an appreciable proportion of these 18 events, the above suggested distribution would have been «enriched» prevalently by high Q^*'s and this was not the case.

3·7. – The « anomalous » events observed in the cosmic radiation. Can they be explained in terms of the K^0-modes known to exist at present?

We have already mentioned that $\sim 10\%$ of the K^0 decays in cosmic radiation events are clearly inconsistent with the $K^0_{\pi 2}$ mode. Such « anomalous » cases have been reviewed in more than one occasion by different authors [21, 34, 41]. At present the situation appears to be as follows:

39 of such events have been reported, 28 observed in cloud chamber (Indiana 4 [26], Cal Tech 18 [44], Berkeley 1 [42], Padua-Göttingen 1 [45], Princeton 7 [42], Paris 7 [42]) and 2 in emulsion [12, 13].

Although the nature of the products is not known with certainty in most of the cases observed in cloud chambers, if they are interpreted as π's the corresponding $Q^*(\pi, \pi)$-values are spread over an interval ranging from 6 to ~ 300 MeV. In a $(1/P, \alpha, p_T)$-representation the events are scattered over a large volume and do not seem to be associated with a single decay surface. This suggests that if they are to be interpreted as a homogeneous group they are due to a decay mode involving at least three secondaries.

The distribution of the α's is found to be almost symmetric with respect to $\alpha = 0$, a fact which might indicate approximately equal masses for the charged fragments. (See THOMPSON [21]). However an almost symmetric distribution can be due to asymmetric decays when:

a) The Q's (and consequently the M's) of the selected particles are much larger than the rest mass of the decay products (in this case the apparent $\alpha^* = (m_1^2 - m_2^2)/m^{*2}$ is always small regardless of the values of $m_1^2 - m_2^2$).

b) The two fragments have equal chance of being positive or negative. The expected distribution of points would be the superimposition of two asymmetric distributions symmetrically placed with respect to the origin. Then as long as the statistics are poor it would be difficult to analyze it.

Since little information is available on the nature of the decay products, an analysis of these events can only be made by comparing them with one, or more, known or assumed decay scheme. BALLAM *et al.* [46] (*) considered: 1) $K^0_{\pi 3}$, 2) $K^0_{\mu 3}$, 3) $K^0_{\beta 3}$ and 4) $K^0_{\pi 2,\gamma} \to \pi^+ + \pi^- + \gamma + 212$ MeV suggested by GELL-MANN and PAIS [43].

For each mode the « phase space » energy distribution of the neutral secondary (see Sect. **9**·2) was calculated and compared with the experimental data. On the basis of 27 events till then published, they [46] concluded that

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(*) A very illuminating discussion, which the writers had with dr. G. T. REYNOLDS in connection with this problem, is gratefully acknowledged.

the « anomalous » events could be due to a mixture of all those schemes. In particular:

i) some of the events, but not all, may be due to scheme (1) (*). In fact only 15 of them are kinematically consistent with it; moreover the energy distribution obtained from these events indicates that not even all the 15 cases are likely to be due to (1);

ii) a large fraction of the 27 events is consistent with scheme (2) and (4). (2) seems less likely for particles of spin zero. With regard to (4) the expected distribution of $Q^*(\pi, \mu)$, based on statistical factors only, is not inconsistent with the experimental one;

iii) also (3) is acceptable and probably is responsible for most of them.

REFERENCES (Ch. 3)

[1] G. D. Rochester and C. C. Butler: *Nature*, **160**, 855 (1947).
[2] A. J. Seriff, R. B. Leighton, C. Hsiao, E. W. Cowan and C. D. Anderson: *Phys. Rev.*, **78**, 290 (1950).
[3] R. Armenteros, K. H. Barker, C. C. Butler, A. Cachon and A. H. Chapman: *Nature*, **167**, 501 (1951).
[4] R. W. Thompson, A. V. Buskirk, L. R. Etter, C. J. Karzmark and R. H. Rediker: *Phys. Rev.*, **90**, 329 (1953).
[5] R. Armenteros, K. H. Barker, C. C. Butler and A. Cachon: *Phil. Mag.*, **42**, 1113 (1951).
[6] R. W. Thompson, A. V. Buskirk, L. R. Etter, C. J. Karzmark and R. H. Rediker: *Phys. Rev.*, **90**, 1122 (1953).
[7] H. S. Bridge, Ch. Peyrou, B. Rossi and R. Safford: *Phys. Rev.*, **91**, 362 (1953).
[8] G. T. Reynolds: *Congrés sur le Ray Cosm., Bagneres de Bigorre* (1953), mim. rep. p. 35.
[9] L. Leprince-Ringuet: *The Sixth Rochester Conf.* (1956), mim. rep. p. V-4.
[10] M. Di Corato, C. C. Dilworth and L. Scarsi: *Suppl. Nuovo Cimento*, **12**, 285 (1954).
[11] D. Hirschberg: as quoted by [23].
[12] M. Yasin: *Phil. Mag.*, **46**, 413 (1954).
[13] D. Lal, Yash Pal and B. Peters: *Phys. Rev.*, **92**, 438 (1953).
[14] M. M. Block, E. M. Harth, M. E. Blevins and G. G. Slaughter: *Nuovo Cimento*, **4**, 46 (1956).
[15] E. W. Cowan: *Phys. Rev.*, **94**, 161 (1954).
[16] C. D'Andlau, R. Armenteros, A. Astier, H. C. De Staebler, P. Gregory, L. Leprince-Ringuet, F. Muller, Ch. Peyrou and J. H. Tinlot: *Nuovo Cimento*, **4**, 916 (1956).

(*) This is in agreement with the recent data obtained by the Pasadena group [45], not included in Ballam *et al.*'s analysis.

[17] N. F. Harmon: *Ph. D. Dissertation*, as quoted by [14].
[18] K. Lande, E. T. Booth, J. Impeduglia, L. M. Lederman and W. Chinowski: *Phys. Rev.*, **103**, 1901 (1956).
[19] K. Lande, L. M. Lederman and W. Chinowski: circulated preprint, Feb. 1957.
[20] J. Podolanski and V. R. Armenteros: *Phil. Mag.*, **45**, 13 (1954).
[21] R. W. Thompson: *Progress in Cosmic Ray Physics*, vol. III.
[22] R. W. Thompson, H. O. Cohn and R. S. Flum: *Phys. Rev.*, **83**, 175 (1951).
[23] K. H. Barker: *Proc. Roy. Soc.*, A **221**, 326 (1954).
[24] W. B. Fretter, M. M. May and M. P. Nakada: *Phys. Rev.*, **89**, 168 (1953).
[25] D. B. Gayther: *Phil. Mag.*, **45**, 570 (1954).
[26] R. W. Thomson, J. R. Burwell and R. W. Huggett: *Suppl. Nuovo Cimento*, **4**, 286 (1956).
[27] K. M. Crowe: *The Avogadro Conference on Fundamental Constants* (Turin, Italy, Sept. 1956) and *Nuovo Cimento*, **5**, 541 (1957).
[28] W. H. Arnold, W. Martin and H. W. Wyld: *Phys. Rev.*, **100**, 1549 (1955).
[29] H. Blumenfeld, E. T. Booth, L. M. Lederman and W. Chinowsky: *Phys. Rev.*, **102**, 1184 (1956).
[30] W. B. Fretter, E. W. Friesen and A. Lagarrigue: *Suppl. Nuovo Cimento*, **4**, 569 (1956).
[31] D. I. Page: *Phil. Mag.*, **46**, 103 (1955).
[32] P. G. Astbury: as quoted by Gayther [27].
[33] I. C. Gupta, N. Y. Chang and A. L. Snyder: circulated preprint.
[34] C. Ballario, R. Bizzarri, B. Brunelli, A. De Marco, E. Di Capua, A. Michelini and G. C. Moneti: *The Avogadro Conference on Fundamental Constants* (Turin, Italy, Sept. 1956). Further results recently obtained have been added.
[35] M. Schwartz, J. Steinberger, R. Plano and N. Samios: in press on the *Nuovo Cimento*.
[36] M. Deutschmann: as quoted by Gayther [27].
[37] I. C. Gupta, A. L. Snyder and N. Y. Chang: *Bull. Am. Phys. Soc.*, **1**, 186 (1956).
[38] G. B. Collins: *The Fifth Rochester Conference* (1955), p. 139.
[39] S. L. Ridgway, D. Berley and G. B. Collins: in press on the *Phys. Rev.*
[40] J. E. Osher, B. J. Moyer and S. I. Parker: UCRL 3300. See also the *Sixth Rochester Conference* (1956), p. V-32.
[41] P. J. Astbury: *Nuovo Cimento*, **12**, 317 (1954).
[42] G. T. Reynolds: Lecture held at Rome University (March 1956).
[43] M. Gell-Mann and A. Pais: *Phys. Rev.*, **97**, 1387 (1955).
[44] J. A. Kadyk, G. H. Trilling, R. B. Leighton and C. D. Anderson: circulated preprint.
[45] M. Deutschmann, M. Cresti, W. D. B. Greening, L. Guerriero, A. Loria and G. Zago: *Nuovo Cimento*, **3**, 180 (1956).
[46] J. Ballam, M. Grisaru and S. B. Treiman: *Phys. Rev.*, **101**, 1438 (1956).

CHAPTER 4.

The Λ^0 Particles.

Reference has been made in Sect. 3·1 to a number of experiments which indicated the existence of two types of neutral V's. We shall examine now those V^0 decaying into a positive fragment of protonic mass and a negative lighter fragment. We may anticipate the result of the ensuing analysis by saying that the existence is now well established of a neutral particle designated as Λ^0 which decays according to the scheme

$$\Lambda^0 \to p + \pi^- + (36.9 \pm 0.2)\ \text{MeV}\,.$$

Recently, evidence has been reported for the alternative mode of decay

$$\Lambda^0 \to n + \pi^0 + Q\,,$$

where Q is to be expected to be (40 ± 0.2) MeV. The existence of other modes of decay is not excluded. However, there are clear indications that, if they exist, they are much less frequent than those known ($\lesssim 2\%$; [1]).

4·1. – The $\Lambda^0 \to p + \pi^-$ mode.

4·1.1. *Summary of evidence on the nature of the charged Λ^0 decay products.* – The first convincing indications for a Λ^0 was presented by the Manchester group [2] (*) in the experiment already discussed in Sect. 3·1.

Almost contemporaneously THOMPSON *et al.* [4] observed two V^0's in which the masses of both particles could be determined and found to be close to

(*) A similar suggestion had been made earlier by HOPPER and BISWAS [3] on the basis of a forked track observed in an emulsion. The identity of the tracks was established but they could not exclude that the event was due to a nuclear disintegration, produced by a neutron, in which only two visible particles were emitted.

that of a proton—for the positive—and to that of a light meson—for the negative one. Similar findings were reported by LEIGHTON *et al.* [5], ARMENTEROS *et al.* [6]; additional evidence was subsequently obtained in [7-12]. The main points of this evidence can be summarized as follows:

1) The heavy positive fragment was found to possess, in the majority of the cases which permitted a direct measurement, a mass very close to that of the proton. No evidence of its instability was found: on the contrary an event was reported [10] consisting of a V^0 whose heavy product stopped in the gas of the chamber without producing any visible disintegration.

2) Amongst the negative fragments only π-mesons could be identified. COURANT (as quoted by BRIDGE *et al.* [11]) measured the interaction mean free path of the secondaries from V^0-decays (containing both neutral hyperons and neutral K's) and obtained a value close to the geometric mean free path, thus proving that μ-mesons could not be present in a large proportion amongst them. No definite case of negative secondary of protonic mass was reported.

3) Extensive mass measurements on secondaries mainly of Λ^0's were made by LEIGHTON *et al.* [7] and their results are shown in Fig. 1. They clearly indicated the presence of π-mesons and protons. Although the shape of these curves may well be explained as due to experimental errors in momentum measurements, the presence of particles other than π and p could not be excluded.

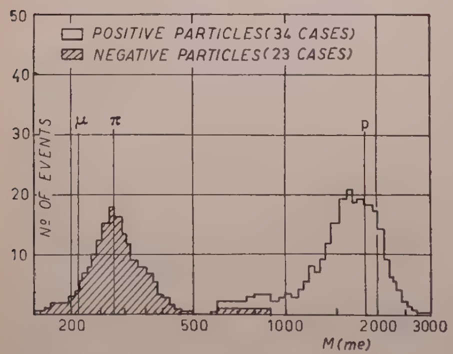


Fig. 1-4·1. – Distribution of the values of the masses of positive and negative Λ_0 secondaries, determined by LEIGHTON *et al.* [7].

Subsequently a number of observations on Λ^0's in emulsion were reported [13-20]. The results obtained by this method are, under many aspects, complementary to those obtained in cloud chamber experiments.

Nuclear emulsions allow mass measurements on individual tracks far more precise than those obtained in cloud chambers. On the other hand the magnetic field is in general of little avail in connection with emulsion work and so the sign of the particles can only be deduced by studying their interaction properties when at rest, whenever this is possible. The origin of a Λ^0 can seldom be traced since Λ^0's live sufficiently long to travel several centimetre. This means, of course, that the line of flight of the V^0's observed can only be determined on the basis of momentum measurements on the secondaries. Finally, an extremely large number of two-branch stars is present in emulsions exposed to high

energy radiation, which can simulate very well a Λ^0 decay. Thus, care has to be taken in selecting the events and undoubtely an element of uncertitude exists, especially if new types of events are to be investigated.

The existence of the Λ^0 was nevertheless confirmed by

1) direct mass estimates on the secondaries of a number of two branch stars, by the scattering and grain counting method;

2) observations on the terminal part of the tracks produced by them, which showed that the fragment of protonic mass was stable and the light one was interacting strongly with matter. Cases in which both fragments stopped in the emulsion were reported [19].

4·1.2. *Evidence for a two-body decay.* – Proof that the spontaneous disintegrations of the neutral particles discussed above, do not involve any neutral secondary, has been obtained as for the K^0 in two indipendent ways: i) by showing that the plane of the secondaries contains the origin of the primary unstable particle, whenever this can be individuated; ii) by showing that the energy release associated with the charged particles is the same for all the events or at least that the individual Q-values are grouped around a single value.

It has also been shown that: iii) if a neutral particle is emitted, this must be one which does not decay into « visible » particles nor interact « visibly » with matter. Let us examine the evidence for the above points:

i) BRUECKNER and THOMPSON [21] have calculated (in an approximate way), the probability distribution of the angle δ, defined as the angle between the plane containing the two charged secondaries and the direction of motion of the neutral primary, for three body decays of a neutral hyperon according to the schemes:

$$\text{(I)} \qquad V^0 \to p + \pi^- + \nu\,,$$

$$\text{(II)} \qquad V^0 \to p + \pi^- + \pi^0\,.$$

It was also assumed that the energy distribution amongst the products was determined by statistical factors only and that the primary V^0 was sufficiently fast so that its velocity and direction were approximately those of the proton. They obtained for these processes the curves reproduced in Fig. 2. The abscissa is measured in terms of the quantity $\gamma\beta\delta$ where $\gamma\beta$ is the reduced momentum of the primary V^0. $\gamma\beta\delta$ is approximately proportional to the transverse component of the momentum of the neutral secondary. The Q-values selected for the two schemes were such as to give an apparent Q^* (see Sect. **9·2**)

consistent with the experiments, supposing that the events of te htype (I) or (II) were erroneously interpreted as two body decays.

The graph shown in Fig. 2 clearly indicates that, even when the neutral secondary is as massive as a π^0, the « uncoplanarity » to be expected is not very large: $4 \sim 5$ degrees on the average.

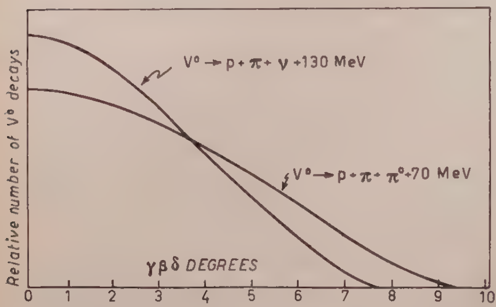


Fig. 2-4·1. – The « uncoplanarity » to be expected for two assumed V^0 decay schemes calculated by BRUECKNER and THOMPSON [21]. $\gamma\beta$ is the reduced momentum of the parent V^0 and δ is the angle between the plane of the charged secondary prongs and the line of motion of the V^0.

Experimental errors in angular determinations are not much smaller in general, and only very precise measurements have to be considered to draw any conclusion. In Fig. 3 (*a*), (*b*) the $\gamma\beta\delta$-distribution obtained by BRIDGE *et al.*[11] is shown, together with Brueckner and Thompson's theoretical curves corresponding to the schemes (I) and (II) (*). In computing the $\gamma\beta\delta$-values plotted in Fig. 3*a*, each event was assumed to be due to a two body decay and the transverse momentum of the proton—which the Authors could not estimate with high precision—was taken as equal to that of the meson. For three body decays, one should reckon the momentum of the hypothetical neutral secondary, but as long as this is not heavier than a π^0, its contribution to the determination of $\gamma\beta\delta$ is on the average small and can be neglected without serious consequences. For Fig. 3*b*, it was assumed that the protons had the maximum momentum consistent with the observations (+). Therefore the values so deduced were in general higher than the true ones; this could however only make the experimental distribution broader than it really was.

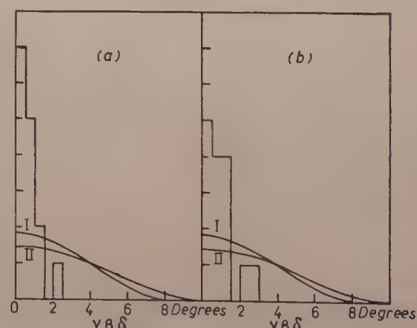


Fig. 3-4·1. – Experimental distribution in the values of $\gamma\beta\delta$ measured by BRIDGE *et al.* [11]. Brueckner and Thompson's theoretical curves are reported for comparison.

Inspection of the two graphs shows

(*) The mass of the primary V^0 was in each case taken of such a value which would make the Q^* identical with the average Q obtained from the experiments.

(+) The momentum determination was based on ionization and residual range measurements.

that in either case ((a) or (b)) the experimental distribution is definitely sharper than expected for a 3-body decay. Analogous results were obtained by GUPTA *et al.* [22].

ii) Under the same assumptions BRUECKNER and THOMPSON calculated also the Q^*-distribution, intended as the distribution of the Q-values which one would obtain interpreting as two-body decays events of type (I) or (II) (*) (Fig. 4). The comparison with the experiments meets here with the same sort of difficulty mentioned in connection with the coplanarity check, in view of the errors in the determination of the Q's which sometimes are as large as — 20 MeV. Furthermore, should a number of discrete Q-values exist, associated with the same secondary particles and separated by a difference not much larger than the individual errors, then the experimental results may well simulate a continuous distribution. In this respect the possibilities in emulsion work are more ample since the determination of the energy of the two secondaries can be made with a precision of less than a few MeV.

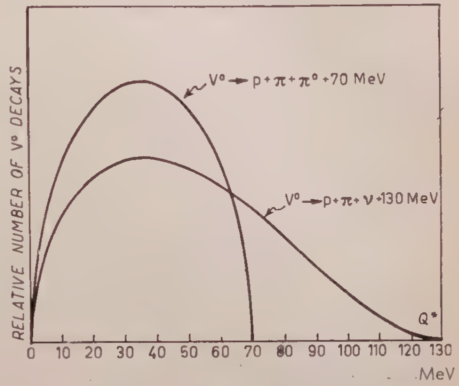


Fig. 4·4·1. – $Q^*(p, \pi)$ distributions for the two processes indicated in the graph, calculated by BRUECKNER and THOMPSON (see text).

Cloud chamber evidence for the existence of a unique Q-value in the region between 34 and 42 MeV was conclusively obtained by THOMPSON *et al.* [23], BRIDGE *et al.* [11], ARMENTEROS *et al.* [2], VAN LINT *et al.* [24] (see Fig. 5), BLUMENFELD *et al.* [25]. Their observations include a total of more than 100 events.

Emulsion observations have been reported by various people, as quoted in paragraph 4·1.1. The Q distribution relative to fifty events reported at the Padua Conference 1954 [26]

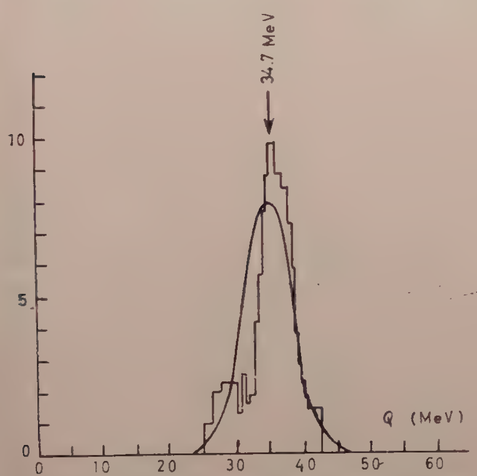


Fig. 5-4·1. – Q-distribution obtained by VAN LINT *et al.* [24] for 19 events. The normalized Gaussian curve corresponds to a probable error of $\pm$ 2.5 MeV.

(*) See footnote (*) at pag. 80.

is shown in Fig. 6: it clearly indicates the existence of a prominent peak on a background of events scattered from 0 to 130 MeV, most probably due to neutron induced 2-prong stars.

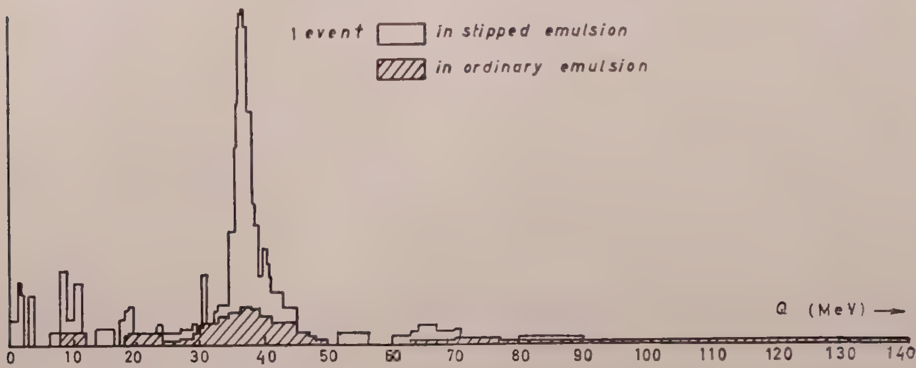


Fig. 6-4·1. – Q-distribution for 50 (p, π^-)-pairs identified in emulsion [26].

iii) Search for direct evidence of a neutral particle emitted in the decay has been carried out by ARMENTEROS *et al.* [6], DEUTSCHMANN [27], FRETTER *et al.* [8], BRIDGE *et al.* [11]. In cloud chambers containing lead plates the presence of γ-rays or π^0's can be revealed through the electronic showers associated with the observed decays. No such event has in fact been observed.

In conclusion, all available evidence indicates the existence of a neutral hyperon decaying into a proton and a π^- only. The existence of other modes of decay of the same hyperon, associated with the emission of two *charged* particles—at present—cannot be excluded. Recent bubble cham-

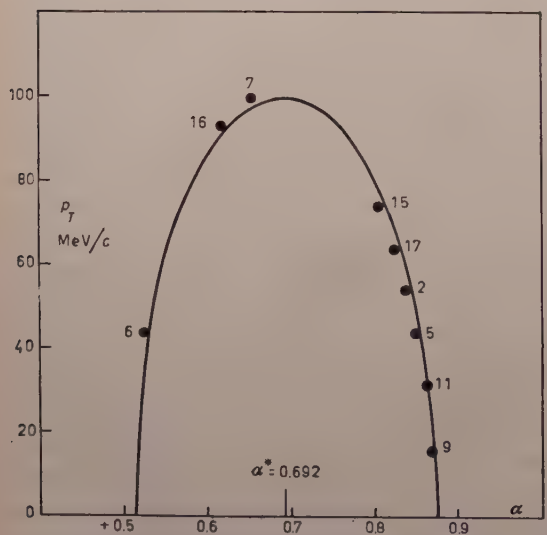


Fig. 7-4·1. – Thompson's Q-plot of FRIEDLANDER *et al.*'s [19] data on Λ^0 [30]. The distribution of the events on the ellipse is noticeably different from that of the cloud chamber observations. THOMPSON (*l. cit.*, page 293) suggested that a strong bias against small α's was introduced by the method of selection based on scanning for π^- stars.

TABLE I-4·1.

Ref.	Source of radiation	Type of detector	Selection criterion	Q (MeV)
[9]	Cosmic rays	Mag. Cl. Ch.	Decays associated with a heavily ionizing proton-like fragment.	42^{+3}_{-2}
[28]	»	»	Decays identified on the basis of the Q-values.	36 ± 1
[25]	Cosmotron (Brookhaven)	»	Decays whose positive secondary had an ionization twice minimum. Selection was made on the basis of Q as determined by momenta and angles.	36 ± 1
[11]	Cosmic rays	Multipl. Cl. Ch.	Decays in which one of the prongs had π-mesonic mass and the other was definitely heavier than a π-meson. Mass estimates were based on ionization and residual range measurements.	37 (most probable value)
[29]	»	Mag. Cl. Ch.	Decays whose positive secondaries were identified as protons by momentum and droplet counting measurements.	35.9 ± 1
[7]	»	»	Decays in which mass estimates, based on momentum and ionization measurements yielded for the secondaries $m_+ > 700\ m_e$. Only $Q < 50$ MeV were selected.	35 ± 3
[30, 23]	»	»	Decays identified by comparison with calculated Q-curves in a α-p_T-diagram. All selected events satisfied the condition $\delta Q \leqslant 7$ MeV, δQ being the experimental error in Q.	37 ± 1
[24]	»	»	Decays associated with a heavily ionizing positive secondary whose mass was consistent with the proton mass and for which the probable error in Q was less than $\pm$ 5 MeV.	34.7 ± 1
[19]	»	Emulsion	Two prong stars whose secondaries were identified as a π^- and a proton respectively. The value of Q was calculated on 10 events whose measured Q resulted between 34 and 40 MeV.	36.9 ± 0.2

ber experiments [1] indicate however that if they exist their relative abundance is less than $\sim 2\%$ with respect to the (p, π^-) mode.

4·1.3. *The mass and the Q-values of the* Λ^0. – In Table I a series of measurements of Q-values for the $\Lambda^0 \to p + \pi^-$ mode is reported.

A weighted average, based on those associated with the smallest error [24, 25, 28-30], yields, for the cloud chamber data, a Q-value of

$$Q_{\text{Cl. Ch.}} = (35.9 \pm 1)\ \text{MeV}$$

and consequently a mass of

$$M_{\Lambda^0} = (2\,179.4 \pm 2.5)\ m_e\ .$$

Very precise measurements in emulsion work have been performed by FRIEDLANDER *et al.* [19]. The result which is quoted in Table I, was obtained on a number of cases selected for having their Q between 34 and 40 MeV. The average Q on these events was found to be

$$Q_{\text{em}} = (36.9 \pm 0.2)\ \text{MeV}\ ,$$

where the error is only statistical. Accordingly we get

$$M_{\Lambda^0} = (2181 \pm 1)\ m_e\ .$$

4·1.4. *The mean life of the* Λ^0. – The experimental methods used for the determination of the mean life of Λ^0's are the same as those used for the $K^0_{\pi 2}$'s.

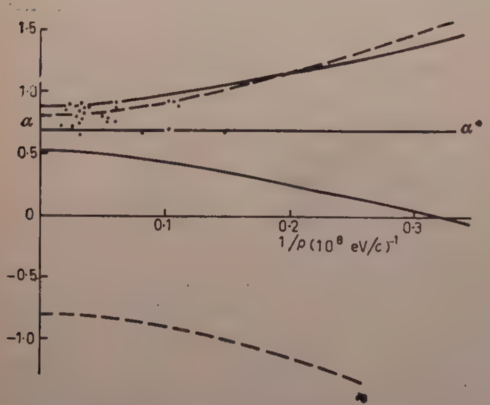


Fig. 8-4·1. – The $(\alpha\text{-}(1/P))$ projection for the events of PAGE and NEWTH. The limiting hyperbolae are those of the Λ^0 (full lines) and $K^0_{\pi 2}$ (dotted lines) decays (see text).

Technical details concerning this type of measurements have been described already in Subsect. 3·4.5.

To what extent the events considered in the early experiments form samples « uncontaminated » by K^0 is difficult to say. The determination by ALFORD and LEIGHTON [31] is based on events discussed by LEIGHTON *et al.* in a previous work [5]. The Q distribution reported by them was spread from 0 to 150 MeV and the Authors looked for systematic differences in lifetime between those having a $Q > 50$ MeV and those having a $Q < 50$ MeV. They

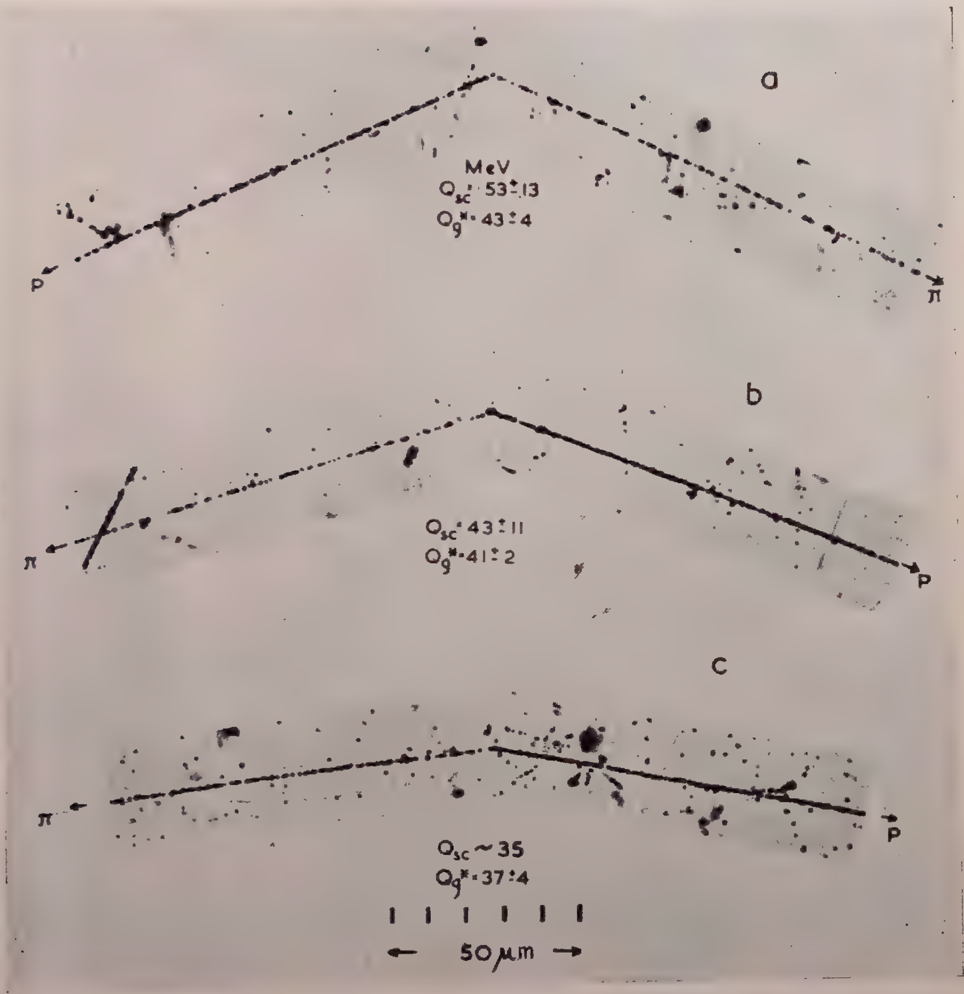


Fig. 9-4·1. – Microphotographs of Λ^0 decays observed in nuclear emulsion [18]. The measured Q-values—obtained from scattering (Q_{sc}) and ionization (Q_{g*}) measurements—are reported underneath each event.

obtained $\tau_H = (1.6 \pm 0.5)\cdot 10^{-10}$ s for $Q > 50$ MeV and $\tau_L = (2.9 \pm 0.8)\cdot 10^{-10}$ s for $Q < 50$ MeV whilst, from all of them $\tau = (2.5 \pm 0.7)\cdot 10^{-10}$ s. They concluded that the difference $\tau_H - \tau_L$ was not significant. BRIDGE *et al.* [11] considered only slow events identified as Λ^0 by the fact that one of the products had a mesonic mass and the other was heavier than a π and obtained $\tau = (3.5 \pm 1.2)\cdot 10^{-10}$ s. PAGE and NEWTH [32] selected 26 events for which the ratio p_+/p_- between the measured momenta of the secondaries was equal or larger than 3. Also the measured quantities p_+, p_- and ionization had to be consistent with each event being a Λ^0-decay. However in most cases only a lower limit for p_+ could be given and the determination of P was then based on the assumption that the event was due to a Λ^0; this procedure may have introduced some unwanted K^0's amongst the events considered. In Fig. 8 the (α-$1/P$)-distribution of the events is reproduced. From that expected for K^0-decays and from the observed number of K^0's associated with values of $\alpha \leqslant -0.5$ (a region which is forbidden to Λ^0's) it is possible to estimate the contamination of K^0's. These Authors reckon that ~ 3 K^0's were introduced in their sample, i.e. $\sim 12\%$ and conclude that their results cannot be seriously distorted. The same method of selection was also used by PAGE [33].

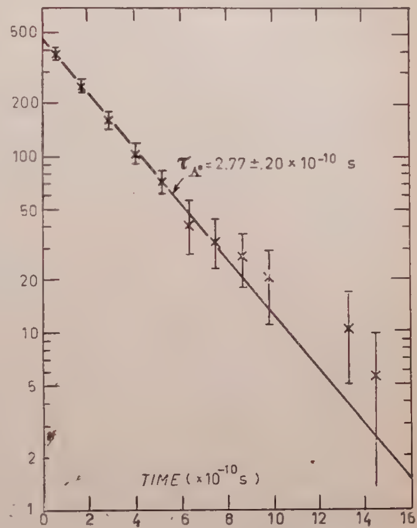


Fig. 10-4·1. – Decay curve for Λ^0's observed by SCHWARTZ *et al.* [1] in a bubble chamber experiment. It is based on 304 events.

ALVAREZ *et al.* [40] analyzed the decay distribution of Λ^0 produced in a hydrogen bubble chamber by slow K$^-$ interacting with protons. The experiments by GAYTHER [34], GUPTA *et al.* [22], BALLARIO *et al.* [35], BLUMENFELD *et al.* [25], SCHWARTZ *et al.* [1] have already been discussed in Subsect. 3·4.5.

Of all those listed in Table II the result of SCHWARTZ *et al.* [1] has the greatest statistical weight. They find

$$\tau_{\Lambda^0} = (2.8 \pm 0.2)\cdot 10^{-10} \text{ s}.$$

The weighted average calculated in all the others listed in Table II is

$$\tau_{\Lambda^0} = (3.4^{+0.4}_{-0.3})\cdot 10^{-10} \text{ s},$$

a value higher than that of [1] but not inconsistent with it. All values of

Table II combined give a best value of

$$\tau_{\Lambda^0} = (3.05 \pm 0.15)\cdot 10^{-10}\,\mathrm{s}.$$

TABLE II-4·1. – Λ^0-*mean life measurements.*

Reference	Source of radiation	Detector	No. of observed events	Mean life 10^{-10} s
[31]	Cosm. rad.	Magnet Cl. Chamb.	63	2.5 ± 0.7
[35]	»	Multiplate »	25	$2.14^{+0.8}_{-0.5}$
[25]	Cosmotron	Magnet »	65	$2.8\ ^{+0.3}_{-.02}$
[11]	Cosm. rad.	Multiplate »	21	3.5 ± 1.2
[36]	»	» »	22	$4.8\ ^{+2.6}_{-1.3}$
[34]	»	» »	21	$4.0\ ^{+3.7}_{-1.2}$
[22]	»	» »	52	$3.6\ ^{+0.8}_{-0.7}$
[33]	»	Magnet »	23	$3.6\ ^{+1.1}_{-0.7}$
[32]	»	» »	26	$3.7\ ^{+3.9}_{-1.3}$
[40]	Bevatron	» H-Bubble Chamb.	25	3.25 ± 0.6
[1]	Cosmotron	» C_3H_8-Buble Chamb.	304	2.8 ± 0.2

4·2. – The $\Lambda^0 \to n + \pi^0$ mode. The branching ratio $f_\Lambda = R(\Lambda^0 \to n + \pi^0)/R(\Lambda^0)$.

Reference has been already made (Sect. 3·5) to a number of experiments for the detection of γ-rays arising from the neutral decay of heavy unstable particles [37-39, 1]. For the first three [37-39] the interpretation involves a number of assumptions regarding the angular and energy distribution of the unstable particles, supposed to be responsible for the observed γ-intensity. The fourth [1] provides us with the clear evidence of some bubble chamber pictures in which electron pairs are associated with the simultaneous production of a K^0 particle. The dynamics of the events are all consistent with an explanation in terms of associated production of Λ^0-K^0 pairs,

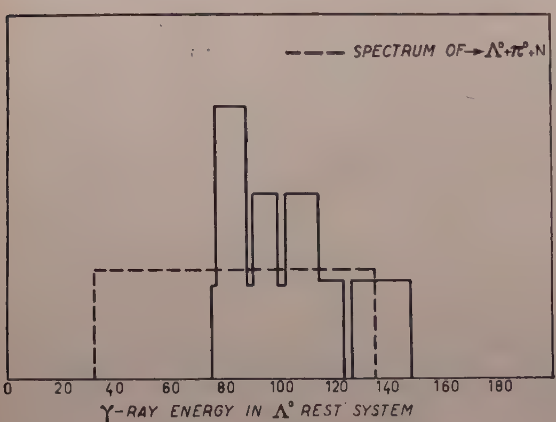


Fig. 1-4·2. – Energy distribution for five γ-rays interpreted as produced in the neutral decay mode of the Λ^0. (According to SCHWARTZ *et al.* [1]).

the Λ^0 undergoing subsequently a neutral decay leading to the emission of γ's.

The number of the observed «associated pairs» reported by [1] is five. Their energy distribution (see Fig. 1) is obviously inconsistent with a decay scheme such as $\Lambda^0 \to n + \gamma$ which would produce monoenergetic γ's of 165 MeV. A scheme $\Lambda^0 \to n + \pi^0$ would produce a flat distribution from 32 to 134 MeV which is not inconsistent with the data.

From the number of associated γ's and of the charged Λ^0-decays observed in the chamber, one can deduce (see Sect. **3**·5) the branching ratio $f = R(\Lambda^0 \to n + \pi^0)/R(\Lambda^0)$ for the relative frequency of occurrence of Λ^0 neutral decays. Alternatively, from the number of observed K^0's unaccompanied by a Λ^0 and from the knowledge of the Λ^0-lifetime, the same ratio can be independently estimated.

Combining the two results the value

$$f = 0.32 \pm 0.05$$

was found.

REFERENCES (CH. 4)

[1] M. SCHWARTZ, J. STEINBERGER, R. PLANO and N. SAMIOS: to be published.
[2] R. ARMENTEROS, K. H. BARKER, C. C. BUTLER, A. CACHON and A. H. CHAPMAN: *Nature*, **167**, 501 (1951).
[3] V. D. HOPPER and S. BISWAS: *Phys. Rev.*, **80**, 1099 (1950).
[4] R. W. THOMPSON, H. O. COHN and R. S. FLUM: *Phys. Rev.*, **83**, 175 (1951).
[5] R. B. LEIGHTON, S. D. WANLASS and W. L. ALFORD: *Phys. Rev.*, **83**, 843 (1951).
[6] R. ARMENTEROS, K. H. BARKAS, C. C. BUTLER and A. CACHON: *Phil. Mag.*, **42**, 1113 (1951).
[7] R. B. LEIGHTON, S. D. WANLASS and C. D. ANDERSON: *Phys. Rev.*, **89**, 148 (1953).
[8] W. B. FRETTER, M. M. MAY and M. P. NAKADA: *Phys. Rev.*, **89**, 168 (1953).
[9] R. ARMENTEROS, K. H. BARKER, M. S. COATES and M. G. SOWERBY: *Phil. Mag.*, **44**, 861 (1953).
[10] D. D. MILLAR and D. I. PAGE: *Phil. Mag.*, **44**, 1049 (1953).
[11] H. S. BRIDGE, CH. PEYROU, B. ROSSI and R. SAFFORD: *Phys. Rev.*, **91**, 362 (1953).
[12] W. B. FRETTER and E. W. FRIESEN: *Phys. Rev.*, **92**, 1089 (1953).
[13] M. DANYSZ: *Bull. d. l'Acad. Pol. Sci.*, **1**, 177 (1953).
[14] A. BONETTI, R. LEVI SETTI and B. LOCATELLI: *Congres Int. Bagnères de Bigorre* (1953), mimeogr. rep. p. 171.
[15] E. AMALDI, C. CASTAGNOLI, G. CORTINI and A. MANFREDINI: *Nuovo Cimento*, **10**, 1351 (1953).
[16] M. TEUCHER: *Congr. Int. Bagnères de Bigorre*, mimeogr. rep. p. 172.
[17] B. PETERS: *Congr. Int. Bagnères de Bigorre*, mimeogr. rep. p. 173.
[18] M. YASIN: *Phil. Mag.*, **45**, 413 (1954).

[19] M. W. Friedlander, D. Keefe, M. G. K. Menon and M. Merlin: *Phil. Mag.*, **45**, 533 (1954).
[20] M. Ceccarelli, M. Grilli and B. Vitale: *Suppl. Nuovo Cimento*, **12**, 305 (1954).
[21] K. A. Brueckner and R. W. Thompson: *Phys. Rev.*, **87**, 390 (1952).
[22] J. C. Gupta, W. Y. Chang and A. L. Snyder: circulated pre-print. See also *Bull. Am. Phys. Soc.*, **1**, 186 (1956).
[23] R. W. Thompson, A. V. Buskirk, L. T. Etter, C. J. Karzmark and R. H. Rediker: *Phys. Rev.*, **90**, 329 (1953).
[24] V. A. J. Van Lint, G. H. Trilling, R. B. Leighton and C. D. Anderson: *Phys. Rev.*, **95**, 295 (1954).
[25] H. Blumenfeld, E. T. Booth, L. M. Lederman and W. Chinowsky: *Phys. Rev.*, **98**, 1203 (1955).
[26] C. Castagnoli, M. W. Friedlander, M. Merlin and M. Teucher: *Suppl. Nuovo Cimento*, **12**, 461 (1954).
[27] M. Deutschmann: *Zeits. Naturfor.*, **7a**, 142 (1952).
[28] W. H. Arnold, W. Martin and H. W. Wyld: *Phys. Rev.*, **100**, 1207 (1955).
[29] E. W. Cowan and A. Guess: as quoted by R. W. Thompson [30].
[30] R. W. Thompson: *Progress in Cosmic Ray Physics*, vol. **3**, ch. 5.
[31] W. L. Alford and R. B. Leighton: *Phys. Rev.*, **90**, 622 (1953).
[32] D. I. Page and J. A. Newth: *Phil. Mag.*, **45**, 38 (1954).
[33] D. I. Page: *Phil. Mag.*, **45**, 863 (1954).
[34] D. B. Gayther: *Phil. Mag.*, **45**, 570 (1954).
[35] C. Ballario, R. Bizzarri, B. Brunelli, A. De Marco, E. Di Capua, A. Michelini and G. C. Moneti: *The Avogadro Conference on Fundamental Particles* (Turin, Italy, Sept. 1956).
[36] M. Deutschmann: as quoted by [32].
[37] G. B. Collins: *The Fifth Rochester Conference* (1955), mimeogr. rep. p. 139.
[38] S. L. Ridgway, D. Berley and G. B. Collins: to be published.
[39] J. E. Osher, B. J. Moyer and S. I. Parker: UCRL 3300 (A).
[40] L. W. Alvarez, H. Bradner, P. Falk Vairant, J. D. Gow, A. H. Rosenfeld, F. T. Solmitz and R. D. Tripp: *Nuovo Cimento*, **5**, 1026 (1957).

CHAPTER 5.

The $\Sigma^{\pm}$-particles.

5·1. – The decay modes of the charged Σ's.

The first example of charged hyperons was found and analysed by the Genoa and Milan groups [1]. A microphotograph of its terminal section is reproduced in Fig. 1. Measurements on the grain density variation and on the scattering indicated that the particle responsible for track J was moving as indicated by the arrow. The end of this track is associated with a lightly ionized track interpreted as due to a fast particle emitted from the point where track J ends.

Direct measurement on the 15 mm long J track yielded for the mass a value of $(2\,500 \pm 345)\,m_e$ thus excluding the possibility of a K^+-decay. Little information could be obtained from the secondary light track which could not be followed through the adjacent emulsions.

The information obtainable from the observed event, although scarce, allowed the discoverers to say that it was most probably due to the decay of an unstable particle of hyperprotonic mass. Subsequent observations [2-4] provided further support for this view and allowed the nature of the secondary particles to be established, identifying them as π-mesons always associated with the same energy. Furthermore, an event was found by the Milan group [5], which was interpreted as due to a particle of mass $(2\,300 \pm 800)\,m_e$ decaying into a charged secondary having a residual range of $1670\,\mu m$ in emulsion. By scattering and grain density measurements its mass was found to be $(2\,030 \pm 500)\,m_e$. Since it did not show any decay product at the end of its range, it was assumed to be a proton of energy (18.7 ± 0.18) MeV. In support of this evidence two similar events were subsequently found, one in Rome [6] and the other in Padua [7], definitely proving the existence of the decay into a proton and a single neutral particle.

The nature of the latter could not be established precisely although on the basis of momentum and energy considerations, it could be deduced that most probably it was a π^0.

The similarity of the mass and Q values obtained for either type of events, strongly suggested that either mode was indeed due to the same type of particle which could decay according to either of the following schemes (if positive)

$$\Sigma^+ \to \pi^+ + n$$

$$\to \pi^0 + p$$

and

$$\Sigma^- \to \pi^- + n$$

if negative.

In the cases reported above, the sign of the charge could be established with certainty only for the protonic decays. Σ's of both signs were observed by FOWLER *et al.* [8] who used a diffusion cloud chamber to analyze the interactions produced by π^- of ~ 3 GeV at Brookhaven laboratory. Fig. 2 shows an event interpreted as the simultaneous creation of a $\Sigma^-(\to \pi^- + n)$ (track a) and K^+ (the latter escaping before decaying). Additional evidence was given by the Turin group [9] who found a hyperon decaying in flight into a π which stopped in the emulsion and interacted with a nucleus.

The large number of events since accumulated have beautifully confirmed these early evidences.

5·2. – The branching ratio $f_{\Sigma^+} = R(\Sigma^+ \to p + \pi^0)/R(\Sigma^+ \to \pi^+ + n)$.

All available data indicate that the ratio f_{Σ^+} is close to unity. FRY *et al.* [10] find $f = 26/22 = 1.18$ and BARKAS *et al.* [11] $f = 13/13$.

5·3. – The masses and the Q-values of the charged Σ's.

Owing to their short lifetime (see Sect. 5·4) Σ particles cannot travel too far from their point of creation. In general observations on them are limited only to those which are created inside (or very close to) the sensitive medium in which they are detected. Consequently determinations based on magnetic analysis and range measurements as those described in connection with K-particles are in this case very difficult and have not been performed.

With emulsions the following methods have been used:

a) Simultaneous measurements of two of the following parameters: scattering, ionization, residual range.

b) Total release of energy associated with the disintegration of the Σ's decaying at rest.

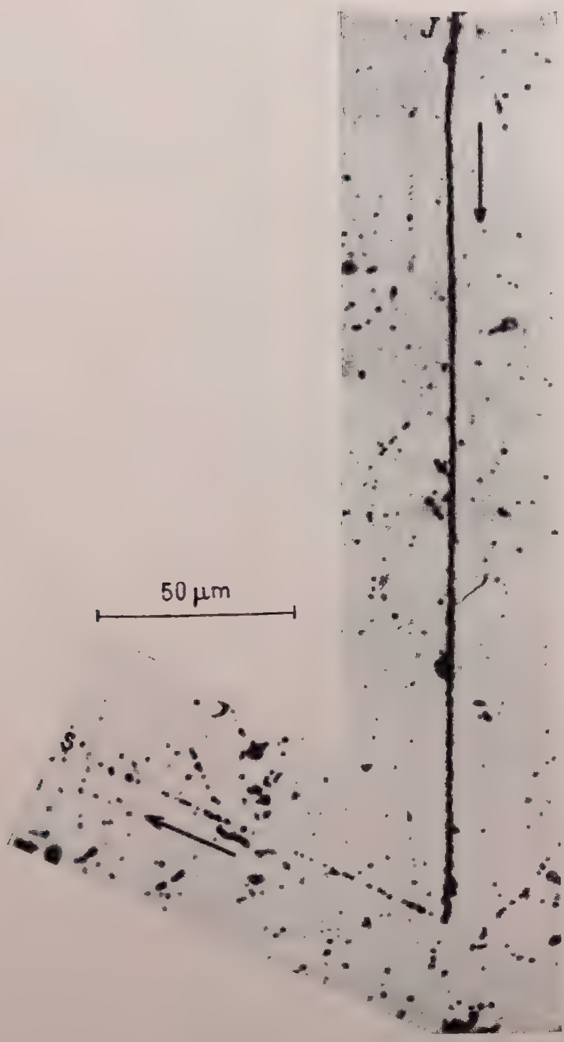


Fig. 1-5'1. – The first example of $\Sigma^+ \to \pi^+ + n$ decay observed in emulsion [1].

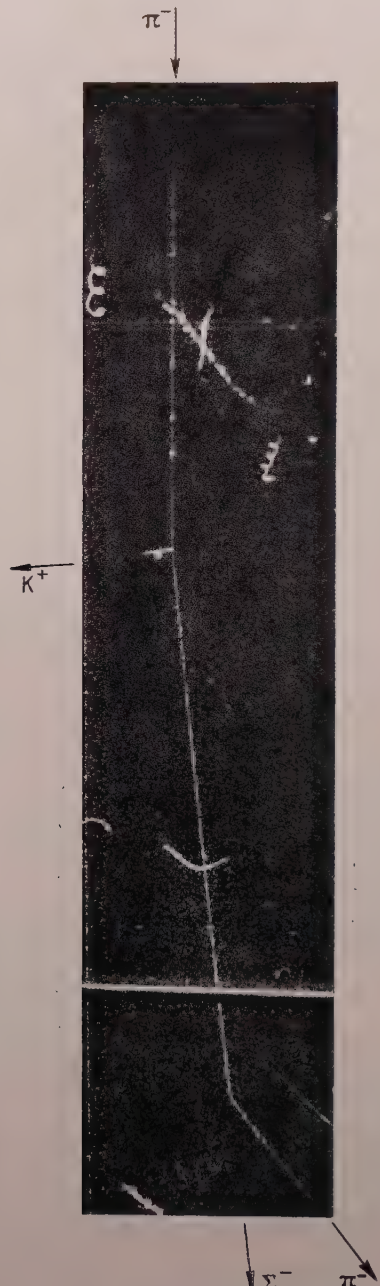


Fig. 2-5·1. – Simultaneous creation of a K^+ and a Σ^- by a 3 GeV π^- in a diffusion chamber. (After Fowler *et al.* [8]).

c) Energy balance in K^- absorption by protons leading to the emission of a charged Σ and of a π of opposite sign, according to the scheme

$$K^- + p \to \Sigma^{\pm} + \pi^{\mp} .$$

Method *a*) is by far the less precise. In very favourable conditions (i.e. a flat track, absence of distortion, even development) a value of the mass can be obtained with a precision of $\pm 5\%$. In general such a precision is not attained and the majority of the results published so far are associated with an error of about 10% or more.

For method *b*) the precision depends, practically, only upon the determination of the energy of the secondary charged particle. The most favourable case is obviously that of the decay of a Σ into a proton and a neutral π^0. The range of the proton being less than 2 mm ((1669.4 ± 8) µm [12]) it will fluctuate around the mean value with a root mean square deviation of $\sim 2\%$ which corresponds to an uncertainty in energy of $\sim 1\%$ on individual events. This can easily be reduced to a negligible amount by averaging on a number of events. However, account is to be taken of other systematic sources of errors such as uncertainty in the range-energy relation and in the density of the emulsion. Averaging over ten events FRY *et al.* [12] are able to give the Q of the disintegration (see Table I) with a precision of $\sim 0.4\%$, from which the mass of the Σ^+ is calculated to 0.04%. Σ^-'s interact strongly with matter and when brought to rest they are captured before they decay. The same method could however be applied to $\Sigma^- \to \pi^- + n$ decays in flight: the determination of the Q then requires the knowledge of the primary velocity at decay which—in general—involves large errors (for example for an event associated with a $\beta_\Sigma \sim 0.3$, $\delta Q/Q \sim 0.2$. The error obviously increases with increasing β_Σ). It may be possible sometimes, to deduce β_Σ from the dynamical analysis of the primary interaction, whenever the latter can be precisely individuated.

Through method *c*) the mass of the Σ^- can be directly deduced from the direct comparison of the ranges of Σ^+'s and Σ^-'s ejected in the reactions:

(i) $$K^- + p \to \Sigma^- + \pi^+ + Q^- ,$$

(ii) $$K^- + p \to \Sigma^+ + \pi^- + Q^+ .$$

Useful information is now beginning to come from bubble chamber experiments. STEINBERGER [13] (see also [14]) has reported the determination of the Q of a $\Sigma^- \to \pi^- + n$ decay in flight observed in a propane bubble chamber. The Σ^- was produced by a 1.3 GeV π^- in a reaction

$$\pi^- + p \to \Sigma^- + K^+ .$$

The π^-, ejected in the subsequent decay of the Σ^-, stopped in the chamber, so that its energy was deduced with high precision from its range.

A summary of the latest mass values is given in Table I. Of the mass values listed here, the last [16] is computed on a number of events found at Berkeley plus others found by the Wisconsin group [15].

TABLE I-5'3. – $\Sigma^\pm$-*mass determinations.*

Ref.	Detector	Method	Mass (m_e)
[12]	Emuls.	from $\Sigma^+ \to p+\pi^0+Q$; $Q = (116.08 \pm 0.47)$ MeV	$M_{\Sigma^+} = 2\,327 \quad \pm 1$
[14, 13]	Bub. Ch.	from $\Sigma^- \to n+\pi^- +Q$ (in flight); $Q = (116 \pm 2.6)$ MeV	$M_{\Sigma^-} = 2\,343 \quad \pm 5.2$
[17]	Emuls.	from i) - ii) (see text); $M_{\Sigma^-} - M_{\Sigma^+} = (14 \pm 6)$ m_e	$M_{\Sigma^-} = 2\,341 \quad \pm 6$
[15]	»	from i) - ii) (see text); $M_{\Sigma^-} - M_{\Sigma^+} = (15.9 \pm 2.9)$ m_e	$M_{\Sigma^-} = 2\,342.9 \;\; \pm 3.2$
[16]	»	from i) - ii) (see text);	$M_{\Sigma^-} = 2\,341.7 \;\; \pm 1.8$
		Average	$M_{\Sigma^-} = 2\,342.06 \pm 1.5$

5'4. – The mean lives of the charged Σ's.

There is a good deal of evidence that the mean life of Σ^+'s and Σ^-'s differ from one another (see Table I). The latest, very precise, results obtained by the bubble chamber group at Berkeley [18] indicate that the negative Σ live about twice as long as the positive ones. Their mean lives were calculated on 27 Σ^+ and 44 Σ^- produced in $K^+ + p$ capture events and decaying in the chamber. The values reported in Table I were obtained by fitting an exponential on the observed time distribution. Possible evidence for the existence of two different mean lives for each variety was sought for but not found.

Within the experimental errors all the other results on identified events agree with them. Results on $\Sigma^\pm$ decays in flight indicate a somewhat smaller value. Cosmic ray data [19-21] may be distorted in an uncontrolled manner by the loss decays (although it is to be noted that the effect should be in the opposite direction since fast $\Sigma^+ \to p + \pi^0$ decays are more often lost than slow ones; such events would appear as a slightly scattered tracks very rarely associated with a noticeable change of ionization). More puzzling is the result of the Wisconsin group [10] who obtains $(0.32^{+0.11}_{-0.07}) \cdot 10^{-10}$ s for $\Sigma^\pm$ in flight,

and $(0.96^{+0.37}_{-0.21})\cdot 10^{-10}$ s for identified $\Sigma^+ \to p+\pi^0$, all ejected in K^--interactions in emulsion. Every non minimum track from them was followed to the end of its range thus selecting—as far as one can see—an unbiassed sample of events. FRY *et al.* estimated that the probability that the difference be only a statistical fluctuation is $\lesssim 1\%$. In either case their procedure was based on the application of the maximum likelihood method (see Sect. 9·3). Considering the great interest which is associated with the precise knowledge of the decay rates, more work will be required to clarify this point.

TABLE I-5·4. – *Charged Σ mean life determinations.*

Ref.	Primary rad.	Detector	Method (*)	τ (10^{-10} s)
$\Sigma^\pm$				
[19]	Cosm. Rays	Emulsion	ML and RT methods; taking Σ-decaying in flight and from rest.	$0.5<\tau<3$
[20]	»	»	ML method - Decays in flight only.	$0.5^{+3.3}_{-0.33}$
[21]	»	»	ML method - Decays in flight only.	$0.35^{+0.15}_{-0.11}$
[10]	K^- abs.	»	ML method - Decays in flight only.	$0.32^{+0.11}_{-0.07}$
Σ^+				
[18]	K^- abs.	H bubble Ch.	Σ^+ decaying in flight.	0.86 ± 0.17
[10]	»	Emulsion	ML method - $\Sigma^+\to p+\pi^0$ in flight and at rest.	$0.96^{+0.37}_{-0.21}$
			ML method - $\Sigma^+\to\pi^++n$ and $\Sigma^+\to p+\pi^0$ only in flight.	$0.47^{+0.44}_{-0.15}$
Σ^-				
[18]	K^- abs.	H bubble Ch.	Σ^- decaying in flight only.	1.83 ± 0.26
[14]	1.3 GeV π^-	C_3H_8 bubble Ch.	Σ^- decaying in flight only.	$1.4^{+1.6}_{-0.5}$
[10]	K^- abs.	Emulsion	From the estimated number of Σ^- in $\Sigma^\pm\to\pi^\pm+n$ decays in flight.	2.5 ± 0.8
[22]	Cosm. Rays	Cl. Ch.	V^-'s which are neither K^- nor Ξ^-.	$1.5^{+0.38}_{-0.25}$

(*) ML = Maximum likelihood method; RT = residual time method (see Sect. 9.3).

REFERENCES (CH. 5)

[1] A. BONETTI, R. LEVI SETTI, M. PANETTI and G. TOMASINI: *Nuovo Cimento*, **10**, 345 (1953).

[2] M. CECCARELLI and M. MERLIN: *Nuovo Cimento*, **10**, 1207 (1953).

[3] B. ROSSI and H. S. BRIDGE: *Congr. Inter. Bagnères de Bigorre*, mim. rep. p. 107.

[4] M. M. SHAPIRO, D. T. KING and N. SEEMAN: *Phys. Rev.*, **93**, 914 (1954).

[5] A. BONETTI, R. LEVI SETTI, M. PANETTI and G. TOMASINI: *Nuovo Cimento*, **10**, 1736 (1953).

[6] C. CASTAGNOLI, G. CORTINI and A. MANFREDINI: *Nuovo Cimento*, **12**, 464 (1954).

[7] B. BALDO, G. BELLIBONI, M. CECCARELLI and B. VITALE: *Suppl. Nuovo Cimento*, **2**, 289 (1954).

[8] W. B. FOWLER, R. P. SHUTT, M. THORNDIKE and W. L. WHITTEMORE: *Phys. Rev.*, **93**, 861 (1954).

[9] A. DEBENEDETTI, C. M. GARELLI, L. TALLONE and M. VIGONE: *Nuovo Cimento*, **12**, 952 (1954).

[10] W. F. FRY, J. SCHNEPS, G. M. SNOW, M. S. SWAMI and D. C. WOLD: to be published.

[11] W. H. BARKAS, W. F. DUDZIAK, P. C. GILES, H. H. HECKMANN, F. W. INMAN, O. J. MASON, N. A. NICKOLS and F. M. SMITH: UCRL 3627.

[12] W. F. FRY, J. SCHNEPS, G. A. SNOW and M. S. SWAMI: *Phys. Rev.*, **103**, 226 (1956).

[13] J. STEINBERGER: *The 6th Rochester Conference* (1956), mimeogr. rep. p. VI-23.

[14] R. BUDDE, M. CHRÉTIEN, J. LEITHER, N. P. SAMIOS, M. SCHWARTZ and J. STEINBERGER: *Phis. Rev.*, **103**, 1827 (1953).

[15] W. F. FRY, J. SCHNEPS, G. A. SNOW, M. S. SWAMI and D. C. WOLD: *Phys. Rev.*, **104**, 270 (1956).

[16] W. W. CHUPP, S. GOLDHABER and F. H. WEBB: UCRL 3584.

[17] W. W. CHUPP, G. GOLDHABER, S. GOLDHABER and F. H. WEBB: UCRL 3044.

[18] L. W. ALVAREZ, H. BRADNER, P. FALK-VAIRANT, J. D. GOW, A. H. ROSENFELD, F. T. SOLMITZ and D. R. TRIPP: *Nuovo Cimento*, **5**, 1026 (1957).

[19] E. AMALDI, C. CASTAGNOLI, G. CORTINI and C. FRANZINETTI: *Nuovo Cimento*, **12**, 668 (1954).

[20] C. DAHANAYAKE, P. E. FRANCOIS, Y. FUJIMOTO, P. IREDALE, C. J. WADDINGTON and M. YASIN: *Nuovo Cimento*, **1**, 888 (1955).

[21] J. H. DAVIES, D. EVANS, P. H. FOWLER, P. E. FRANCOIS, M. W. FRIEDLANDER, R. HILLER, P. IREDALE, D. KEEFE, M. G. K. MENON, D. H. PERKINS and C. F. POWELL: *Suppl. Nuovo Cimento*, **4**, 470 (1956).

[22] G. H. TRILLING and G. NEUGEBAUER: to be published.

Chapter 6.

The Σ^0-particle.

According to the theory of Gell-Mann and Nishijima (see Ch. 11) a unit isotopic spin should be assigned to the Σ particles. This implies that a neutral Σ exists. It was noted that such a particle would decay very rapidly according to the scheme $\Sigma^0 \rightarrow \Lambda^0 + \gamma$ since the latter reaction does not violate the conservation of the strangeness (Sect. **11**·6).

Both decay products being neutral and the mean life of the Σ^0 being so short, such a decay can be proved to have occurred only in those exceptional cases in which the parent interaction is observed together with the visible decay of the Λ^0 and the materialization of the γ into an electron pair and, at the same time, a detailed dynamical analysis of events can be carried out to exclude other explanations.

Already early indications obtained by Fowler *et al.* [1] and Walker [2] consistently favoured the existence of such a particle, although they were not clear cut proof.

Fowler *et al.*'s evidence will be reported in Sect. **17**·2 being more pertinent to the question of the associated production.

The event, observed by Walker [2] in a hydrogen diffusion cloud chamber, lent itself to a rather complete analysis. The disappearance of a 1.1 GeV, π^- track was seen in a picture in which also a Λ^0 and a K^0 were identified. Measurements on the individual momenta of the charged products of latter particles, permitted the Author to establish their direction of motion which was found to pass through the point in which the π^- track disappeared, but indicated, at the same time that the lines of flight of the three particles were not coplanar. Assuming the primary interaction to be

$$\text{(i)} \qquad \pi^- + p \rightarrow \Sigma^0 + K^0$$

and the decay of the Σ^0 to be responsible for the observed momentum unbalance and, moreover, knowing the energy of the primary and of the K^0, the vanished momentum and energy could be calculated. They were found to be (50 ± 30) MeV/c and (47 ± 25) MeV respectively, consistent with the

hypothesis that a Σ^0 was created and subsequently decayed as predicted by the theory.

More definite results have been reported by ALVAREZ *et al.* [3] from the study of K^- and Σ^- interactions with protons in a hydrogen bubble chamber. When a Σ^- is brought to rest in that medium, it is captured by a proton and may produce a Λ^0 through either of the schemes

$$\text{(ii)} \qquad \Sigma^- + p \to \Lambda^0 + n$$

$$\text{(iii)} \qquad \Sigma^- + p \to \Sigma^0 + n \to \Lambda^0 + \gamma + n\,,$$

(ii) will produce monochromatic Λ^0's of 36.9 MeV in energy; (iii) is energetically possible if $M_{\Sigma^-} > M_{\Sigma^0}$; if so, it will produce a continuous spectrum of Λ^0's, whose limits depend on the mass difference $M_{\Sigma^-} - M_{\Sigma^0}$.

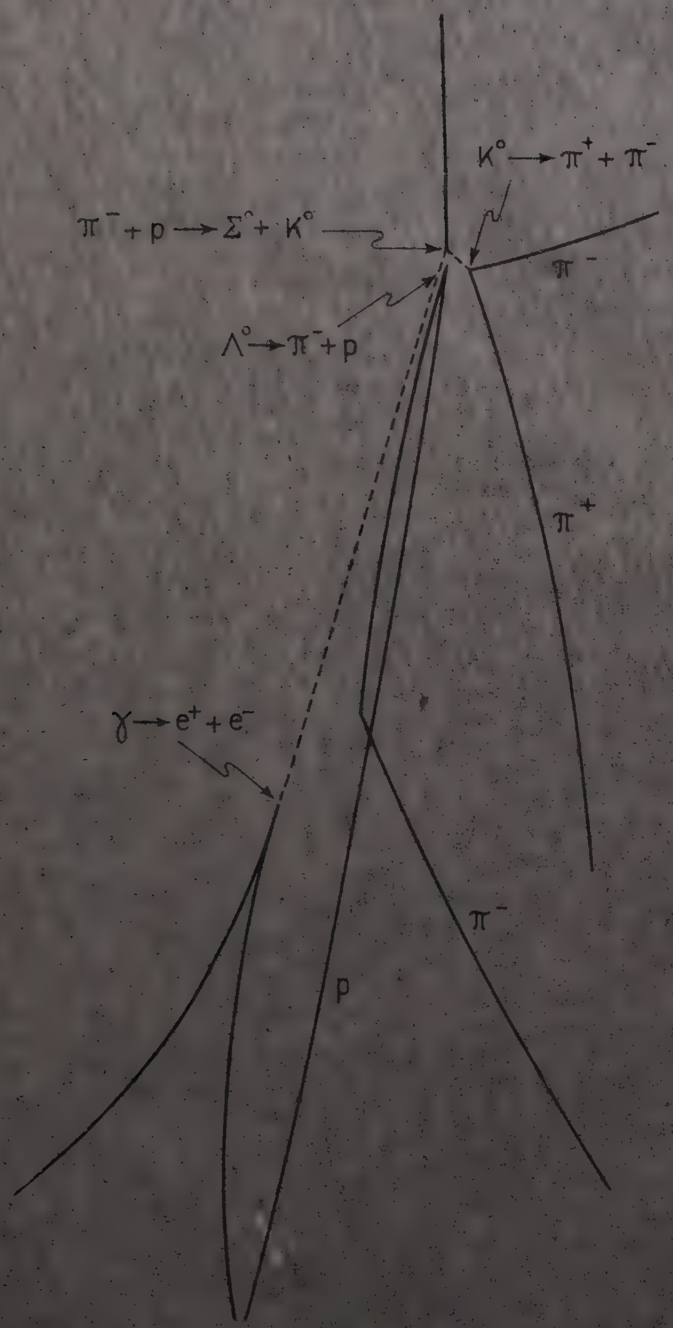
From two cases unconsistent with (ii) and interpretable as (iii) these authors deduce

$$1.7 \leqslant M_{\Sigma^-} - M_{\Sigma^0} \leqslant 22.7 \text{ MeV}\,.$$

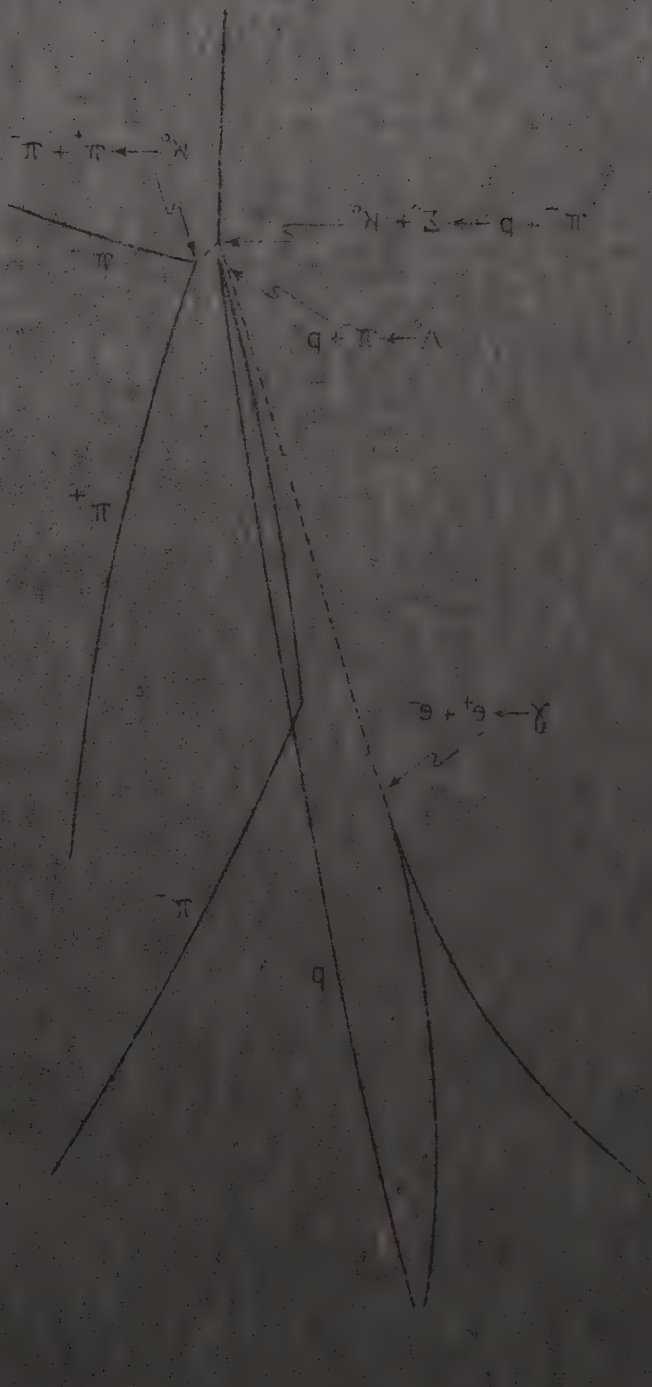
Decisive evidence for the reaction (i) has been recently obtained by PLANO *et al.* [4]. It is based on three propane-bubble-chamber photographs in which the disappearance of an energetic π^- track is associated with the presence of a K^0, a Λ^0 and an electron-positron pair. Two of these events (one of which is shown in Fig. 1) probably resulted from π^- interaction with free protons, and the third with carbon. In the first two the dynamics of the process in their various steps are so precisely defined to allow the determination of more parameters than needed. If the events are interpreted in terms of scheme (i) all the independent angular and momentum determinations are consistent with each other, and exclude other hypothetical explanations. The Q of the Σ^0 —as obtained averaging over the three cases—was (73 ± 3.5) MeV, corresponding to a mass for the Σ^0 of $(2\,323 \pm 7)\,m_e$.

REFERENCES (CH. 6)

[1] W. D. FOWLER, R. P. SHUTT, A. M. THORNDIKE and W. L. WHITTEMORE: *Phys. Rev.*, **93**, 861 (1954).
[2] W. D. WALKER: *Phys. Rev.*, **98**, 1407 (1955).
[3] L. W. ALVAREZ, H. BRADNER, P. FALK-VAIRANT, J. D. GOW, A, H. ROSENFELD, F. T. SOLMITZ and R. D. TRIPP: *Nuovo Cimento*, **5**, 1026 (1957).
[4] R. PLANO, N. SAMIOS, M. SCHWARTZ and J. STEINBERGER: *Nuovo Cimento*, **5**, 216 (1957).



$K^0 \to \pi^+ + \pi^-$
$\pi^- + p \to \Sigma^0 + K^0$
π^-
$\Lambda^0 \to \pi^- + p$
π^+
$\gamma \to e^+ + e^-$
π^-
p



K⁰ → π⁺ + π⁻
π⁻ + p → Σ⁰ + K⁰
Λ⁰ → π⁻ + p
γ → e⁺ + e⁻
π⁺
π⁻
p

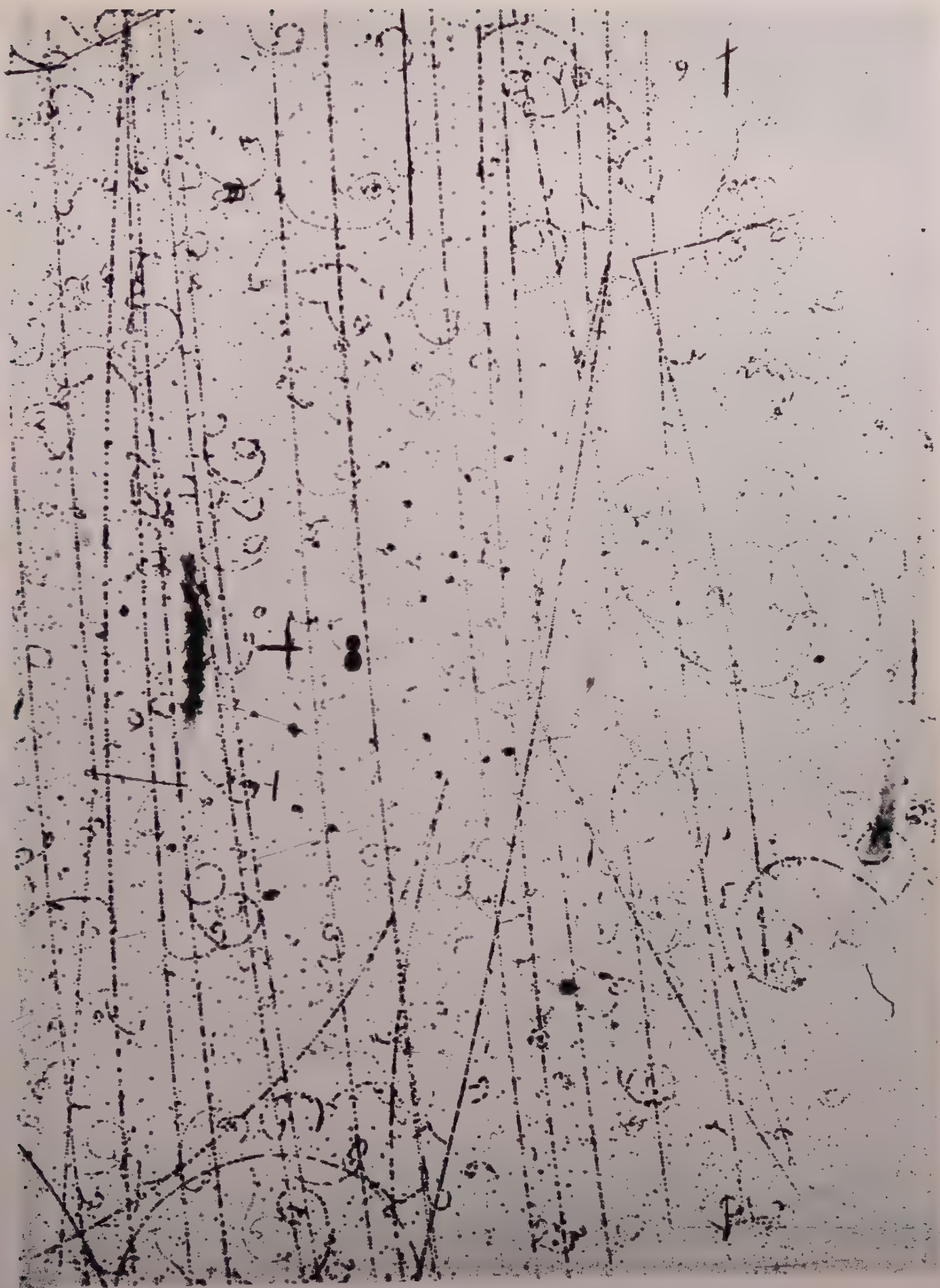


Fig. 2-6. – Simultaneous production of a γ-ray, a Λ^0 and a K^0 in a $\pi^- + p$ interaction. This event has been found consistent only with the assumption that a Σ^0 was created which subsequently decayed into a $\Lambda^0 + \gamma$ [4].

CHAPTER 7.

The Ξ-particles.

7·1. – The decay mode of the Ξ⁻-particles (cascade decays).

The first example of an unstable particle decaying into a V^0 and a light meson was reported by the Manchester group [1]. In a cloud chamber photograph (Fig. 1) they observed an unstable negative particle decaying in flight into a light meson of momentum (73 ± 20) MeV/c. In the same photograph a V^0 cloud be seen, the apex of which was at 1.25 cm from the point of decay of the other particle. Moreover the line of flight of the V^0 passed through the point at which the decay of the negative particle was seen to occur at least within the accuracy allowed by the measurements.

This strongly suggested that the V^0 was generated in the decay of the unstable negative particle. Unfortunately the V^0 could not be identified as a Λ^0 or a K^0 and therefore the value of the mass of the primary particle was uncertain. Assuming that the V^0 was in fact a Λ^0 the Q of the disintegration was found to be ~ 60 MeV and the primary mass $(2500 \div 2800)\, m_e$. This hypothesis was later confirmed by three events found by ANDERSON *et al.* [2] and by the Paris group [3].

Up to the present date 16 examples have been reported, all negative and produced by cosmic rays, which leave no doubt as to the existence of a particle decaying according to the scheme

$$\Xi^- \to \pi^- + \Lambda^0 . \tag{i}$$

No example of positive or neutral Ξ has ever been observed. It is difficult to think why positive Ξ should not be detected, if they existed in comparable number and had a comparable mean life. Ξ^0 would probably decay into all neutral particles and such a decay would be much more difficult to detect.

7·2. – The mass of the Ξ^-.

Details of individual measurements on the 16 events mentioned above are reported in Table I.

Direct mass measurements on the track of the primary have not been possible on the events observed in cloud chamber. The mass can be calculated from the Q-value when the mode of decay can be established.

TABLE I-7·2. – Ξ^- *masses and* Q *determinations.*

Ref.	Detector	Direct mass measurements	Q (MeV)	Remarks
[2]	Cloud. Ch.	—	60 ± 15	Average value from two events
[1]	»	—	~ 60	
[4]	»	—	63 ± 9	
[5]	»	—	67 ± 12	
[6]	»	—	66 ± 6	
[7, 8]	»	—	72 ± 8	
»	»	—	45 ± 16	
»	»	—	63 ± 4	
»	»	—	52 ± 13	
»	»	—	68 ± 7	
	»	—	67 ± 12	
[9]	Emulsion	$2\,500 \pm 900$	72 ± 28	
[10]	»	$2\,640 \pm 300$	71 ± 5	
[11]	»	$2\,500 \pm 500$	63 ± 27	
[12]	»	$2\,200 \pm 250$	59 ± 11	

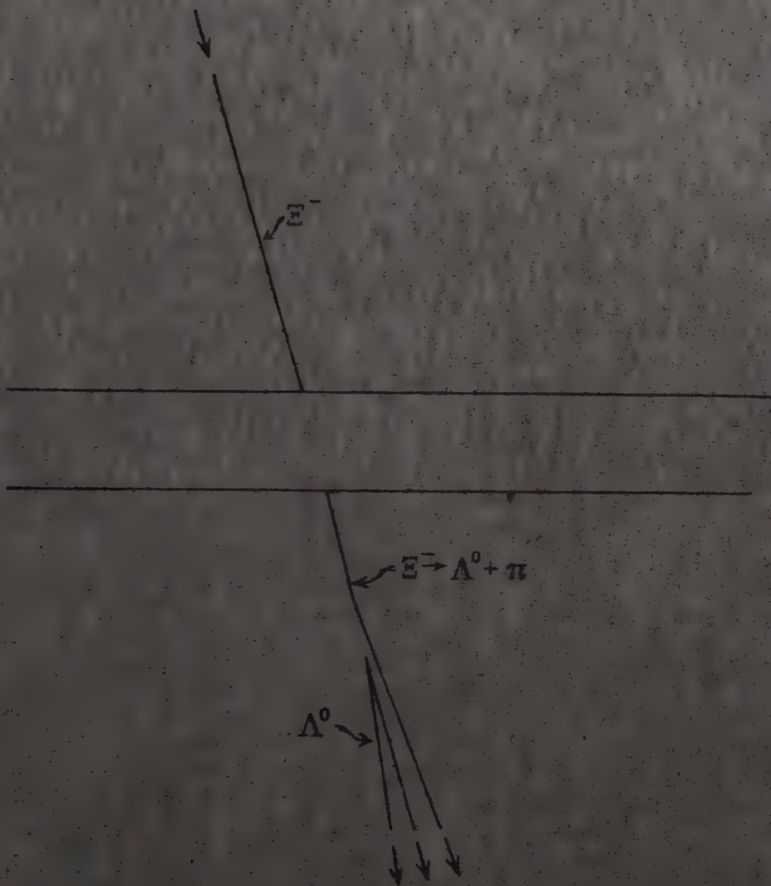
In emulsion, on the contrary, direct estimates are often possible but the values given in Table I indicate that, in general, the uncertainty involved is appreciable ((20 ~ 30)%) and in none of the cases listed there the identity of the primary could be established unambiguously from direct mass measurements only. Since—in addition—in emulsion the associated Λ^0 which is emitted from the disintegration is almost impossible to detect, the identification of the events reported [9-12] was based only on the determination of the Q-value for the assumed scheme (i).

The average value of Q, as obtained from the data of Table I is

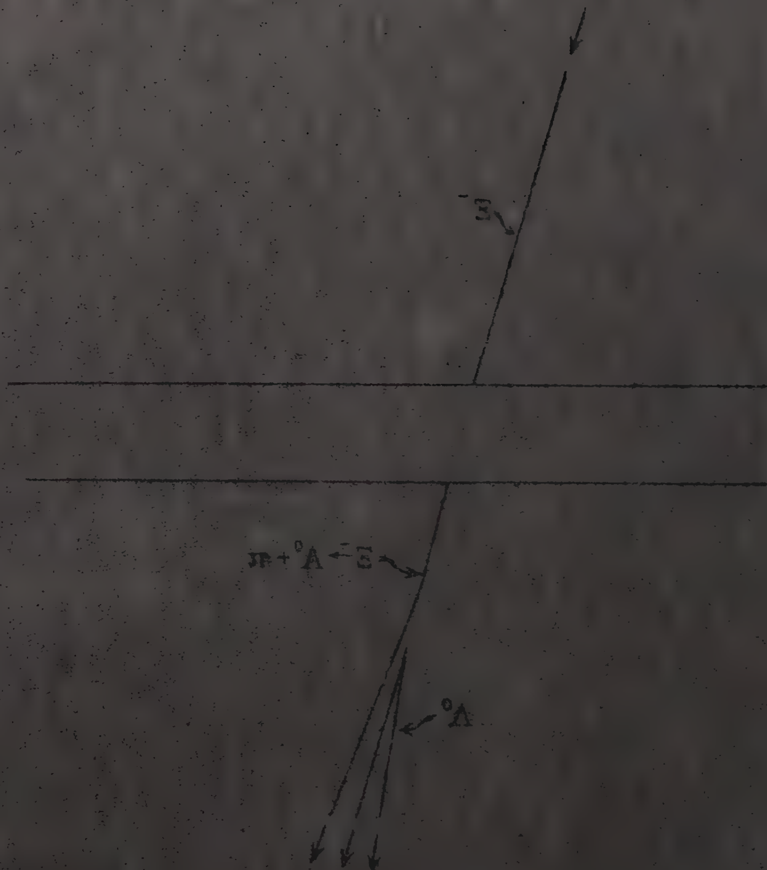
$$Q = 65.4 \pm 2.2\ ,$$

which gives for the mass of the Ξ^- the value

$$M_{\Xi^-} = (2\,583 \pm 5.5)\ \mathrm{m_e}\ .$$



Ξ^-
$\Xi^- \to \Lambda^0 + \pi$
Λ^0



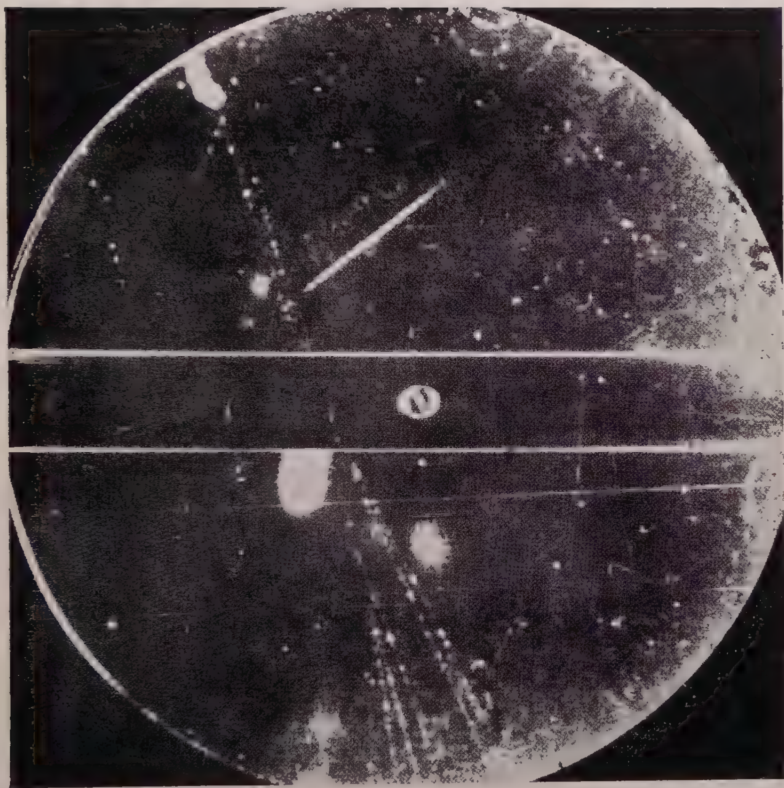


Fig. 1·73. – The first reported example of « cascade decay » [1].

7·3. – The mean life of the Ξ.

TRILLING *et al.* [8] have estimated a lower limit for the Ξ^- mean life on their six cases listed in Table I. They used Bartlett's statistical method (Sect. 9·3) suitably modified to take into account the fact that, in cloud chamber observations, a Ξ^- is identified when the secondary Λ^0 is seen to decay in the same picture. They obtain, to a 95% confidence level

$$\tau_{\Xi^-} \geqslant 1.8 \cdot 10^{-10}\ \text{s}\ .$$

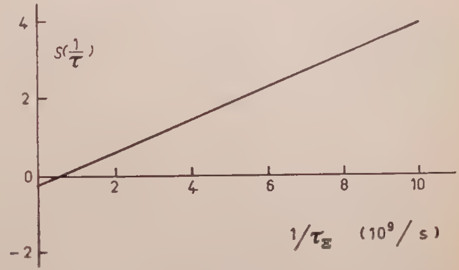


Fig. 2-7·3. – $S(1/\tau)$-plot for six Ξ^- decays observed by TRILLING *et al.* [8].

An upper limit could not be estimated by this method, since the value of $1/\tau$ was found to be zero for $S = -0.2$ (see Fig. 2). A crude evaluation was attempted by the same Authors by establishing an upper limit for the number of non decaying Ξ^- going through the chamber; they find

$$\tau_{\Xi^-} \lesssim 2 \cdot 10^{-8}\ \text{s}\ .$$

An estimate was also made by DAHANAYAKE *et al.* [11] on the basis of two events found in emulsion and it yielded

$$\tau_{\Xi^-} \sim 10^{-10}\ \text{s}\ .$$

REFERENCES (CH. 7)

[1] R. ARMENTEROS, K. H. BARKER, C. C. BUTLER, A. CACHON and C. M. YORK: *Phil. Mag.*, **43**, 597 (1952).
[2] C. D. ANDERSON, E. W. COWAN, R. B. LEIGHTON and V. A. G. VAN LINT: *Phys. Rev.*, **92**, 1089 (1953).
[3] R. ARMENTEROS, B. GREGORY, A. LAGARRIGUE, L. LEPRINCE-RINGUET, F. MULLER and CH. PEYROU: *Suppl. Nuovo Cimento*, **12**, 324 (1954).
[4] H. W. ARNOLD: *Thesis* (1955); see also H. W. ARNOLD, J. J. BALLAM, G. K. LINDEBERG and V. A. J. VAN LINT: *Phys. Rev.*, **98**, 838 (1955).
[5] E. W. COWAN: *Phys. Rev.*, **94**, 161 (1954).
[6] W. B. FRETTER and E. W. FRIESEN: *Phys. Rev.*, **96**, 853 (1954).
[7] J. D. SORRELS, R. B. LEIGHTON and C. D. ANDERSON: *Phys. Rev.*, **100**, 1457 (1955).
[8] G. H. TRILLING and G. NEUGEBAUER: circulated pre-print.
[9] P. H. BARRETT: *Phys. Rev.*, **94**, 1328 (1954).
[10] C. CASTAGNOLI, G. CORTINI and A. MANFREDINI: *Nuovo Cimento*, **2**, 565 (1955).
[11] C. DAHANAYAKE, P. E. FRANCOIS, Y. FUJIMOTO, P. IREDALE, C. J. WADDINGTON and M. YASIN: *Nuovo Cimento*, **1**, 888 (1955).
[12] M. W. FRIEDLANDER, D. KEEFE and M. G. K. MENON: *Nuovo Cimento*, **1**, 482 (1955).

Chapter 8.

Heavier hyperons.

Eisenberg [1] has reported one case favouring the existence of hyperons heavier than any hitherto observed. A particle (track K_1; see Fig. 1) is emitted from a 5+11p star (P). After 5.72 mm it suffers a 10° deflection (point A) and after an additional range of 24.2 mm (track K_2) it produces a disintegration (S) leading to the emission of 5 ionizing particles which carry away a « visible » energy of (209 ± 4) MeV.

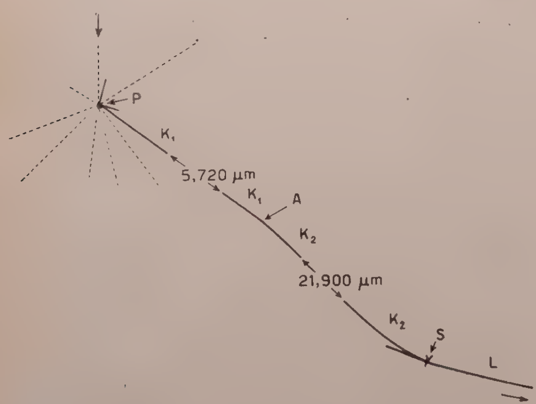


Fig. 1-8. – Schematic drawing of Eisenberg's event.

Careful scattering and ionization measurements indicate that track K_1 and track K_2 could not be due to the same particle. Track K_2 was due to a particle of mass $(840 \pm 190)\ m_e$ [2], presumably a K^- interacting at the end of its range with a nucleus of the emulsion. Track K_1 on the other hand was produced by a particle of mass $(3\,220 \pm 700)\ m_e$. Furthermore, from range and ionization measurement it appeared that the K particle responsible for track K_2 was faster (at A) than the particle responsible for track K_1.

This event can be interpreted either as the decay of a very massive

$$Y^- \rightarrow K^+ + \text{one or more neutral particles} + Q$$

or as the interaction of a hyperon with a nucleus, leading to the emission of a K^- and a number of neutral particles, for example

$$Y^- + n \rightarrow K^- + \text{neutrals} + Q$$

a type of interaction which has been observed by other research workers [3].

Eisenberg points out, however, that the energy released in the supposed interaction taking place at A is too high to identify Y^- with any of the known hyperons.

Interpreting the event as due to a decay of a new very massive hyperon into 2 particles only he gets $Q \sim 5$ MeV, a value which does not depend on the assumption regarding the mass of the neutral secondary.

Also the Wisconsin group [4] has reported on a similar event observed in nuclear emulsion. In their case the primary particle was seen to decay from rest ejecting a 42 MeV K-meson; the latter was interpreted to be negative since it did not decay into any visible secondary.

Assuming the primary to be a hyperon, its mass has to be $\geqslant 2890\ m_e$. Unfortunately no direct mass estimate was possible on its 44 μm long track and no definite conclusion could be reached with regard to its identity: it could infact be due to a K-meson or to an « anomalous » hyperfragment (see Sect. **21·9**) although the latter interpretation seemed unlikely.

Neither case offers sufficiently safe evidence to be worth discussing. Further work is required to clarify the whole question.

REFERENCES (Ch. 8)

[1] Y. Eisenberg: *Phys. Rev.*, **96**, 541 (1954).
[2] Y. Eisenberg: *Rep. Pisa Conference* (1955), Mimeog. rep. 329.
[3] M. M. Block and D. T. King: *Phys. Rev.*, **97**, 1415 (1955).
[4] W. F. Fry, J. Schneps and M. S. Swami: *Nuovo Cimento*, **2**, 346 (1955).

CHAPTER 9.

Appendix.

In the following pages a brief survey is reported of some methods of analysis which have been employed in research on unstable particles. It is intended to enable a reader, who is unfamiliar with them, to follow the discussion of some of the experiments described in the previous chapters. Those who wish to obtain a more detailed knowledge of these methods will have to consult the original literature.

Sect. **9·1** deals with the dynamical analysis of two-body decays; in Sect. **9·2** phase space integrals for K-three body decays are reported and in Sec. **9·3** statistical methods of analysis of experimental data are briefly discussed in connection with mean life estimates.

9·1. – Dynamical analysis of two body decays.

We have seen in Chapters **3** and **4** how rarely individual cloud chamber observations of the decay of unstable particles have afforded an unambiguous determination of their decay scheme, i.e., of the masses M of the primary and m_i of the secondaries. This is especially true for cosmic ray particles, most of which are observed to decay when fast: consequently most of the particles involved produce minimum ionization tracks and their masses cannot be determined. Not unfrequently, however, quantities—such as for example the momenta P and p_i of the primary and secondary particles and the angles Φ_i between any two of them—can be estimated. These are not sufficient to identify individually their decay schemes, but this may be possible on the basis of a statistical analysis of a number of cases. It can be shown that the choice for the values which each P, p_i, Φ_i, can assume is limited by momentum and energy equations which depend on the decay scheme. Consequently, from the distributions of the values of P, p_i and Φ_i, as experimentally determined for a given group of events, the masses M and m could be determined.

The purpose of the discussion which follows is to see how this can be done. The problem has been treated in detail by THOMPSON [2] and PODOLANSKI and ARMENTEROS [1]; what is reported below is only a summary of their results.

We shall consider two body decays of neutral particles: extension to charged ones is—however—immediate.

9·1.1. *Fundamental formulae and useful parameters.* – Suppose that for each event the momenta p_+ and p_- of the secondaries and the angle Φ between them (see Fig. 1-3·4) have been determined. The application of energy and momentum conservation laws shows that the above quantities have to satisfy the equation

$$e_+e_- - p_+p_- \cos\Phi = \tfrac{1}{2}(M^2 - m_1^2 - m_2^2)\,, \tag{1-9·1}$$

where $e_\pm = \sqrt{p_\pm^2 + m_\pm^2}$ is the total energy of either secondary in the laboratory system. In a $(p_+p_-\Phi)$-space this equation represents a surface (the « decay surface ») (*) and equation (1) tells us that individual events due to the *same* 2-body decay scheme will be represented in this space by points placed on this surface. Given a sufficient number of observations, the problem to solve becomes that of finding values for M, m_+, m_- which adjust the decay surface to be as close as possible to the experimental determinations.

Decay surfaces in a $(p_+p_-\Phi)$-space are much too complicated to be of any help in practical applications. The following parameters have been introduced, which greatly simplify the task:

(i) The inverse of the primary momentum $1/P$.

(ii) The Manchester α-parameter,

$$\alpha = \frac{p_{+l} - p_{-l}}{P}\,, \tag{3-9·1}$$

where p_{+l} and p_{-l} are the longitudinal components of the momenta p_+ and p_- of the decay products in the laboratory system (see Fig. 1-3·4). It is easy to prove that

$$\alpha = \frac{p_{+l}^2 - p_{-l}^2}{P^2} = \frac{p_+^2 - p_-^2}{P^2} = -\frac{\sin^2\Phi_+ - \sin^2\Phi_-}{\sin^2\Phi} = -\frac{\sin(\Phi_+ - \Phi_-)}{\sin\Phi}\,, \tag{4-9·1}$$

(*) Decay surfaces are also called « Q-surfaces » where Q is intended to be the sum of the kinetic energies of the secondaries in their c.m.-system. Since the conservation of energy imposes that

$$M = m_+ + m_- + Q\,, \tag{2-9·1}$$

the value of the Q is uniquely determined by the decay scheme.

and that (*)

(5-9·1) $$\alpha = \alpha^* + \frac{2p^*}{P}\frac{E}{M}\cos\Phi^*\,,$$

where

(6-9·1) $$\alpha^* = \frac{m_+^2 - m_-^2}{M^2} = \frac{e_+^* - e_-^*}{M}\,;$$

E is the primary total energy and p^* the momentum of either secondary in the decay center of mass system. It follows that if Φ^* is isotropically distributed, the average value of α is equal to α^*.

(iii) The transverse momentum p_T (see Fig. 1-3·4)

(7-9·1) $$p_T = p_+ \sin\Phi_+ = p_- \sin\Phi_- = p^* \sin\Phi^*\,.$$

(iv) Podolanski-Armenteros parameter ε,

(8-9·1) $$\varepsilon = \frac{2p_T}{P} = \frac{2\sin\Phi_+\sin\Phi_-}{\sin\Phi}\,.$$

In terms of $1/P$, α, p_T equation (1) reads

(9-9·1) $$p_T^2 + \frac{(\alpha-\alpha^*)^2}{4(1/P^2 + 1/M^2)} = p^{*2}\,,$$

and in terms of $1/P$, α, ε,

(10-9·1) $$\frac{\varepsilon^2}{M^2/P^2} + \frac{1}{E^2/P^2}(\alpha-\alpha^*)^2 = \varepsilon^{*2}\,,$$

where $\varepsilon^* = 2p^*/M$ and E is the total energy of the primary particle. It can be easily verified that

(11-9·1) $$m_+ = \frac{1}{2}M\{(1+\alpha^*)^2 - \varepsilon^{*2}\}^{\frac{1}{2}} = \frac{1}{2}M\left\{(1+\alpha^*)^2 - \frac{4p^{*2}}{M^2}\right\}^{\frac{1}{2}},$$

(12-9·1 $$m_- = \frac{1}{2}M\{(1-\alpha^*)^2 - \varepsilon^{*2}\}^{\frac{1}{2}} = \frac{1}{2}M\left\{(1-\alpha^{*2}) - \frac{4p^{*2}}{M^2}\right\}^{\frac{1}{2}}.$$

(*) Quantities with an asterisk are referred to the system in which the primary particle was at rest.

9·1.2. *The « decay surface » in the* $(1/P, \alpha, p_T)$*-space.* – Equations (9) and (10) each define a family of surfaces respectively in the space $(1/P, \alpha, p_T)$ and $(1/P, \alpha, \varepsilon)$. Let us consider here the former representation. The practical problem consists in determining from the distribution of the experimental points the correct values of the parameters α^*, M, P^*; from these and equation (11) and (12), m_+ and m_- are immediately deduced.

In Fig. 1*a*, the decay surface for the Λ^0's is shown in isometric projection.

The use of three dimensional representations is unpractical for obvious reasons: two dimensional projections on the $(1/P, \alpha)$ plane or in the (α, p_T) plane are more convenient.

(i) *Projection on the* $(1/P, \alpha)$ *plane.* – The intersections of the surface with planes $p_T = \text{const}$ are hyperbolae, whose shape is visualized in Fig. 1. All the points related to the corresponding decay mode must fall inside the region defined by the branches of the limiting hyperbola $(p_T = 0)$. (See Fig. 2*a*).

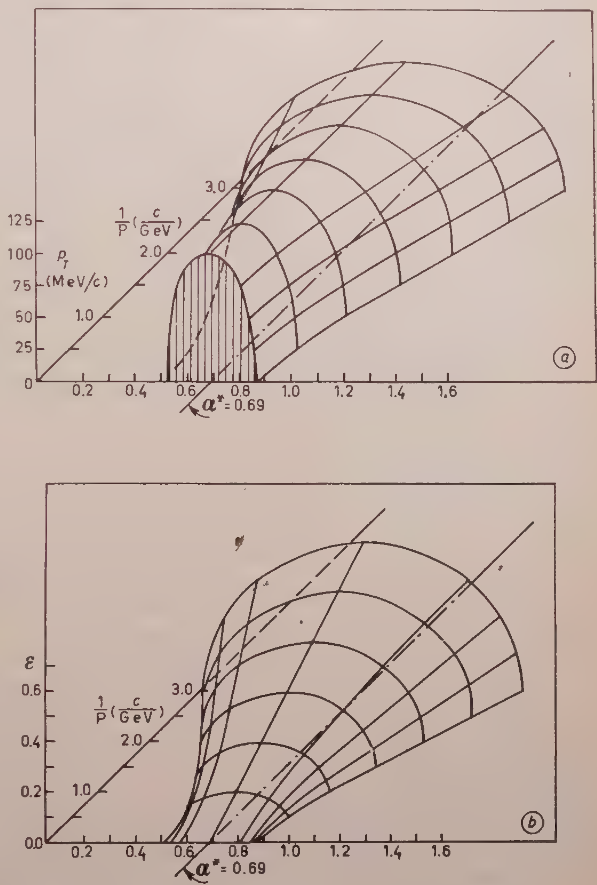


Fig. 1-9·1. – Isometric projections of the Λ^0 decay surface: *a*) in the $(1/P, \alpha, p_T)$-space and *b*) in the $(1/P, \alpha, \varepsilon)$-space. (From R. W. Thompson [2]).

(ii) *Projection on the* (α, p_T) *plane.* – Intersections with the planes $1/P = \text{const}$ are ellipses. Their centre has the co-ordinates $(\alpha^*, 0)$ and the length of the two semi-axes is p^* and $2p^*/M\beta$ respectively. The distribution of the points on each ellipse is immediately obtained when the distribution in terms of p_T is known. It follows from (5) that *isotropically distributed* events tend to crowd the upper segment of each ellipse. From Fig. 1 it can be seen that for $1/P$ sufficiently small the shape of these projections does not change appreciably with P. For practical purposes the limiting ellipse $(1/P=0)$

can be used when $P \geqslant 1$ GeV. For slower primaries the event can be « transferred » to the limiting ellipse, displacing it on the decay surface along the intersection with planes $p_T = \text{const}$. The (α, p_T)-projection will be displaced horizontally from the measured value of α to the value $\alpha' = \beta(\alpha - \alpha^*) + \alpha^*$.

An example of application of this projection to the analysis of V⁰ events may be seen in Fig. 3. The separation between Λ^0 and $K^0_{\pi 2}$ needs not be stressed. It can equally well be seen that for $\alpha \sim 0.7$ the interpretation may not be unambiguous. It may be interesting to notice that if the quantity $\frac{1}{2}M(\alpha - \alpha^*)$ is plotted instead of α, the ellipses are deformed to become circles (*) of radius p^*, indipendent of β, i.e. the surface has become a cylinder. In particular the circular projection gives a more vivid picture of the decay since the circle represents the locus of the terminal point of the vector p^* in the center of mass system (see equation (5)).

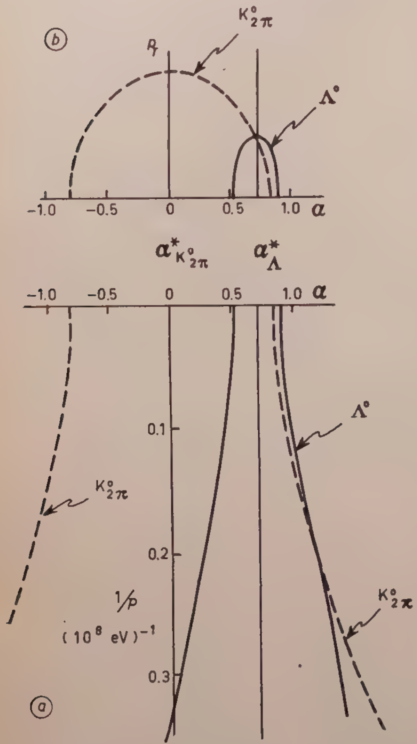


Fig. 2-9·1. – Intersections of the Λ^0 decay surface shown in Fig. 1*a* with: (*a*) the $(1/P, \alpha)$-plane and (*b*) the $(p_T; \alpha)$-plane. Those for the $K^0_{\mu 3}$ decay are also reported (dotted lines).

9·1.3. *The « decay surface » in the* $(1/P, \alpha, \varepsilon)$*-space.* – We now turn to the representation in terms of $1/P$, α and ε. The projection on the $(1/P, \alpha)$ plane needs not be discussed again since it is identical with (i) (9·1.2).

(iii) *The projection on the* (α, ε) *plane* is on the other hand essentially different. The intersection of planes $1/P = \text{const}$, projected on the $(1/P = 0)$-plane are confocal ellipses, whose semiaxes are equal to $\varepsilon^* M/P$ and ε^*/β respectively. The first one tends to zero when $P \to \infty$, i.e. the ellipses tend to degenerate into a straight segment on the α-axis (Fig. 1.*b*).

One of the most interesting properties of this projection is that the position of an event is determined by the angles Φ_+ and Φ_- only. To prove it let us consider the vector (see Fig. 4)

$$\boldsymbol{R} = \frac{\boldsymbol{p}_+ - \boldsymbol{p}_-}{P},$$

(*) See K. H. Barker [4].

which has components

$$R_l = \frac{p_{+l} - p_{-l}}{P} = \alpha \quad \text{parallel to } \boldsymbol{P},$$

and

$$R_T = \frac{2p_T}{P} = \varepsilon \quad \text{perpendicular to } \boldsymbol{P}.$$

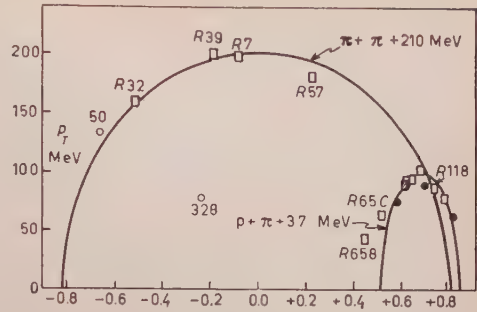


Fig. 3-9·1. – (α-p_T) plot for V^0 events. The curves are the limiting ellipses for the Λ^0 and $K^0_{\pi 2}$ decay processes. Square and circles represent individual measurements on cosmic ray events observed in cloud chamber. (After THOMPSON [2]).

If in the (α, ε)-plane the vector $2\boldsymbol{p}_+/P$ is applied to the point $S_1(\alpha = -1, \varepsilon) = 0$ at an angle with respect to the α-axis equal to the angle Φ_+, and the vector $2\boldsymbol{p}_-/P$ on the point $S_2(\alpha = +1, \varepsilon = 0)$ at an angle Φ_-, $\boldsymbol{R}$ is represented by a vector having its origin at $\varepsilon = 0$, $\alpha = 0$ and its terminal where the vectors $2\boldsymbol{p}_+/P$ and $2\boldsymbol{p}_-/P$ intersect. It follows that without any previous knowledge of the individual momenta, the values of α and ε can be determined graphically if Φ_+ and Φ_- are known.

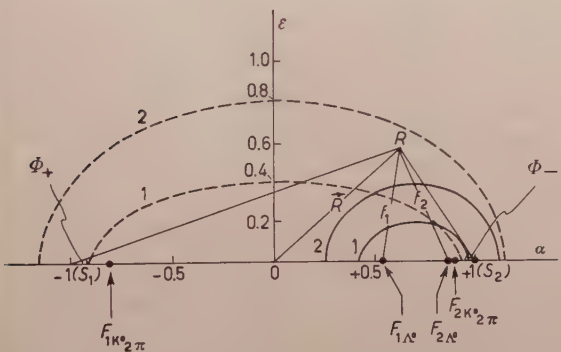


Fig. 4-9·1. – Projections on the (α, ε)-plane of iso-P lines of the Λ^0 decay surface (full lines). The same for $K^0_{\pi 2}$ decays are also reported (dotted lines).

Furthermore PODOLANSKI and ARMENTEROS have shown that if f_1 and f_2 are the distances between an event R, and the foci of the ellipses (Fig. 3) the angle Φ^* of emission in the centre of mass system is given by (*)

$$\cos \Phi^* = \frac{f_1 - f_2}{2\varepsilon^*},$$

(*) When no magnetic field is available the sign of the secondary particles is often unknown and two assignments are possible for f_1 and f_2. For $K^0_{\pi 2}$ this does not involve any difficulty since the two secondary masses are equal and the exchange of Φ_+ with Φ_- would only exchange R with its symmetric image with respect to the ε-axis, *on the same ellipse*. Thus the only consequences would be the exchange of f_1 with f_2 which would leave the calculated value of β unchanged and the substitution of Φ^* with $\pi - \Phi^*$ which is immaterial. For Λ^0 decays it would give different values of f_1 and f_2 as the Λ^0 ellipses are not symmetric with respect to the ε-axis. Often however, the choice can be made on the basis of ionization measurements or other indications.

and the velocity of the primary

$$\beta = \frac{2\varepsilon^*}{f_1 + f_2}\,.$$

An example of application of the above procedure was described in Sect. 3·4.5.

9·2. – Three body decays. Momentum and Q-distributions.

When more than two particles are produced in the disintegration process of an unstable particle, conservation laws do not impose single values for the individual parameters p_i and e_i, and the representations discussed in the previous section are of little avail. Momentum and energy distributions are determined by the properties of the interactions responsible for the decay and by the statistical factors giving the density of the final states.

The formulae reported below give the energy and momentum distributions for the secondaries ejected in three body decays as *determined by statistical factors only*.

9·2.1. *Momentum distribution.* – Let m_i $(i=1, 2, 3)$ be the mass of the i-th decay product, p_i its momentum in center of mass system of the primary particle and e_i its total energy in the same system; also let M be the mass of the primary and $Q = M - \sum m_i$. The momentum distribution of any of the three particles is then given by

$$\text{(1-9·2)}\quad f(p_i)\,\mathrm{d}p_i = $$
$$= \text{const} \times p_i^2\,\mathrm{d}p_i\,\frac{\mathrm{d}}{\mathrm{d}Q}\int \mathrm{d}\boldsymbol{p}_j \int \mathrm{d}\boldsymbol{p}_k\,\delta(\boldsymbol{p}_i + \boldsymbol{p}_j + \boldsymbol{p}_k)\,U(M - e_i - e_j - e_k)\,,$$

where $\delta(x)$ and $U(x)$ represent the well known delta and step functions.

The explicit form of (1) in the relativistic case is (Bloch [5])

$$\text{(2-9·2)}\quad f(p_i)\,\mathrm{d}p_i = \text{const} \times p_i^2 \left\{1 - \frac{2(m_j^2 + m_k^2)}{M_{jk}^{*2}} + \frac{(m_j^2 - m_k^2)^2}{M_{jk}^{*4}}\right\}^{\frac{1}{2}} \cdot$$
$$\cdot \left\{3(M - e_i)^2 \left(1 - \frac{(m_j^2 - m_k^2)^2}{M_{jk}^{*4}}\right) - p_i^2 \left(1 - \frac{2(m_j^2 + m_k^2)}{M_{jk}^{*2}} + \frac{(m_j^2 - m_k^2)^2}{M_{jk}^{*4}}\right)\right\},$$

having put M_{jk}^* for $\sqrt{(M - e_i)^2 - p_i^2}$. The maximum value allowed for p_i is

$$(p_i)_{\max} = \frac{1}{2M}\{M^2 - (m_i + m_j + m_k)^2\}^{\frac{1}{2}}\{M^2 - (m_i - m_j - m_k)^2\}^{\frac{1}{2}}\,.$$

9·2.2. *The Q^*-distribution.* – Let us consider the case of a neutral particle which decays into one neutral and two charged secondaries, having masses m_1, m_2 and m_3 respectively. As m_1 cannot be seen—its presence can be guessed if the line of motion of the primary is established—the event will appear as a V⁰ decay associated with an apparent $Q = Q^*_{ij}$ and an apparent primary mass M^*_{ij}, where

$$Q^*_{ij} = \sqrt{(e_i + e_j)^2 - (\boldsymbol{p}_i + \boldsymbol{p}_j)^2} - m_i - m_j = \sqrt{(M - e_k)^2 - p_k^2} - m_i - m_j\,,$$

$$M^*_{ij} = Q^*_{ij} + m_i + m_j = \sqrt{(M - e_k)^2 - p_k^2}\,.$$

The Q^*-distribution is derived immediately from the p-distribution given above [5]

$$(3\text{-}9{\cdot}2) \qquad F(Q^*_{ij})\,\mathrm{d}Q^*_{ij} = G(M^*_{ij})\,\mathrm{d}M^*_{ij} =$$

$$= \text{const}\cdot\frac{M^*_{ij}\,\mathrm{d}M^*_{ij}}{M^5}\,(M^2 + m_k^2 - M^{*2}_{ij})\,(M^2 - (M^*_{ij} + m_k)^2)^{\frac12}\,(M^2 - (M^*_{ij} - m_k)^2)^{\frac12}\cdot$$

$$\cdot\left(1 - \frac{2(m_i^2 + m_j^2)}{M^{*2}_{ij}} + \frac{(m_i^2 - m_j^2)^2}{M^{*4}_{ij}}\right)^{\frac12}\cdot\left\{3(M^2 - m_k^2 + M^{*2}_{ij})^2\left(1 - \frac{(m_i^2 - m_j^2)^2}{M^{*4}_{ij}}\right) - \right.$$

$$\left. - (M^2 - (M^*_{ij} - m_k)^2)\,(M^2 - (M^*_{ij} + m_k)^2)\cdot\left(1 - \frac{2(m_i^2 + m_j^2)}{M^{*2}_{ij}} + \frac{(m_i^2 - m_j^2)^2}{M^{*4}_{ij}}\right)\right\}.$$

$$0 \leqslant Q^*_{ij} \leqslant M - (m_i + m_j + m_k).$$

9·3. – Statistical analysis of lifetime estimates.

Statistical methods for estimation of the mean life of the new unstable particles have been considered both in connection with cloud chamber and emulsion experiments [6-14]. The most complete discussion is contained in the articles of Bartlett [7, 8], which are chiefly related to the study of unstable particles in cloud chamber photographs.

An application of Bartlett's statistical treatment to the analysis of emulsion data can be found in [9, 10, 12].

9·3.1. *Bartlett's maximum likelihood procedure.* – Let us consider a cloud chamber picture of an event, due to the decay of an unstable particle, for which the following quantities can be determined:

1) The *path length* (l_i) from the entry of the particle in the chamber to the point of decay;

2) The *potential path length* (L_i), i.e. the path which the particle would have covered *inside the chamber*, had it not decayed (see Fig. 1*a*).

If the velocity of the primary can be deduced, these quantities can immediately be translated into *time of flight* t_i and *potential time of flight* T_i.

Let us suppose, at first, that the chamber does not contain any plate inside the sensitive volume. Considering that the time available for the unstable

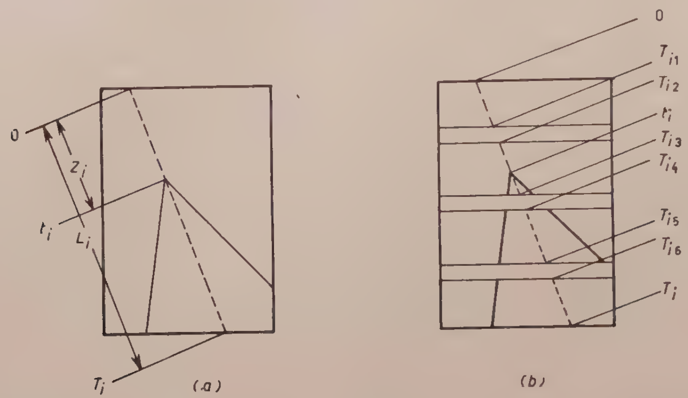


Fig. 1-9·3.

particle to decay inside the chamber was T_i, the normalized probability for the decay to be observed within a certain interval $\mathrm{d}t_i$ inside the chamber, is

$$\mathrm{d}p_i = f_i \mathrm{d}t_i = \frac{\exp[-t_i/\tau]\,\mathrm{d}t_i/\tau}{1-\exp[-T_i/\tau]}, \qquad 0 \leqslant t_i \leqslant T_i\,, \tag{1-9·3}$$

τ being the meanlife to be determined. If n observations of n events are available, the probability that the first decay happens in the interval $\mathrm{d}t_1$, the second in $\mathrm{d}t_2$, and so on, is obviously

$$\mathrm{d}\mathcal{P} = \prod_i^{1\dots n} \mathrm{d}p_i = \prod_i^{1\dots n} \frac{\exp[-t_i/\tau]\,\mathrm{d}t_i/\tau}{1-\exp[-T_i/\tau]} = F\,\mathrm{d}t_1\,\mathrm{d}t_2 \dots \mathrm{d}t_n\,. \tag{2-9·3}$$

The estimation of τ from a set of experimental data $(t_i\,|\,T_i)_{i=1,2,\dots n}$ can be made by the « maximum likelihood » method (H. Cramér [15], Ch. 32 and 33).

The *likelihood equation*

$$\frac{\partial}{\partial\tau} \ln F(t_1 \dots t_n;\, T_1 \dots T_n\,|\,\tau) = 0\,, \tag{3a-9·3}$$

solved with respect to τ gives a *maximum likelihood estimate* for it. Equation (3), in our particular case, is to be written

$$\sum_i^{1\dots n}\left(\frac{t_i}{\tau} - 1 + \frac{T_i \exp[-T_i/\tau]}{\tau(1-\exp[-T_i/\tau])}\right) = 0\,, \tag{3b-9·3}$$

an equation which can be solved by interpolation or by successive approximations. It can be shown (see CRAMÉR, loc. cit.) that under certain general conditions—satisfied in the present case—the likelihood equation has a solution $\hat{\tau}$ which is an *asymptotically normal* estimate of τ, distributed around the true value τ_0 with an asymptotic variance σ given by

$$(4\text{-}9{\cdot}3)\qquad \frac{1}{\sigma^2} = \sum_i E\left(-\frac{\partial^2 \ln f_i}{\partial\tau^2}\right) = \sum_i \left\{-\int\left(\frac{\partial^2 \ln f_i}{\partial\tau^2}\right)_{\tau=\tau_0} f_i(t_i\,|\,\tau_0)\,\mathrm{d}t_i\right\},$$

and that the variables $(\partial \ln f_i/\partial\tau)_{\tau=\tau_0}$ converge in probability to zero with an asymptotic distribution associated with a variance

$$(5\text{-}9{\cdot}3)\qquad \sigma'^2 = E\left(\frac{\partial \ln f_i}{\partial\tau}\right)^2_{\tau=\tau_0} = \int\left(\frac{\partial \ln f_i}{\partial\tau}\right)^2 f_i(t_i\,|\,\tau_0)\,\mathrm{d}t_i =$$

$$= \int\left(\frac{1}{f_i}\frac{\partial f_i}{\partial\tau}\right)^2 f_i(t_i\,|\,\tau_0)\,\mathrm{d}t_i = \int\left[\left(\frac{1}{f_i}\frac{\partial f_i}{\partial\tau}\right)^2 - \frac{1}{f}\frac{\partial^2 f_i}{\partial\tau_i^2}\right] f_i(t_i\,|\,\tau_0)\,\mathrm{d}t_i = -E\left(\frac{\partial^2 \ln f_i}{\partial\tau^2}\right)_{\tau=\tau_0},$$

since $\int(\partial^2 f_i/\partial\tau^2)\,\mathrm{d}t_i = 0$. It follows then that $\sum_i^{1\ldots n}(\partial \ln f_i/\partial\tau)_{\tau=\tau_0}$ is asymptotically normal, with a mean value zero and variance $\sigma''^2 = \sum\sigma'^2 = \sum_i^{1\ldots n}\left(-E(\partial^2 \ln f_i/\partial\tau^2)_{\tau=\tau_0}\right)$. Then the variable

$$(6\text{-}9{\cdot}3)\qquad \frac{\sum\left(\dfrac{\partial \ln f_i}{\partial\tau}\right)_{\tau=\tau_0}}{\left\{\sum\left(-E\left(\dfrac{\partial^2 \ln f_i}{\partial\tau^2}\right)_{\tau=\tau_0}\right)\right\}^{\frac{1}{2}}},$$

is asymptotically normal with zero mean value and variance 1. Following Bartlett's treatment we consider the function

$$(7a\text{-}9{\cdot}3)\qquad S(\tau) = \frac{\sum_i^{1\ldots n}\left(\dfrac{t_i}{\tau} - 1 + \dfrac{T_i}{\tau}\dfrac{\exp[-T_i/\tau]}{1-\exp[-T_i/\tau]}\right)}{\left\{\sum_i^{1\ldots n}\left(1 - \dfrac{T_i^2}{\tau^2}\dfrac{\exp[-T_i/\tau]}{(1-\exp[-T_i/\tau])^2}\right)\right\}^{\frac{1}{2}}},$$

which for $\tau = \tau_0$ is equivalent to (6) and has zero mean and unit variance. Obviously $S(\hat{\tau}) = 0$ since $\hat{\tau}$ is the root of equation (3b). It can be seen that if plotted as a function of $1/\tau$, the function $S(1/\tau)$ is nearly linear in the neighbourhood of $1/\hat{\tau}$, a fact which makes the graphical interpolation an easy task. As long as S can be considered—at least approximately—as a variable having gaussian distribution, the *limits of confidence* to be associated with a determination of $\hat{\tau}$ can be determined as follows. Let P be the probability that $\hat{\tau}$ differs from the true (unknown) value τ_0 in either direction by more than θ

times the standard deviation. As S has unit variance, the limits of confidence for τ, associated with a probability P, are given by the roots of the two equations

$$S(\tau) = \pm\,\theta(P)\,.$$

For the convenience of the reader values of $\theta(P)$, as defined by the equation

$$\sqrt{\frac{2}{\pi}}\int_{\theta(P)}^{\infty}\exp\left[-\eta^2/2\right]\mathrm{d}\eta = P(|\hat{\tau}-\tau_0|>\theta(P))\,,$$

have been reported in Table I.

TABLE I-9·3.

$$P(|\hat{\tau}-\tau_0|>\theta_P) = \sqrt{\frac{2}{\pi}}\int_{\theta(P)}^{\infty}\exp\left[-\eta^2/2\right]\mathrm{d}\eta\,.$$

P	$\theta(P)$	P	$\theta(P)$	P	$\theta(P)$	P	$\theta(P)$
1	0	.70	.385 3	.40	.841 6	.10	1.644 9
.95	.062 7	.65	.453 8	.35	.934 6	.05	1.960 0
.90	.125 7	.60	.524 4	.30	1.036 4	.01	2.575 8
.85	.189 1	.55	.597 8	.25	1.150 3	.001	3.290 5
.80	.253 3	.50	.674 5	.20	1.281 6	.000 1	3.890 6
.75	.318 6	.45	.755 4	.15	1.439 5	—	—

In Fig. 2 an example is shown of a mean life determination by this method. The function S related to the times of flight t_i and to the potential times T_i of eleven V^+ observed in a cloud chamber was calculated by FRETTER *et al.* [16] (Fig. 2) for assumed values of

$$\frac{1}{\tau} = 0;\ 0.5;\ 1;\ 2\ \ldots\cdot 10^9\ \mathrm{s}^{-1}\,.$$

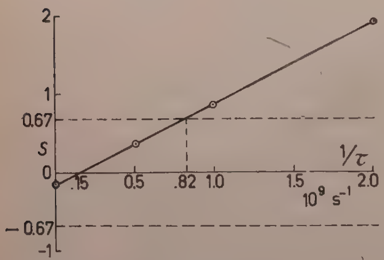


Fig. 2-9·3. – Example of a $S(1/\tau)$-plot for the determination of the mean life of unstable particles, by the maximum likelihood method. The graph refers to 11 V^+ measured by FRETTER *et al.* [16].

As predicted S was found an almost linear function of $1/\tau$. Adjacent points could then be jointed by straight segments and the intersection with the line $S(1/\tau) = 0$ gave the most likely value for the lifetime of the V^+.

A lower confidence limit, corresponding

to a probability 50% was determined. $P = 0.5$ corresponds to $\theta(P) = 0.67$ (see Table I). The equation $S(1/\tau) = 0.67$ was solved graphically (see Fig. 2) and $(1/\tau)_{\text{low. lim.}}$ was found to be $0.82 \cdot 10^9$, corresponding to $\tau_{\text{low. lim.}} = 1.12 \cdot 10^{-8}$ s. The data did not enable these authors to determine an upper limit for the same level of confidence. For $S = -0.14$ one gets in fact $1/\tau = 0$, i.e. $\tau = \infty$.

The assumption that S be normal is closer to the truth, the higher the number of observations n is. In general however S is not normal—in fact it has a positive third moment and therefore a positive skewness coefficient γ. In our case (see BARTLETT [7], p. 251-4)

$$\gamma = \frac{\mu_3}{\sigma''^3} = \frac{\sum_i^{1\dots n}\left(2 - \frac{T_i^3}{\tau^3}\frac{\exp[-T_i/\tau](1+\exp[-T_i/\tau])}{(1-\exp[-T_i/\tau])^3}\right)}{\left\{\sum_i^{1\dots n}\left(1 - \frac{T_i^2}{\tau^2}\frac{\exp[-T_i/\tau]}{(1-\exp[-T_i/\tau])^2}\right)\right\}^{\frac{3}{2}}}.$$

BARTLETT suggests the use of the *amended quantity*

$$S' = S + \tfrac{1}{6}\gamma(S^2 - 1) \tag{7b-9·3}$$

in place of S in equation (7a).

It was pointed out by NEWTH (see PAGE and NEWTH [14]) that the maximum likelihood equations have to be modified when the chamber contains plates since a particle decaying inside a plate would not, in general, be considered. The corresponding potential time T_i and time of flight t_i of particles observed in the chamber are to be calculated from the sum of the various T_{ij} corresponding to the points of entry of the i-th particle in the j-th plate and of the exit from the $(j-1)$-th plate (see Fig. 1b).

The probability of decaying in a *visible* part of a chamber containing s plates is

$$1 - \exp[-T_{i1}/\tau] + \exp[-T_{i2}\tau] - \exp[-T_{i3}/\tau] + \dots +$$
$$+ \exp[-T_{is}/\tau] - \exp[-T_i/\tau] = 1 - \exp[-T_i/\tau] + \sum_j^{1\dots s}(-1)^j \exp[-T_{ij}/\tau]$$

instead of $1 - \exp[-T_i/\tau]$ which appeared in the denominator of (1). The distribution function $F(t_1, t_2 \dots; T_1, T_2 \dots | \tau)$ has to be modified into

$$F'(t_i; T_i; T_{ij}|\tau)\,\mathrm{d}t_i \dots = \prod_i^{1\dots n} f'\,\mathrm{d}t_i =$$
$$= \prod_i^{1\dots n}\frac{1}{\tau}\frac{\exp[-T_i/\tau]\,\mathrm{d}t_i}{1 - \exp[-T_i/\tau] + \sum_j^{1\dots s}(-1)^j\exp[-T_{ij}/\tau]}.$$

The likelihood equation becomes

$$\sum \frac{\partial \ln f_i'}{\partial \tau} = \frac{1}{\tau} \sum \left(\frac{t_i}{\tau} - 1 + \frac{T_i \exp[-T_i/\tau] - \sum_j (-1)^j T_{ij} \exp[-T_{ij}/\tau]}{\tau(1 - \exp[-T_i/\tau] + \sum_j (-1)^j \exp[-T_{ij}/\tau])} \right) = 0 .$$

Taking

$$U_i = \frac{T_i}{\tau}, \qquad U_{ij} = \frac{T_{ij}}{\tau}, \qquad u_i = \frac{t_i}{\tau},$$

$$A_i = 1 - \exp[-U_i] + \sum_j (-1)^j \exp[-U_{ij}]$$

$$B_i = U_i \exp[-U_i - \sum_j (-1)^j U_{ij} \exp[-U_i]$$

$$C_i = U_i^2 \exp[-U_i] - \sum_j (-1)^j U_{ij}^2 \exp[-U_{ij}], \qquad (I_1)_i = 1 - \frac{C_i}{A_i} - \frac{B_i^2}{A_i^2},$$

Bartlett finds

$$S(\tau) = \sum_i \left(u_i - 1 + \frac{B_i}{A_i} \right) \Big/ \{\sum_i (I_1)_i\}^{\frac{1}{2}} .$$

The skewness has been calculated by DI CAPUA [17] who finds

$$\gamma = \sum_i \left(2 + \frac{D_i}{A_i} + \frac{3B_iC_i}{A_i^2} - \frac{2B_i^3}{A_i^3} \right) \Big/ \{\sum_i (I_1)_i\}^{\frac{3}{2}} ,$$

where the symbols have the same meaning as above and

$$D_i = \sum_j (u_{ij}^3 \exp[-u_{ij}] - U_{ij}^3 \exp[-U_{ij}]) .$$

9·3.2. *Applications of the maximum likelihood method to emulsion observations.* – As far as the application of the method discussed in the previous paragraph is concerned, emulsions mainly differ from cloud chambers in having a much higher stopping power. Charged particles living 10^{-8} s or longer, once entered inside an emulsion volume of sufficiently large size to bring them to rest, are usually seen to decay after stopping. Unless one disposes of high intensities of well collimated and analyzed beams of such particles (see, in this connection, the experiments on charged K's) the emulsion can only help to establish a lower limit for their mean life.

Singly charged particles, having a mean life shorter than $\sim 10^{-9}$ s, will be seen to decay in flight in emulsion in a proportion which will be higher the shorter their mean life is. If the analysis is confined to those which decay

in flight, Bartlett's method can be applied unaltered; but if all the information derivable from all the events is wanted, the theory will have to be modified. This problem arose in connection with the Σ particles which were seen to decay almost in equal numbers—in flight and from rest.

The analysis can be carried out as follows [9, 12]. Let us distinguish three types of events:

a) those in which the unstable particles decay from rest;

b) those in which they decay in flight but their position and velocity at the moment of decay are such that they would have been brought to rest inside the emulsion volume had they not decayed;

c) those in which they decayed in flight and are not included in *b*).

Events of type *a* and *b* are obviously to be associated with an infinite « potential time ». Nevertheless they supply us with different information. For the *a* type all we know is that they decayed after a time t_a which is the slowing down time (or moderation time) inside the emulsion. For the *b* type we know the exact time t_b of decay from the moment they entered the emulsion stack. This is also true for the *c* type but for these the potential time T_c is not infinite.

The likelihood function, modified to take into account these differences is [12]

$$\mathrm{d}\mathcal{P}' = F\,\mathrm{d}t_{n_a+1}\,\mathrm{d}t_{n_a+2}\,\ldots\mathrm{d}t_n\int_{t_1}^{\infty}\mathrm{d}t_1\int_{t_2}^{\infty}\mathrm{d}t_2\,\ldots\int_{t_{n_a}}^{\infty}\mathrm{d}t_{n_a}P(t_1,t_2,\ldots t_n;\infty\ldots\infty T_{n-n_c}\ldots T_n\,|\,\tau) =$$

$$= \prod_i^{1\ldots n_a}\exp[-t_i/\tau]\prod_k^{1\ldots n_b}\frac{1}{\tau}\exp[-t_k/\tau]\,\mathrm{d}t_k\prod_l^{1\ldots n_c}\exp[-t_l/\tau](\tau(1-\exp[-T_l/\tau]))\,\mathrm{d}t_l\,,$$

where n_a, n_b and n_c are the numbers of events of type *a*, *b* and *c* respectively.

Following the procedure already discussed above the most probable value of τ is found by putting $\partial/\partial\tau \ln F' = 0$ which leads to

$$\hat{\tau} = \frac{1}{n_b+n_c}\left[\sum_1^{n_a}{}_i\,t_i + \sum_1^{n_b}{}_k\,t_k + \sum_1^{n_c}{}_l\left(t_l + \frac{T_l}{\exp[T_l/\tau]-1}\right)\right] \tag{8-9·3}$$

Bartlett's $S(\tau)$-function is here given by the expression

$$S(\tau) = \frac{\sum_i^{1\ldots n_a}\frac{t_i}{\tau} + \sum_k^{1\ldots n_b}\frac{t_k}{\tau} - (n_b+n_c) + \sum_l^{1\ldots n_c}\frac{t_l}{\tau} + \sum_l^{1\ldots n_c}\frac{T_l}{\tau}\exp[T_l/\tau]-1)^{-1}}{\left[n_b+n_c-\sum_l^{1\ldots n_c}\left(\frac{T_l}{\tau}\right)^2\exp[-T_l/\tau](1-\exp[-T_l/\tau])^{-2}\right]^{\frac{1}{2}}}\,, \tag{9-9·3}$$

$\hat{\tau}$ is the value of τ which makes $S(\tau) = 0$. Again S is a nearly linear function of $1/\tau$ and has a standard deviation equal to 1. Limits of confidence for the value of $\hat{\tau}$ are derived as before (see Subsect. **9**·3.1.).

9·3.3. *The residual time method.* – In the deduction of equations (8) and (9) no account has been taken of the fact that the probability of observing the decay of a particle in emulsion is a function of its velocity.

Decays of particles at rest are in fact much easier to detect than those of particles in flight. In the latter case the difference of ionization between the primary and secondary particle may be very small and the angle in the laboratory frame of reference also small. Such an event may easily be taken for a single scattering of a non decaying particle and the observer may fail to register it altogether. The importance of this « kinematic bias » becomes apparent if one takes an extreme case, as for instance

$$n_b = n_c = 0$$

which means: all decays in flight are lost. Then, from (8) we get $\tau = \infty$.

To take into account this bias AMALDI [9] has introduced the « probabilities of detection » $p(\beta)$ of a decay of a particle having a velocity β, which howevery, are most difficult to evaluate.

To overcome this difficulty a different method was suggested by AMALDI *et al.* [10] based on the following idea. Let us consider the simple case of a monoenergetic source of unstable particles. The life time distribution of the particles emerging from this source is obviously

$$\mathrm{d}N = N \exp[-t/\tau]\,\mathrm{d}t/\tau\,,$$

where N is the intensity of the source and $\mathrm{d}N$ is the number of those decaying in the interval $t - t+\mathrm{d}t$, t being measured from the instant of creation. Let $\bar{t}$ be the time necessary for each of these particles to be brought to rest in the emulsion if no decay takes place. The number of decays at rest will be

$$N_R = N\int_{\bar{t}}^{\infty} \exp\left[-\frac{t}{\tau}\right]\frac{\mathrm{d}t}{\tau} = N\exp\left[-\frac{\bar{t}}{\tau}\right].$$

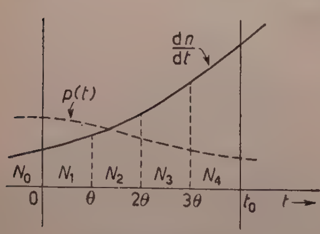


Fig. 3-9·3.

In view of the extension to non-monoenergetic sources which we shall mention later it is convenient to place the origin of the time axis at the point where our monoenergetic particles would have

stopped had they not decayed. Then the particles are assumed to be created at the time $t = -\bar{t}$. We shall subdivide the time interval $0 - \bar{t}$ into a number k of subintervals (in Fig. 3, $k = 4$) so that the length of each of them will be $\theta = \bar{t}/k$, and we shall number them starting from $t = 0$. The number of particles decaying in the k-th interval (the 4-th in Fig. 3) will be

$$N_k = N\left(1 - \exp\left[-\frac{\theta}{\tau}\right]\right),$$

and the subsequent $(k-1)$-th interval

$$N_{k-1} = (1 - N_k)\left(1 - \exp\left[-\frac{\theta}{\tau}\right]\right) = N \exp\left[-\frac{\theta}{\tau}\right]\left(1 - \exp\left[-\frac{\theta}{\tau}\right]\right),$$

and in the $(k-r)$-th one

(10-9·3) $$N_{k-r} = N \exp\left[-r\frac{\theta}{\tau}\right]\left(1 - \exp\left[-\frac{\theta}{\tau}\right]\right).$$

It follows that the number of those which decay at rest is

$$N_0 = N \exp[-t/\tau]$$

and, for example,

(11-9·3) $$\frac{N_1}{N_0} = \frac{\exp[-(k-1)\theta/\tau]}{\exp[-k\theta/\tau]}(1 - \exp[-\theta/\tau]) = \exp[\theta/\tau] - 1\,,$$

and in general, for $k > 1$

(12-9·3) $$N_{k+1}/N_k = \exp[\theta/\tau]\,.$$

In conclusion, from a monochromatic source one can only expect a distribution of decays described by the simple exponential expressions given above. If the observed distribution differs from it, this can only be due to an experimental bias.

The « geometric bias » which was automatically eliminated in Bartlett's method may play her an important role. However in the case of the hyperons, which have a very short lifetime it is not very important. In fact the length of path of hyperons observed in emulsions is rarely longer than 2 or 3 cm. The experiments from which the overwhelming part of the data have been obtained, have been performed using large stacks of emulsions so that the « geometric bias » is certainly small. The truth of this is also apparent from the following remark. So far the discussion has been limited to the case of

a mono-energetic source, which is rarely the case in any experiment on Σ particles. On the other hand we can always select a number of particles created at an energy $E > E_0$ and consider them from the point at which their energy was E_0. In this way our experimental situation does not differ from that of a monochromatic source. If E_0 is chosen so that the residual range is small compared with the dimensions of the stack, the « geometrical bias » which may favour short living particles in comparison with those living a longer time, will be negligible.

On the other hand the detection efficiency $p(\beta)$ mentioned above, can be calculated. We can associate with each of the k intervals a single p_k, since they refer to particles of the same velocity and of the same ionization. Introducing the p's into equation (11) and (12) one gets

$$\frac{n_1}{n_0} = \frac{p_1}{p_0} \exp\left[(\theta/\tau) - 1\right], \tag{13-9·3}$$

$$\frac{n_{k+1}}{n_k} = \frac{p_{k+1}}{p_k} \exp\left[\theta/\tau\right], \tag{14-9·3}$$

where the symbols $n_k = N_k p_k$ have been introduced to represent the numbers of the events in fact observed. If θ is taken sufficiently small one can safely assume $p_1 = p_0 = 1$ and a first approximation value for τ is obtained from (13)

$$\frac{1}{\tau} = \frac{1}{\theta} \ln\left(1 + \frac{n_1}{n_0}\right),$$

which in turn may be used to estimate the efficiencies. For example [10]

$$p_2 = \frac{n_2}{n_1(1 + n_1/n_0)}; \qquad p_3 = \frac{n_3}{n_1(1 + n_1/n_0)^2}; \quad \text{etc.}$$

The p's so calculated may then be used in connection also with the maximum likelihood method.

When the source of particles is not monochromatic, then their energy spectrum at creation must be known. For hyperons, unfortunately, it is not. Considering the simple case of a uniform distribution $f(E) = G(t)$ const (where G is the number of particles created per unit time interval, the time being measured as discussed above), AMALDI *et al.* [10] obtain

$$N_0 = G\tau + (\overline{N} - G\tau) \exp\left[-\theta/\tau\right],$$

$$N_1 = G\theta + (\overline{N} - G\tau)\left(1 - \exp\left[-\theta/\tau\right]\right).$$

REFERENCES (Ch. 9)

[1] J. Podolanski and R. Armenteros: *Phil. Mag.*, **44**, 13 (1953).
[2] R. W. Thompson: *Progress in Cosmic Ray Physics*, vol. III.
[3] R. W. Thompson, A. V. Buskirk, L. R. Etter, C. J. Karzmark and R. H. Rediker: *Phys. Rev.*, **90**, 329 (1953).
[4] K. H. Barker: *Proc. Roy. Soc.*, A **221**, 328 (1954).
[5] M. M. Block: *Phys. Rev.*, **101**, 796 (1956).
[6] J. G. Wilson and C. C. Butler: *Phil. Mag.*, **43**, 993 (1952).
[7] M. S. Bartlett: *Phil. Mag.*, **44**, 249 (1953).
[8] M. S. Bartlett: *Phil. Mag.*, **44**, 1407 (1953).
[9] E. Amaldi: *Suppl. Nuovo Cimento*, **2**, 253 (1955).
[10] E. Amaldi, C. Castagnoli, G. Cortini and C. Franzinetti: *Nuovo Cimento*, **12**, 668 (1954).
[11] M. Annis, W. Cheston and H. Primakoff: *Rev. Mod. Phys.*, **25**, 818 (1950).
[12] C. Castagnoli, G. Cortini and C. Franzinetti: *Suppl. Nuovo Cimento*, **12**, 279 (1954).
[13] J. A. Newth: *Suppl. Nuovo Cimento*, **11**, 290 (1954).
[14] D. I. Page and J. A. Newth: *Phil. Mag.*, **45**, 38 (1954).
[15] H. Cramér: *Mathematical Methods of Statistics*, (Princeton, 1946).
[16] W. B. Fretter, E. W. Friesen and A. Lagarrigue: *Suppl. Nuovo Cimento*, **4**, 569 (1956).
[17] E. Di Capua: private communication.

PART II

THE THEORETICAL PROBLEMS

Introduction.

The following part will be a review of the attempts which have been made to interpret the behaviour of the new particles. As soon as they were discovered such particles posed new kinds of theoretical problems: the first one was that of being produced easily and of having long lifetimes. This problem, which in the following will be called the « paradox » strong production slow decay will be clarified in the next chapter **10**. Its solution led to the idea, particularly emphasized by PAIS, of the associated production of the new particles. A particular scheme of associated production was presented independently by GELL-MANN and NISHIJIMA, a scheme which is based on extending the attribution of an isotopic spin to the new particles. Such scheme was proposed at a time in which the experimental data on the new particles were very few, and, since then, it has received remarkable experimental confirmations. It will be developed in Chapter **11**, where we shall also deal, very briefly, with some attempts to give a mathematical expression to the scheme of Gell-Mann and Nishijima.

The Chapter **12** will consider the problem of the charged heavy mesons; do the different kinds of decays observed for the charged heavy mesons correspond to the same or to different particles? Here the interesting fact appears that the two decay modes $K^+_{\pi 2}$ and $K^+_{\pi 3}$ seem to imply different spin and/or parity, for the decaying particle though, inside the experimental errors, they correspond to the same mass, lifetime, and several other characteristics. This problem still has not had a definite solution; among the many proposals which have been made, and which are described in the Chapter **12**, we recall here the one which appears now to be the most probable solution: the assumption, advanced by LEE and YANG, of the non conservation of parity in the weak interactions, an hypothesis which has already received remarkable experimental confirmations in the field of the β and μ decay phenomena.

The mentioned conclusion that $K^+_{\pi 2}$ and $K^+_{\pi 3}$ correspond, most probably, to different spins and/or parities is reached with the analysis of the $K^+_{\pi 3}$ decay; this will be considered, following DALITZ, in the Chapter **13**.

The Chapter **14** reports on the situation of the neutral heavy mesons; in particular the consequences of the fact that, in a scheme like the one of Gell-Mann and Nishijima, the neutral heavy mesons are represented by complex rather than by real fields, are discussed as indicated by GELL-MANN and PAIS.

The problem of determining the spins of the hyperons and the methods which have been suggested for this, is considered in the Chapter **15**.

Finally the Chapter **16** reports on the question of the interactions responsible for the decays of the new particles; here the situation is certainly not improved with respect to that of accounting for the pion, muon and neutron decay which already could not be considered completely satisfactory; this situation, too, is summarized in the Chapter **16**.

We shall close this summary of the material presented in this part with some nomenclature:

1) The hyperons and heavy mesons will be called « new particles ». All the other particles will be denoted as « old » or « ordinary » particles.

2) The neutrino, electron, muon will be called leptons, a word already in use.

3) The nucleons and hyperons will be sometimes called « baryons », a name introduced by PAIS.

CHAPTER 10.

The Paradox Strong Production-Slow Decay.

10·1. – Why the new particles are strange.

The new particles, whose properties have been extensively described in the previous sections, have in common an apparently curious property: that of being copiously produced in pion-nucleon (or nucleon-nucleon) collisions on the one hand (compare the values of the cross-sections listed in Sect. **18**·1) and of being remarkably stable on the other [1-5 and especially 6]. Such a property is not easily explainable; one would think that if the interaction between one of the new particles, say a Λ^0, and the pions and nucleons is such as to lead to an appreciable rate of production, then the same interaction should lead to a strong rate of decay, unless some very special mechanism is operating.

We may illustrate this point with an example [7] (*): *assume* that the production of the Λ^0 takes place according to the reaction:

$$\pi^- + \mathrm{p} \rightarrow \Lambda^0 + \pi^0 \,. \tag{1-10·1}$$

To agree with the experimental production cross-section of the Λ^0, the matrix element for such a production process must then be, for a pion energy of ~ 1.5 GeV, $\sim \frac{1}{3}$ of the matrix element for the elastic scattering process of the pion from the proton. From what we know about the strength of the

(*) The following argument can be made more quantitative [2] introducing, e.g., in addition to the usual pion-nucleon interaction, an interaction of the kind:

$$H = if\bar{\psi}_{\Lambda^0}\gamma_5\psi_{\mathrm{N}}\varphi^*_{\pi} + \mathrm{h.c.}$$

between the Λ^0, the nucleon and the pion fields. The lifetime for the decay of a Λ^0 into p and π^- is then given by:

$$\tau^{-1} = (f^2/4\pi)(1/M_{\Lambda^0})q(E_q - M_{\mathrm{p}}) \,,$$

where q is the momentum of the decay products in the rest system of the Λ^0; M_{Λ^0} and M_{p} are the masses of Λ^0 and proton,

To get the correct order of magnitude for the decay lifetime of the Λ^0, one must have $f^2/4\pi \sim 10^{-11}$; on the other hand to give rise to a reasonable production cross section $f^2/4\pi$ should be of the order of one to ten.

Yukawa interactions between pions and nucleons it is clear that also the matrix element for the virtual transition:

$$\text{(2-10.1)} \qquad n \to \Lambda^0 + \pi^0 ,$$

must have more or less the same order of magnitude; one also would think that the matrix element for the « almost » reverse reaction:

$$\text{(3-10.1)} \qquad \Lambda^0 \to n + \pi^0 ,$$

should have the same order of magnitude; but, if so, the decay of the Λ^0 through the process (3) (or equivalently through the process $\Lambda^0 \to n+\pi^0 \to \to p+\pi^-$) would be much too fast; in fact assuming the matrix element for the decay transition (3) to have the same order of magnitude as the matrix element for (2), the lifetime of the Λ^0 should be $\sim 10^{-22}$ s, a factor 10^{12} times smaller than the observed one ($\sim 3\cdot 10^{-10}$ s).

The situation is quite similar, of course, for any of the new particles. For example *assume* that the K^0 are produced in a reaction of the kind:

$$\text{(4-10.1)} \qquad \pi^- + p \to n + K^0 .$$

Reaction (2) is in this case substituted by:

$$\text{(5-10.1)} \qquad n \to n + K^0$$

and a fast decay of the K^0 might be expected to take place according to the two step process:

$$\text{(6-10.1)} \qquad K^0 \to n + \bar{n} \to 2\pi$$

leading again to a discrepancy by a factor $\sim 10^{12}$ between the observed and the calculated lifetime.

Notice that in the above deductions there is: *a*) the hypothesis that (1) or (4) are the reactions in which the production of the Λ^0 or respectively K^0 takes place, *b*) the hypothesis that the matrix element for the process (3) has the same order of magnitude as the matrix element for the process (2), or that the matrix element for the first step of process (6) has the same order of magnitude as the one for the process (5).

In order not to get the discrepancy mentioned above one of the two hypotheses must be wrong. This will be examined in the next section in greater detail; we just say here that the two assumptions appeared when the new particles have been discovered, both very natural; this is the reason why the new particles have been called « strange particles ». It must be however emphasized that, from a fundamental point of view, they cannot be regarded as more strange than the proton or the electron [8].

10·2. – Fast and slow processes.

We shall in the following conventionally call « strong » the interactions responsible (*) for the creation of the new particles; and « strong » the processes or reactions produced by such interactions; the order of magnitude of strong decay process is expected from the arguments of the past section, to be not much smaller than 10^{-22} s; we shall then speak of a « fast » decay process. The fact that usually the decay of a new particle takes place in a time longer than this by a factor $10^{10} \div 10^{12}$, will be expressed conventionally by saying that such decay is « slow ».

Using this nomenclature, the conclusions of the last section may be summarized by saying that the fact that the production processes of the new particles are strong, implies that also the decay processes should be fast unless one of the assumptions made in the argument of the last section is incorrect.

We first examine (following [7]) the possibility that the assumption *b*) is incorrect; that is we examine here the possibility that the matrix element for the decay process (3-**10**·1) is not of the same order of magnitude as the matrix element for the production process (2-**10**·1); we shall fix our attention on the Λ^0 but similar arguments hold, of course, for the other particles.

The first point of view we assume is the following: the reaction leading to the strong production of Λ^0 is effectively (1-**10**·1), but there is some mechanism which produces a considerable slowing down of the decay process (3-**10**·1).

In looking for the mechanism in question we first observe that the matrix elements for the production and decay processes refer to rather different energies, the production taking place at energies of several hundreds MeV and the decay implying an energy release of just ~ 37 MeV. Therefore, also if, at the same energy, the two processes (2-**10**·1) and (3-**10**·1) (being « almost » one the reverse of the other) have the same or nearly the same matrix element, it may happen that the matrix element in question depends so strongly on the energy that it is much smaller at the low decay energies, than at the high energies corresponding to the production.

Therefore a possible way out of the paradox strong production-slow decay is to assume [9, 7, 2] that the relevant matrix elements depend strongly on the energy; one may then ask why there should be such a strong energy dependence: it is easy to realize and it will be shown in detail in the next section, that a strong energy dependence of the matrix elements may be obtained if

(*) By this word « responsible » we want to imply that if such strong interactions were absent the rates of the production processes would be reduced by at least a factor 10^{10}; we do not want to imply that such rates would be exactly zero (compare Sect. **11**·8). A more satisfactory separation between strong and slow processes will appear later in the frame of the Gell-Mann Nishijima scheme.

the new particles have high spin values. The attribution of high spin values to the particles may thus provide a possible solution to the paradox.

The following remark may be appropriate to illustrate clearly, on an analogy, what is the kind of mechanism involved; consider a nucleus, initially in its ground state, bombarded by a beam of energetic γ-rays; consider a γn process in which, say, a neutron is expelled from the nucleus, the residual nucleus being left in an excited state of energy E_1; assume that such state has a high value of the spin but is energetically not very distant from the ground state E_0 of the residual nucleus; the neutron having taken a large part of the γ-ray energy, which is assumed to be sufficiently larger than the threshold energy for the process in question. A process like the one described above, may take place with an appreciable cross-section, which does not depend very much on the spin of the excited state, provided the energy of the γ-ray giving rise to the process is sufficiently high.

However if we now fix our attention on the γ decay of the excited state to the ground state its lifetime turns out to be very long due to the fact that the order of the γ-multipole to be emitted is high and the energy difference $E_1 - E_0$ is small: the level E_1 is a metastable one and the excited state thus formed is an isomeric state; as we have seen the process of excitation of such an isomeric state may be considered a « strong » one; yet the decay is a « slow » process. This is produced essentially by the high spin of the state in question.

Considering now the reaction (1-**10**.1) and substituting in the description of the above process, the bombarding π^- to the bombarding γ, the bombarded proton to the initial nucleus, the outgoing π^0 to the outgoing neutron and the Λ^0 to the excited isomeric state, we have a description of how a high spin mechanism may produce the desired effect in solving the paradox strong production-long lifetime of the new particles.

10.3. – The high spin hypothesis.

The analogy of the last section makes it clear that, on assuming a high spin for the Λ^0, an energy dependence of the matrix elements may be obtained such as to make the decay process much slower than would « a priori » be expected [9]. Here a more quantitative examination of this question will be made, always referring to a Λ^0 for definiteness and assuming that the production takes place according to the reaction (1-**10**.1).

The hamiltonian governing the production process, as well as the decay process contains two different interactions; one is the ordinary Yukawa interaction between pions and nucleons, the other is an interaction operator (like the one in the footnote of Sect. **10**.1) which connects the pion, nucleon and Λ^0 fields; the total transition probabilities are built, by perturbation theory,

essentially as products of matrix elements of the above operators; it is of course the matrix element of the (n, π, Λ^0) interaction, which is very strongly energy dependent if the Λ^0 has a high spin; our purpose here will be to show that at the energy involved in the decay, this matrix element M_{decay} is much smaller than the one $M_{\text{prod.}}$ at the production energy if the Λ^0 has a high spin.

The process in question will be treated by the perturbation theory. The production process (1-10·1) is a two step process; we consider only the sequence of the transitions:

(1-10·3) $$\pi^- + p \xrightarrow[a]{} n \xrightarrow[b]{} \Lambda^0 + \pi^0$$

disregarding, for simplicity, the other possible paths. In (1) step a is induced by the YUKAWA, and step b the (n, π, Λ^0) interaction.

The decay is represented by:

(2-10·3) $$\Lambda^0 \xrightarrow[b]{} n + \pi^0 \xrightarrow[a]{} \pi^- + p\,.$$

Also here steps a and b are respectively Yukawa and (n, π, Λ^0) steps.

All what we have to compare are the matrix element for step b both in the production and in the decay process; these are the quantities previously indicated by $M_{\text{prod.}}$ and M_{decay}. The ratio $|M_{\text{decay}}|^2/|M_{\text{prod.}}|^2$ will then give a reasonable idea of the order of magnitude of the ratio between the rates of decay and production. The above matrix elements will be proportional to the amplitude of the Λ^0, π^0 wave (process (1)) or respectively n, π^0 wave (process (2)) in the region in which the interaction between the Λ^0, n, π^0 is strong; the proportionality factor will be not very different in the two cases.

It is in the spirit of the perturbation treatment to approximate such waves by plane waves. For the process (1) the wave in question will be $\exp[i\boldsymbol{p}\cdot\boldsymbol{r}]$, where $\boldsymbol{p}$ is the relative momentum of the Λ^0 and π^0: $\boldsymbol{p} = \frac{1}{2}(\boldsymbol{p}_{\Lambda^0} - \boldsymbol{p}_{\pi^0})$; for the process (2) the wave is $\exp i[\boldsymbol{q}\cdot\boldsymbol{r}]$, where $\boldsymbol{q}$ is the relative momentum of the n and π^0. On account of the conservation of the total angular momentum, the amplitudes which we have to consider are just the projections of the above plane waves with the appropriate orbital angular momentum l; if S is the spin of the Λ^0, then we have $l = S \pm \frac{1}{2}$, according to the parity of the Λ^0; for definiteness we assume $l = S - \frac{1}{2}$ in the following.

The amplitudes $A_l(pr)$, $A_l(qr)$ of the above waves at a distance r are spherical Bessel functions (apart from an unimportant proportionality factor):

(3-10·3) $$A_l(qr) \sim J_l(qr)\,, \qquad A_l(pr) \sim J_l(pr)\,.$$

Call R the radius of the interaction region between the Λ^0 and the nucleon; R is perhaps comprised in the interval $\hbar/Mc < R < \hbar/mc$, where M is the

nucleon and m the pion mass; let us take here

$$R = 0.5 \frac{\hbar}{mc}, \tag{4-10·3}$$

remarking however that relatively small variations of R may be important in calculating the following Table I.

We remark at this point that if, for a given l, k is sufficiently larger than the value k_0 defined by:

$$k_0 = \frac{l}{R},$$

then the function $J_l(kR)$ is of order of magnitude of unity in the interaction region, that is for $r \lesssim R$.

Assume now that p, which is the momentum of the produced Λ^0 in the center of mass system, is sufficiently larger than a p_0 defined by:

$$p_0 = \frac{S - \frac{1}{2}}{R} \simeq \frac{S}{R}. \tag{5-10·3}$$

Then the amplitude of the wave produced, which is, as said above, proportional to $M_{\text{prod.}}$, is close to unity in the interaction region.

The amplitude $J_l(qr)$ of the wave emitted in the decay process, is, on the contrary, much smaller than one, in the interaction region, if $S \gg 1$. This is due to the fact that qR is now < 1 (its value is ~ 0.35 for the choice (4) of R; q is of course determined by the decay energy of 37 MeV); assuming $S \gg 1$ we may approximate $J_l(qr)$ in the interaction region (at $r = R$) by:

$$\frac{(qR)^{S-\frac{1}{2}}}{2S!!}, \tag{6-10·3}$$

a formula which is valid for $S \gg qR$. Expression (6), which is proportional to M_{decay} is much smaller than one, the more so the smaller is R and the larger S; the physical reason is that it is difficult for a wave with a small momentum to traverse a high centrifugal barrier.

The ratio between the square of M_{decay} and the square of $M_{\text{prod.}}$ is (for a production energy larger than the one corresponding to the p_0 given by (5)):

$$\frac{|M_{\text{decay}}|^2}{|M_{\text{prod.}}|^2} \sim \frac{(qR)^{2S-1}}{(2S!!)^2}. \tag{7-10·3}$$

Such ratio is tabulated in the Table I below for several spin values with the choice (4) of R.

TABLE I-10·3. *The values of* $(qR)^{2S-1}/(2S!!)^2$ *for several spin values* $[R = 0.5(\hbar/m_\pi c)]$.

S	3/2	5/2	7/2	9/2	11/2	13/2
$(qR)^{2S-1}/(2S!!)^2$	$1.3\cdot10^{-2}$	$6.6\cdot10^{-5}$	$1.6\cdot10^{-7}$	$1.2\cdot10^{-10}$	$2.6\cdot10^{-13}$	$1.85\cdot10^{-15}$

These values represent (compare the text) the ratios between the rates of decay and production of a Λ^0 for several spin values of the Λ^0, when the production takes place at an energy such that the condition (5-10·3) is satisfied. In view of the roughness of the calculation it is not expected that the figures are reliable to more than a factor 100.

We see therefore that, if a spin, say, 11/2 is assumed for the Λ^0 then we might get a large production cross-section, at an energy of 1.5 GeV or higher of the Λ^0 in the centre of mass system and, at the same time, the observed long lifetime in the decay; this shows that the high spin hypothesis is a mechanism which produces a strong energy dependence of the matrix elements, thus permitting, in principle, to solve the paradox strong production-slow decay.

10·4. – Criticisms of the high spin hypothesis.

Though the discussion of the previous section gives just orders of magnitude, we have presented it in some detail to make clear the mechanism involved; however, in view of the fact that the high spin mechanism has not been successful in the explanation of many aspects of the behaviour of the new particles, we shall not continue here the discussion in detail, but simply summarize a few points worth considering, and, in particular those facts which do not find an adequate explanation using this model [7].

First it is clear that the same mechanism which stabilizes the Λ^0 may explain the long lifetime of the K^0 or any other new particle provided it has a high spin.

Furthermore we remark that, when stabilizing a particle against a fast decay, we have to stabilize it, not only against a fast decay into nucleons and pions, but also against, for example, a fast electromagnetic decay; this is not difficult for the high spin model, because the γ decay is slowed down for the same reason which is effective in the particle decay, namely the high multipolarity of the radiation involved. So, there is no serious objection on those accounts.

However the following are objections to the model:

1) The associated production of the new particles is now an established fact; for example the reaction

$$\text{(1-10·4)} \qquad p + n \to \Lambda^0 + K^0 + p$$

is now well established, but, as far as we know, no case has been observed of a reaction like:

$$\text{(2-10·4)} \qquad p + n \to \Lambda^0 + \Sigma^+ ,$$

which would be energetically favoured and should have been seen; notice that there is no reason why in a high spin model such a reaction should not take place; on the contrary one should think that, at the same available energy, a reaction like (2) should be much more frequent than a reaction like (1-**10**·1), near threshold [7].

In fact, near threshold, only components of the incoming wave with a low orbital momentum are effective; now the final Σ^+ and Λ^0 in reaction (2) having both a high spin, may conserve the total angular momentum being in a state of relative motion with a low orbital momentum; therefore the amplitude of the outgoing wave will be large; on the contrary the state of relative motion of the two final particles in reaction (1-**10**·1) must have a high orbital momentum to compensate for the spin of the produced Λ^0, hence the amplitude of the outgoing wave will be reduced.

2) No reason is given for the fact that, at least at low energies, the production of K^+ is much more frequent than that of K^- (compare Sect. **18**·1).

3) More generally we may say that the absence of many processes, and the existence of others, does not find an adequate explanation in the high spin model. The processes considered under 1) and 2) are just two examples but there are now many others.

4) Let us consider [7] the « cascade particle », which decays according to the scheme:

$$\Xi^- \to \Lambda^0 + \pi^- ;$$

to explain the small decay constant is necessary to attribute to that particle a spin $\sim 25/2$, to prevent its direct decay into the state $n + \pi^-$ which should be also energetically favoured (this is—at least—unaesthetic!).

5) The observed ratio between the mesonic and the non-mesonic decay of a Λ^0 bound in an hyperfragment (compare Sect. **15**·7), as well as the order of magnitude of the lifetime of the hyperfragments [10], can be hardly reconciled with a high spin of the Λ^0.

On the other hand the high spin hypothesis deserved some consideration, when the theoretical aspects of the new particles were first investigated, mainly on account of the following reason: in discussing the high spin mechanism no mention has been made of the effective structure of the new particles; therefore the most simple thing would be in such a model to regard them simply as metastable excited states of the pion nucleon field, the

word metastable having the same meaning as, say, in the α decay with the difference that the centrifugal barrier, and not the Coulomb barrier produces here the metastability. Therefore in the high spin model there is no need of introducing new «elementary» particles, besides the nucleons and the mesons. We may add that some calculations have been done by the Tamm-Dancoff method [11] in order to see whether, for instance, a pseudoscalar meson theory with a reasonable coupling constant may account for the metastable excited states in question. Due to the general difficulties of meson field theory such calculations do not allow to reach more definite conclusions than the elementary arguments already presented and we refer to the original literature. We shall close this section remarking that, for the reasons mentioned above and for many others which will become apparent from the following, the high spin model has to be regarded as unsatisfactory; in the following sections an alternative, entirely different and much more satisfactory approach to the problem of the new particles will be presented.

The problem of trying to determine directly the spins of the new particles through the examination of angular correlations and other effects will be dealt with in Chapters **13** and **15**.

10·5. – The scheme of associated production.

In Sect. **10**·3 we have seen that a possible but unsatisfactory way to solve the apparent paradox strong production-long lifetime could be that of assuming a strong energy dependence of the matrix elements, hence of attributing high spins to the new particles. However, as mentioned in Sect. **10**·1, there is another possibility of avoiding the paradox which consists in rejecting the other hypothesis which was made in arriving at the paradox.

In other words: the strong energy dependence of the matrix elements from the energy, hence the high spin model, was a necessity if we assumed that the production of Λ^0 took place according for example to the reaction $\pi^- + p \to \to \Lambda^0 + \pi^0$ or any other similar reaction, which might, when inverted give rise to a fast decay. We now reject this assumption; more precisely we assume that the processes which lead to a strong production of the new particles are not at all processes like (1-**10**·1), but instead processes which, by their very nature, may not, when inverted, be responsible for their decay. To give an example of such a possible process, consider for instance the reaction:

$$\pi^- + p \to \Lambda^0 + K^0 \tag{1-10·5}$$

and assume that the strong production of a Λ^0 from a π^- p collision does not take place at all from reaction (1-**10**·1), but takes instead place from the reaction (1), so that, each time a Λ^0 is produced, a K^0 is produced together with it (associated production). If this is the case or, more generally, if each

new particle interacts strongly with the pion nucleon field only if there is at least another new particle present (here, of course, present means « present at the point or volume of interaction ») then the interaction which is responsible for the production cannot have any importance in the decay, that is it cannot lead to a fast decay as was the case with (2-**10**·1), because at the moment of decay the new particle is alone.

The interaction responsible for the decay must then be another quite different interaction, and the coupling constant of such an interaction may be chosen to be sufficiently small as to give rise to a decay as slow as we like. Production and decay of the new particles are thus completely unrelated [1, 3-6].

Therefore, if a reasonable explanation can be found of why the strong interactions (the ones responsible for the strong production and the strong absorption) should be such as to produce the new particles only in pairs or more, then a possible solution to the paradox is given. Of course in order that such a solution be the correct one it is necessary that the experiment shows that the production of the new particles is always associated; a single case of clearly unassociated production would be sufficient to make the explanation untenable. The evidence on this point will be discussed in Part III.

Moreover one has to be sure also that the electromagnetic interactions are not such as to give rise to too fast decays of the new particles; in fact the other interactions which are responsible for the decay may be chosen as weak as one likes it, so to say, « ad hoc » to give the observed rates of decay; but the electromagnetic interactions are always present and the scheme must be constructed in such a way that γ decays do not ruin the metastability of the particles.

10·6. – The introduction of a new quantum number.

In the past section it has been shown on the particular example of the reaction (1-**10**·5) how a scheme of associated production may solve the paradox; in this section we simply want to reformulate the same ideas in a more general way, independently of the particular case of the reaction (1-**10**·5); this may be done as follows [6].

We assume that three quite different kinds of interactions are operating between the particles with which we are dealing:

1) « Strong » interactions, responsible among other things for the production of the new particles and having the property of operating only if at least two particles participate in a process (*) (hence giving rise to associated

(*) Attention must be paid to the fact that this does not mean that *all* the reactions implying the participation of at least two particles are strong.

production of the new particles, but completely ineffective for the decay of a new particle into old ones); their order of magnitude is equal to that of the pion nucleon interaction.

2) « Weak » interactions, which may produce among other things the decays of the new particles at the low rates and in the modes indicated by the experiment; their order of magnitude will turn out to be similar to that of the β decay interactions.

3) « Electromagnetic » interactions, which couple every charged particle with the photons; with respect to them the scheme must be so constructed that the decay of the new particles with the intervention of real or virtual photons are not so fast as to imply too short lifetimes for the new particles.

The experimental fact that the strong processes never imply the participation of leptons indicates that the strong interactions are confined to pions, nucleons, and new particles, leptons do not intervene in the strong interactions. On the contrary the weak interactions imply also the participation of leptons, as it is evident from the fact that many decay modes of the new particles have leptons as final products.

It is clear that the above distinction of three kinds of interactions was already present in the physics of the old particles. Thus the strong interactions were those among pions and nucleons, the weak interactions were those responsible for the β decay, μ decay, π decay and so on.

Therefore in the following we shall include in the strong interactions also the usual pion-nucleon Yukawa-like-interaction and in the weak interactions the β like interactions just mentioned. The scheme so constructed includes all the particles, the old and new ones. The only new feature which is introduced is that the strong interactions when acting on the new particles have the property of operating only when at least two new particles participate in a reaction. It is this property which solves the paradox, as we did see in the past section.

The question which arises at this point is then: why do the strong interactions have this property?

The idea on which are based almost all the attempts to give reasons of this property of the strong interactions is essentially the following: besides the usual quantum numbers, like the parity, the spin and the charge, another quantum number is attributed to the particles which intervene in the strong interactions, both the old (pions and nucleons) and the new ones (new particles). The assumption is then introduced that the strong interactions are such that the total new quantum number of a system of reacting particles is conserved; in other words the « strong interactions » are invariant not only with respect to the well known groups (extended Lorentz group and

charge conjugation) but also with respect to some other operation which just corresponds to this new conservation law.

By choosing the strong interactions and assigning the values of the above quantum number in a convenient way it is then possible to arrange things so that in the strong reactions in which new particles are produced from ordinary particles, two new particles at least have to come out in order that the total new quantum number for the final state is equal to the total new quantum number for the initial state.

On the contrary the weak interactions are supposed not to conserve the new quantum number, thus permitting the decay of the new particles to take place as already explained.

REFERENCES (CH. 10)

[1] Y. NAMBU, K. NISHIJIMA and Y. YAMAGUCHI: *Progr. Theor. Phys. (Japan)*, **6**, 615, 619 (1951).
[2] K. NISHIJIMA: *Progr. Theor. Phys. (Japan)*, **9**, 414 (1953).
[3] H. MIYAZAWA: *Progr. Theor. Phys. (Japan)*, **6**, 631 (1951).
[4] S. ONEDA: *Progr. Theor. Phys. (Japan)*, **6**, 633 (1951).
[5] H. SUURA: *Progr. Theor. Phys. (Japan)*, **6**, 893 (1951).
[6] A. PAIS: *Phys. Rev.*, **86**, 633 (1952).
[7] M. GELL-MANN and A. PAIS: *Proc. of the Glasgow Conference on Nuclear and Meson Physics* (London, 1955).
[8] A. PAIS: *Proc. of the Rochester Conf.* (1955).
[9] E. FERMI and R. P. FEYNMAN: unpublished, quoted in [7].
[10] R. GATTO: *Nuovo Cimento*, **1**, 372 (1955).
[11] B. P. NIGAMI and H. P. NOYES: *Phys. Rev.*, **99**, 989 (1956).

CHAPTER 11.

The Gell-Mann and Nishijima scheme.

11·1. – The isotopic spin as the new quantum number.

It has been pointed out in the last section that the property of the strong interactions to give rise only to associated production, is something for which a reason must be given; and that a possible explanation is obtained if the strong interactions between the particles, old and new ones, with which we are dealing, are chosen in such a way as to give rise to the conservation of some new quantum number. This may be done in several ways and each way corresponds to some « classification scheme » for the particles. We refer to the literature (*) for several classification schemes which have been proved to be not completely adherent to the experimental facts and we discuss in the following only the classification scheme proposed independently by GELL-MANN [2, 4, 5] and NISHIJIMA [6-8]; this scheme has been very successful, up to now in interpreting the experimental facts.

This classification scheme is based on the assumption that the notion of isotopic spin has to be extended to the new particles (×) so that to all the particles intervening in the strong interactions (pions, nucleons and each of the new particles) a value of the isotopic spin has to be assigned.

After one has assigned in a way which we shall describe the value of the isotopic spin t and of the third component of the isotopic spin t_3 to each particle the following assumptions are made (+):

(*) In particular we may mention a classification scheme due to Pais ([1], [2], in this last ref. designated as III) based on requiring invariance under rotations in a four dimensional isotopic spin space, and a similar scheme by Salam and Polkinghorne [3]. The scheme of Pais predicts the existence of too many particles (for instance it predicts a Λ^0, Λ^+, two Σ triplets, of which one contains a doubly charged Σ^{++}, two Ξ doublets) giving no reason for the non existence of these particles. The scheme of Salam and Polkinghorne has no inconvenients for the hyperons but probably some inconvenients for the K-mesons since it predicts that only one $K^+_{\pi 3}$ is produced together with Ξ which appears now very improbable; also a K^- might, according to the assignments of [3], be absorbed in flight through the reaction $K^- + p \to \Xi^- + \pi^+$ or at rest by a nucleus through the reaction $K^- +$ « N » $+$ « N » $\to \Xi^- + N + \pi$ (« N » means bound nucleon), which have not, up to now, been observed.

(×) Compare for an older attempt D. C. PEASLEE: *Phys. Rev.*, **86**, 127 (1952).

(+) Small letters will be used for quantities referring to single particles, capital ones for quantities referring to a system of particles. Of course T is obtained from the various t by the usual rules of composition of the vectors, and T_3 is simply the algebraic sum of the various t_3's.

1) The interactions which conserve T and T_3 are strong and are indeed the only strong ones. They are confined to pions, nucleons and new particles.

2) In addition weak interactions exist violating the conservation of T and T_3. They include also the leptons.

3) Of course the electromagnetic interactions too are present.

The order of magnitude of these various kinds of interactions has been already mentioned (Sect. **10**·6).

It may then be shown that the strong interactions give rise necessarily to associated production, or more generally have the property to be operative only if at least two new particles intervene in a process.

The decay procesess are then due to the weak interactions. It may also be shown [5] that under some simple assumptions the electromagnetic interactions which of course violate the conservation of the total T, nevertheless cannot give rise, alone, to a fast decay of the new particles so that the production and decay processes are completely separated as mentioned in Sect. **10**·5.

It appears that to assign an isotopic spin to the new particles is, in a certain sense, almost a necessity; in fact the new particles interact strongly with the pions and the nucleons so that, in any reaction involving real pions and nucleons only, there may be a contribution from virtual intermediate states in which new particles are present. For example an elastic scattering of a pion and a proton may take place through (compare (1-**10**·5))

$$\pi^- + p \to \Lambda^0 + K^0 \to \pi^- + p .$$

If we want to conserve the isotopic spin in such a reaction between the initial and the final state, it must be conserved in each intermediate step, so that an isotopic spin has to be attributed to the Λ^0-K^0 pair. We may then go further, and say that both the Λ^0 and the K^0 have an isotopic spin. One realizes, therefore, that if one believes in the charge independence of the pion-nucleon system and if one uses reaction (1-**10**·5), an established experimental fact, one is naturally led to attribute isotopic spins to the new particles. Of course, it is a matter of personal taste to believe or not in the charge independence of the pion-nucleon system; although such charge independence is well established at low energies (*), little is known experimentally at energies such that virtual transitions to a Λ^0-K^0 state might play a role.

In any case in the next section we shall proceed, following the Gell-Mann–

(*) Among the experimental confirmations of the charge independence for the nucleon-nucleon and pion-nucleon system at low energies we may mention here: *a*) the equality of the np and pp effective ranges and scattering lengths; *b*) the existence of isobaric multiplets among the light nuclei; *c*) Hildebrand's comparison of the rate of the $n+p \to d+\pi^0$, $p+p \to d+\pi^+$ reactions.

Nishijima scheme, to assign t and t_3 values to the new particles, making use of a certain number of established experimental facts. The criterion for this assignment will be analogous to the criterion with which one assigns isotopic spins to the levels of nuclei; in that case, if a nuclear state is found with the same properties in $2t+1$ isobars, but not in $2t+3$, then to this nuclear state a value t for the isotopic spin is attributed, and a value of $t_3 = (N - Z)/2$. In the case of nuclei in assigning in this way the value of the isotopic spin, account has to be taken, of the fact that, due to the Coulomb repulsion or to an excessive neutron excess, some level may be lost because of its instability. Here we shall assume that no phenomenon of this kind occurs, so that if a particle has just $2t+1$ partners having the same mass, spin parity and so on, then t is the value of the isotopic spin for the particle in question; this means that in counting the number of partners of a given particle we shall always assume that no doubly charged particle exists until it will be observed; however one has to keep in mind the possible existence of neutral partners, which may escape experimental detection; this produces a certain amount of indefiniteness in assigning the value of the isotopic spin which will be discussed for each case separatly; also the assignment of t_3 needs in some cases a discussion; for the moment the only assumption will be that, 1) t has the properties of an angular momentum operator so that t_3 may vary, once t has been established from $-t$ to t; 2) to each value of t_3 corresponds, biunivocally, a value for the charge.

11·2. – Particle-antiparticle conjugation.

Before going on it is necessary to point out two properties which the theory under construction is supposed to have; these are: 1) the invariance with respect to particle-antiparticle conjugation, 2) the conservation of the baryonic number. In this section we shall shortly clarify the first property, in the next (Sect. **11·3**) the second one.

Assuming for a moment that only the electromagnetic interactions are present we want to postulate that for any particle with a given mass and spin, the theory predicts the existence of an antiparticle with exactly the same mass and spin; if the particle is charged the antiparticle has the opposite charge and is, therefore certainly distinct from the particle; if the particle is neutral the antiparticle may either coincide with the particle (as the π^0 or the photon) or not (as the neutron). The invariance with respect to particle-antiparticle conjugation (often called also charge conjugation) means now that the evolution of a closed system of particles and antiparticles is left unchanged when all the particles are exchanged with all the antiparticles and viceversa.

The same properties are assumed to hold also when the strong interactions

are introduced; then the invariance with respect to particle-antiparticle conjugation implies in addition to the properties mentioned above that, if to a particle an isotopic spin t has been assigned, the same t value must be assigned to the antiparticle; as far as the t_3 values are concerned it may be shown, considering the transformation properties under rotations of a tensor or of a spinor, and of their complex conjugates, *that particle and antiparticle have opposite values of* t_3.

Finally invariance under particle-antiparticle conjugation is supposed to be true (*) also when the weak interactions are introduced; in addition to the properties mentioned above, one will thus have the equality of the lifetimes of particle and antiparticle, if these are unstable, and if they decay with a definite lifetime.

11·3. – Conservation of the baryonic number.

As is well known, the nucleons satisfy a conservation law in the sense that the number of nucleons minus the number of antinucleons remains constant. It is this law which ensures the stability of the Universe. Now the experimental fact that each time an hyperon appears in a reaction a nucleon or another hyperon disappears and viceversa suggests that the above law has to be completed so as to include the hyperons. Therefore calling N_+ the total number of nucleons plus the total number of hyperons and N_- the total number of antinucleons plus the total number of antihyperons, we shall assume that in any reaction the quantity $N = N_+ - N_-$ remains constant:

(1-**11**·3) $$N = N_+ - N_- = \text{const}\,.$$

Recalling that we did call baryon every particle belonging to the nucleon-hyperon family and antibaryon any particle belonging to the antinucleon-antihyperon family and defining a « baryonic number » which is $+1$ for a baryon, -1 for an antibaryon and 0 for any other particle and which has the additive property, the law expressed by (1) may also be interpreted as the conservation of the total baryonic number N.

(*) It has recently appeared (compare the Sect. **12**·6) that the weak interactions are generally not invariant under particle-antiparticle conjugation. It has however been shown that nevertheless the mass and the lifetime of particle and antiparticle remain (at least to the first order in the weak interactions) the same (compare Sect. **12**·6 and the references quoted there).

11·4. – Assignment of t and t_3 to the Λ^0 and to the heavy bosons.

We now proceed in a way very close to that of [8] in our program of assigning isotopic spins to the various particles and begin by assigning t and t_3 to the Λ^0; we observe that no charged partner of the Λ^0 exists, that is no charged particle with approximately the same mass. The particles having the mass closest to the one of the Λ^0 are in fact the Σ^+ and Σ^-. However the difference in mass between such particles and the Λ^0 (75 MeV) is too large to be attributable to electromagnetic or similar effects; we are therefore entitled to assume that the Λ^0 is an isotopic spin singlet, that is:

$$t_{\Lambda^0} = 0 \, . \tag{1-11·4}$$

Then t_3, which must not be larger than t, must also be zero:

$$t_{3\Lambda^0} = 0 \, . \tag{2-11·4}$$

Notice that these assignments have already an important implication; that is in this scheme the Λ^0 cannot be thought, not even in principle, as an excited virtual state composed of pions and nucleons. This is evident since any such state, with zero isotopic spin must contain an even number of nucleons + antinucleons; therefore it cannot have baryonic number 1. It will be apparent from the following that in this scheme no one of the new particles may be thought as an excited virtual state of the pion nucleon field; they have to be regarded as « elementary » particles, the meaning of the word « elementary » being the same as for the nucleons and the mesons.

We now pass to assign the isotopic spin to the heavy bosons: consider for this purpose the strong reaction (1-10·5):

$$\pi^- + p \to \Lambda^0 + K^0 \, , \tag{3-11·4}$$

(the following argument is valid for any neutral boson K^0 which is produced *strongly* together with a Λ^0 in a π^- p collision).

Since the isotopic spin of the Λ^0 is zero, and the ones of the π^- and p are respectively 1 and $\frac{1}{2}$ we get, assuming, as mentioned, that in a strong reaction the isotopic spin and its third component have to be conserved: $t_{K^0} = \frac{3}{2}$ or $\frac{1}{2}$ and $t_{3K^0} = -\frac{1}{2}$, where use has been made of the fact that $t_{3\pi^-} = -1$, $t_{3p} = \frac{1}{2}$. Now, no doubly charged boson has been observed; therefore we assume $t_{K^0} = \frac{1}{2}$ so that finally

$$t_{K^0} = \tfrac{1}{2} \, , \qquad t_{3K^0} = -\tfrac{1}{2} \, . \tag{4-11·4}$$

We then proceed as follows: since the K^0 has isotopic spin $\frac{1}{2}$ it must have a charged partner having $t = \frac{1}{2}$, $t_3 = +\frac{1}{2}$; it may be either positively, or negatively charged; we *assume*, for a reason which will be apparent from the following that it is positively charged and call it K^+; so

$$t_{K^+} = \tfrac{1}{2}\,, \qquad t_{3K^+} = \tfrac{1}{2}\,.$$

Moreover, to any particle an « antiparticle » must correspond with exactly the same mass, spin and parity; therefore to the bosons K^+ and K^0 two other bosons K^- and $\overline{K}^0$ must correspond which are the charge conjugate of the previous ones. The isotopic spin of $\overline{K}^0$ and K^- will be the same as the one of K^+ and K^0; as far as the third component of the isotopic spin of $\overline{K}^0$, K^- is concerned, we have from a remark in Sect. **11**.2 that (*):

$$t_{3\overline{K}^0} = -\tfrac{1}{2}\,, \qquad t_{3K^-} = \tfrac{1}{2}\,. \tag{5-11.4}$$

Whether the K^0, K^+, K^-, $\overline{K}^0$, are sufficient to cover the variety of the experimentally observed bosons, or it is necessary to introduce more than one boson « family » is a question which will be discussed in Chapter **13**.

11.5. – Assignment of t and t_3 to the Σ and Ξ particles.

We now consider the particles Σ and Ξ. As will be mentioned in Chapter **17** there is evidence that the reaction:

$$\pi^- + p \to \Sigma^- + K^+\,, \tag{1-11.5}$$

occurs as a strong reaction. If we identify the K^+ of this reaction with the K^+ previously introduced, the isotopic spin of the Σ^- has to be either 2 or 1 or 0. The third component t_3 has to be -1 so that the value of t cannot be zero. The value 2 for t is excluded if we exclude doubly charged particles; there-

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(*) Notice, that, if we assigned instead: $t_{3\overline{K}^0} = +\frac{1}{2}$ in contrast with the mentioned remark, a contradiction would arise. Infact nothing could prevent the reaction:

$$K^0 \rightleftarrows \overline{K}^0\,,$$

to take place as a strong reaction, because K^0 and $\overline{K}^0$ would have the same values of t and t_3: This would imply that also the reaction:

$$n + n \to \Lambda^0 + \Lambda^0\,,$$

should be strong, in contradiction with the fact that it does not conserve T.

fore only the value $t = 1$ remains which implies the existence of two partners of the Σ^-, one with $t_3 = 0$, the other with $t_3 = +1$. We are then led to attribute $t_{\Sigma^+} = 1$, $t_{3\Sigma^+} = 1$ to the Σ^+ and to introduce another neutral particle of the Σ family which we may call Σ^0, having $t_{\Sigma^0} = 1$ and $t_{3\Sigma^0} = 0$. The experimental evidence for this particle has been discussed in Sect. **6**·4. It must be stressed that a small mass as well as lifetime difference between the Σ^- and Σ^+ (and also Σ^0) is entirely compatible with the scheme presented here; the situation is different for the K^+ and the K^- which, being one the charge conjugate of the other, must have exactly the same mass and lifetime.

We finally turn to the cascade particle Ξ^-, whose existence has been a stumbling block for older classification schemes [9].

We assign to it a t value equal to $\frac{1}{2}$ and a t_3 value equal to $-\frac{1}{2}$. Another assignment might have been $t = t_3 = 0$ but the former assignment seems preferable for reasons which will be clear in the following (Sect. **16**·10); if the assignment $t_{\Xi^-} = \frac{1}{2}$ and $t_{3\Xi^-} = -\frac{1}{2}$ is accepted, a Ξ^0 must exist with the same value of t but with $t_3 = \frac{1}{2}$. So far there is no evidence for it, but its experimental detection is difficult so that this lack of evidence is by no means against the proposed assignment.

TABLE I-**11**·5. – *Summary of assignments of t, t_3 and strangeness to the various particles (for the antibaryons, the values of the above quantities are not reported; t is the same, while t_3, u and s are the opposite as for the corresponding baryon).*

			Baryons			
	q	t	t_3	u	s	a
n	0	$\frac{1}{2}$	$\frac{1}{2}$	1	0	0
p	1	$\frac{1}{2}$	$\frac{1}{2}$	1	0	0
Λ^0	0	0	0	0	-1	1
Σ^+	1	1	1	0	-1	1
Σ^0	0	1	0	0	-1	1
Σ^-	-1	1	-1	0	-1	1
Ξ^-	-1	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-2	2
Ξ^0	0	$\frac{1}{2}$	$\frac{1}{2}$	-1	-2	2
			Bosons			
π^+	1	1	1	0	0	0
π^0	0	1	0	0	0	0
π^-	-1	1	-1	0	0	0
K^+	1	$\frac{1}{2}$	$\frac{1}{2}$	1	1	-1
K^0	0	$\frac{1}{2}$	$-\frac{1}{2}$	1	1	-1
K^0	0	$\frac{1}{2}$	$\frac{1}{2}$	-1	-1	1
K	-1	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-1	1

We end this section with the preceding table which reports the proposed assignments (*); for completeness also the old, strongly interacting particles, are included in the table. The columns labelled u, s, and a will be discussed in the next section. For the antibaryons the values of t_3, u, s and a are not explicitely reported but are, as mentioned before, simply the values of the corresponding baryon with opposite sign.

11·6. – Introduction of the strangeness.

To derive in the simplest way the consequences of the proposed classification scheme, it is convenient to introduce for each particle, as in column 5 of the Table I-11·5, the quantity u, defined as twice the deviation of the charge q from the third component of the isotopic spin:

(1-11·6) $$u = 2(q - t_3)\,.$$

It is seen that, with the above assignments, particles of the same charge multiplet have all the same value of u. For example K^+ and K^0 have $u = +1$, neutron and proton have $u = +1$; Σ^+, Σ^0, Σ^- have $u = 0$ and so on. One realizes also easily that if the values of t_3 had been assigned in a different way, this would not have been the case. For example if we had attributed $t_3 = \frac{1}{2}$ to the K^- and $-\frac{1}{2}$ to the K^+, then K^+ and K^0 should have had different values of u.

Obviously the relation (1) which may be also read

(2-11·6) $$q = t_3 + \frac{u}{2}\,,$$

is a simple generalization of the usual relations between charge and t_3 with which we are familiar in the case of nucleons and pions:

(3-11·6) $$q = t_3 \pm \tfrac{1}{2} \qquad (+ \text{ for a nucleon, } - \text{ for an antinucleon})$$

(3′-11·6) $$q = t_3 \qquad \text{pions}\,.$$

(*) If one prefers, one may derive these assignments and all their consequences, to be discussed in the following sections, assuming as GOLDHABER has done (*Phys. Rev.*, **101**, 433 (1956)) the existence of only one new particle family (say the $K^+K^0\ \bar{K}^0K^-$) with $T = \frac{1}{2}$ and assuming that all the other new particles are bound states of a nucleon with one or more $\bar{K}^0$, K^- with such a binding energy as to give rise to the observed masses. For instance $\Sigma^- = (nK^-)_{\text{bound}}$, $\Sigma^+ = (pK^0)_{\text{bound}}$ etc. This avoids considering as elementary all the new particles, but, of course, the difficulty arises as we have bound states in which the binding energy has the order of magnitude of the mass of one of the constituents.

The requirement that different particles of the same charge multiplet have the same value of u is, of course, arbitrary, but has the advantage of the simplicity; one may hope of being able to justify it on the basis of some future theory.

It is now evident, from the fact that q and t_3 have the additive property, that u has it too, in the sense that the total U for a system of particles is the sum of the separate u's; the important point is now that U is conserved in the strong reactions; this is simply a consequence of the fact that the charge Q is always conserved, and T_3 is conserved in the strong reactions; the conservation of U is therefore completely equivalent to the conservation of T_3; however the property which U has of having the same value for all the particles of a given charge multiplet makes the following discussion simpler.

Such discussion becomes even simpler, if we introduce, instead of u the quantity s, called the « strangeness », defined for each particle as:

$$\text{(4-11·6)}\qquad \begin{cases} s = u \mp 1 & \text{(— for baryons; + for antibaryons)} \\ s = u & \text{(for bosons).} \end{cases}$$

The values of s are tabulated in column 6 of the Table I-**11**·5.

For a system of particles, from (2), (3) we obtain

$$\text{(5-11·6)}\qquad S = 2\left(Q - T_3 - \frac{N}{2}\right),$$

where N is the baryonic number; N, as already mentioned (Sect. **11**·3), remains constant in any process; therefore also S remains constant in the same way as U in all the processes which conserve T_3 and it is equivalent to speak of conservation of T_3 or of conservation of U or of conservation of S.

Some Authors [10] use instead of s the quantity a (or A) which is simply equal to $-s$ (or $-S$); a will be called the « attribute », and is reported for the various particles in column 7 of the Table I-**11**·5.

11·7. – Fast processes.

The consequences of the scheme may now be best discussed (following [5]) considering first a hypothetical situation (*) in which only the strong interactions are present and examining the processes which may take place in this situation. We shall call these processes *strong* or *fast*; later we shall introduce the weak

(*) Notice that in this situation the masses of the various particles belonging to each charge multiplet would be probably exactly equal; in other words the mass difference between n, p or $\pi^\pm$ and π^0 or Σ^- and Σ^+ and Σ^0 are probably of electromagnetic origin.

and the electromagnetic interactions. In this section we shall consider only the strong processes. The main points are the following:

1) Consider any collision in which only pions and nucleons intervene: the sum of the s-values is initially zero because the individual s-values of the pion and nucleons are all zero (Table I-11·5); in a strong process the sum of the s-values must be zero also in the final state; by comparing the column 6 of Table I-11·5 and observing that the new particles have all values of s different from zero it follows that at least two new particles have to be produced together. Moreover a new particle, left alone, cannot decay at all, if only the strong interactions are present, into ordinary (pions and nucleons) particles, because S would not be conserved. This shows that the attribution of a convenient t_3 value to the new particles and the assumption that strong processes must conserve T_3 solves the paradox strong production-long lifetime.

It is interesting to notice that for the solution of the paradox it is already sufficient to assume that the strong processes conserve T_3 or, what is the same thing, U or S; there seems to be no need for assuming that also T, the modulus of the isotopic spin, is conserved; all what is necessary is the assignment of a strangeness to each particle, as indicated in column 6 of Table I-11·5 and the postulate that such a strangeness is conserved in the strong processes; this is what has been pointed out by SACHS [10], the concept of an isotopic spin being, according to Sachs' point of view, a secondary one. According to the point of view exposed here, on the contrary, although in the end only the conservation of T_3 plays a role in solving the paradox, also the conservation of T is assumed to hold; whether this is the case or not may only be decided by the experiments, looking for those strong processes which may constitute a specific test of the conservation of T; LEE [11] has indicated a certain number of such processes; other tests of T conservation are indicated (*) in [12] and [13]; some evidence on this point may be derived also from the hyperfragments (Chapter 20); unfortunately there does not seem to be any real process easily accessible which would be allowed assuming just the conservation of T_3, and forbidden assuming also the conservation of T; such a process would of course best discriminate between the two points of view. A process of this kind (far from any experimental possibility) would be $\Lambda^0+\Lambda^0 \to \Lambda^0+\Lambda^0+\pi^0$ which is allowed by the conservation of T_3 (or equivalently of the strangeness) but not by the conservation of T.

(*) Some processes considered by the above Authors are:

$$\begin{cases} K^-+d \to \Sigma^0+\pi^-+p & (A_1) \\ K^-+d \to \Sigma^-+\pi^0+p & (A_2) \end{cases} \qquad \begin{cases} K^-+d \to \Sigma^-+p & (B_1) \\ K^-+d \to \Sigma^0+n & (B_2) \end{cases} \qquad \begin{cases} K^-+{}^4He \to \Sigma^0+{}^3H & (C_1) \\ K^-+{}^4He \to \Sigma^-+{}^3He & (C_2) \end{cases}$$

If T is conserved then one should have $R(A_1)=R(A_2)$; $R(B_2)=2R(B_1)$; $R(C_2)=2R(C_1)$ Here $R(x)$ means: rate of process x.

2) While the fact that the strong interactions conserve T_3, that is S, implies as we have just said, that only the reactions in which at least two new particles intervene may be strong, the reverse is of course not true; not *all* the reactions in which two new particles intervene are such that S may be conserved. A convenient graphical representation of the possible strong reactions is presented in the following Fig. 1.

The particles are represented in several columns according to the value of s. In ordinates the rest masses of the particles are given, in units of the pion mass.

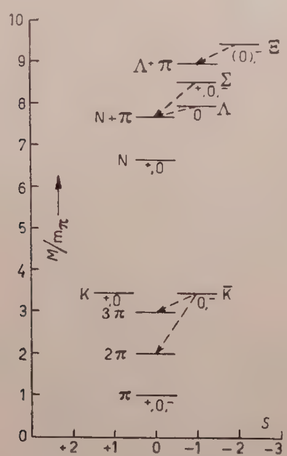

Fig. 1-11·7. – Classification of the pions, K-mesons, nucleons, and hyperons. The vertical scale is the mass in units of the pion mass. Electric charges are indicated, that in parenthesis being suggested but not established. Dashed arrows indicate known decay processes (from [10]).

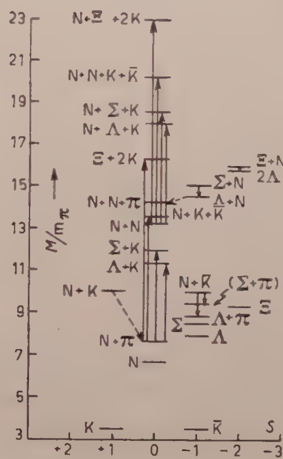

Fig. 2-11·7. – Extension of Fig. 1 to include various reactions. Solid vertical arrows indicate fast processes; for production of particles the arrows point upward, for disintegrations, downward. Threshold or Q value (in the centre of mass system) is indicated by the length of the arrow (from [10]).

Considering a production process in which the initial particles are pions and nucleons, one may say that for any new particle produced among the ones of column $s = -1$, a particle in column $s = +1$ has to be produced; for any particle produced belonging to the column $s = -2$ (at the moment there is only the cascade particle) two particles in the column $s = +1$ have to be produced; and so on. On the contrary in the absorption reactions for any particle absorbed belonging to column $s = -2$ two particles must be produced in column $s = +1$ (plus any number may be produced of course in column $s = 0$); or four particles in column $s = +1$, one in column $s = -1$ and any number in column $s = 0$; and so on.

Fig. 2 is an extension of Fig. 1 in which several strong processes

constructed following the above criteria are indicated; the threshold, in the case of production, and the energy release (both in the centre of mass system) are indicated by the length of the arrows in units of the pion mass; for example a pion hitting a nucleon may give rise to a Λ, K pair with a threshold of ~ 890 MeV; or to a Ξ^- and two K^+ with a threshold of 2.21 GeV and so on; and a Σ interacting with a nucleon may give rise to a Λ^0 plus a nucleon with an additional release of energy of 75 MeV; a Ξ^- interacting with a proton gives rise by a fast process to two Λ^0's with an energy release of ~ 30 MeV; and so on

11·8. – Weak and electromagnetic processes.

We now want to see what happens if besides the strong interactions also the weak ones are present; the processes which may take place only through the intervention of the weak interactions will be called *slow*. Since the weak interactions are assumed not to conserve T_3 (or S), the slow processes will violate the conservation of T_3 (or S).

The decay processes of the new particles may now take place. They have been already reported in Fig. 2-**11**·7 by dashed lines.

Their rate will be determined by the magnitude of the weak interactions; this will be discussed later (Chapter **16**) but we have already said—and we repeat it here—that agreement with the observed rates of decay is obtained if the order of magnitude of the weak interactions is assumed to be not very different from that of the so called Universal Fermi interaction.

Of course the weak interactions may in principle give rise also to production processes of new particles by processes which may violate the conservation of S. However such processes may be neglected in ordinary experimental conditions, having the same order of magnitude of the inverse β decay.

Finally we complete the scheme introducing the electromagnetic interactions. Processes which take place with the intervention of the electromagnetic interactions will be called « electromagnetic ».

We want first to consider a situation in which only the strong and electromagnetic interactions are supposed to be present and show that the conservation of T_3 (that is of S) still holds when some assumptions are made on the form of the electromagnetic interactions, (though of course the conservation of T is not valid). Therefore in such a situation decays of the new particles cannot take place, since such decays take place with changes of S. This is satisfactory because if the strong and electromagnetic interactions alone were able to produce the decays of the new particles, the rates of such decays should probably be much larger than observed.

To prove the above statement, namely that the electromagnetic inter-

actions while violating usually the conservation of T still conserve T_3, the assumption which we make [5] is the following: the lagrangian including the strong and electromagnetic interactions is obtained from the lagrangian of the strong interactions only by the substitution

$$\frac{\partial}{\partial x_\mu}\Phi^i \to \frac{\partial}{\partial x_\mu}\Phi^i - iq_i A_\mu(x)\Phi^i\,, \tag{1-11·8}$$

whenever the operator $\partial/\partial x_\mu$ is applied to a field operator Φ^i; in (1) q_i is the charge annihilated by the field operator Φ^i in question or minus the charge created by the same.

Consider now the original lagrangian of the strong interactions. We may imagine it to be written as a sum of different products of creation and annihilation operators of the various intervening particles, each product containing possibly derivatives $\partial/\partial x_\mu$ of the operators. In each product the numbers of creation and destruction operators Φ^i, Φ^{j+} must be such that the total strangeness is conserved. The fact that the total strangeness is conserved depends indeed only on the above numbers.

But the electromagnetic terms obtained from the product in question through the substitution (1) all contain the same number of creation and destruction operators as the original product, except for the appearence of $A_\mu(x)$ factors. Therefore the lagrangian obtained from the strong interaction lagrangian through (1) also conserves the strangeness.

We notice, that, in principle one might consider more general electromagnetic interactions than the ones obtained by (1). It is possible to construct such interactions in a gauge invariant way, and at the same time in such a way that they do not conserve the strangeness. For instance if ψ_{Σ^+} is the destruction operator of a Σ^+ and ψ_p^+ the creation operator of a proton, an interaction like

$$\lambda\psi_p^+\gamma_4\sigma_{\mu\nu}\psi_{\Sigma^+}F_{\mu\nu} \qquad F_{\mu\nu} = \text{electromagnetic field}, \tag{2-11·8}$$

would be gauge invariant but would not conserve the strangeness. The assumption (1) means that an interaction like (2) has to be excluded « a priori » from the theory. To this assumption GELL-MANN has given the name of « principle of the minimal electromagnetic interaction » meaning by this that the electromagnetic field does not possess any other interaction with the particles except the usual one obtained through (1) which is just that with the usual fourcorrent of charged particles.

Having shown that a theory constructed only through the strong and electromagnetic interactions conserves (under the assumption (1)) the strangeness, the electromagnetic processes may be divided in two kinds:

a) *strong electromagnetic processes* which imply only the strong and electromagnetic interactions or possibly only the electromagnetic ones, and which conserve the strangeness (e.g. photoproduction of new particles);

b) *weak electromagnetic processes* which imply in addition the weak interactions and which do not conserve the strangeness (e.g. radiative decay of a new particle).

Both for processes *a*) and *b*) we may say in a crude way that, usually, their rates will be reduced with respect to those of the corresponding strong or weak processes by a factor of the kind $(e^2/\hbar c)^n$; so the photoproduction of a K^- and Λ^0 from a nucleon is expected to have a cross-section 1/137 that of the corresponding process induced by pions and the decay of a new particle with the emission of a photon is expected to be slower than the one without such emission by a factor 1/137. Of course the above statements are only crude ones, and have to be discussed better in any single case. The point is however that the introduction of the photon (under the assumption (1)) does not change the classification in strong and weak processes and in particular does not lead to fast electromagnetic decays of the new particles.

11·9. – The possibility of including other particles in the scheme.

It may be finally asked [5] whether if another new particle is discovered which decays slowly and is produced abundantly it may be fitted in the scheme; in other words how much space is available in the Gell-Mann and Nishijima scheme for other new particles.

If we confine ourselves to particles with a charge not larger than one, t can assume only the values 0, $\frac{1}{2}$ and 1. The value of u corresponding to each value of the charge for any one of the above t values may then be easily determined.

If we maintain that: 1) only those u values are possible which are the same for the particles in the different charge states of a given isotopic spin and 2) that the charge multiplicity for a given t must be $2t+1$ it appears that the only possible assignments in the Gell-Mann–Nishijima scheme are the following.

If we require that no two different bosons, nor two hyperons may exist which have the same charge and the same strangeness (in which case a strong electromagnetic decay would take place unless slowed by some special mechanism) we see comparing the Table I that only a few places are actually at the disposal of other new particles; namely, both for the bosons and for the hyperons only the assignments in row I and VI; all of these have $u = \pm 2$. We conclude that there is still place in the theory for particles with $u = \pm 2$

and corresponding signs ± 1 of the charge but not more (+). In terms of strangeness, using (4-**11**·6, 5-**11**·6), we may therefore have a negative baryon of strangeness -3 and a positive one of strangeness $+1$, and the corresponding antibaryons; a negative boson with strangeness -2 and a positive one (its charge conjugate) with strangeness $+2$.

TABLE I-**11**·9. – *Possible assignments in the Gell-Mann and Nishijima scheme under the assumptions* 1), 2) (*see text*).

	u	q	t
I	-2	-1	0
II	-1	-1, 0	$\frac{1}{2}$
III	0	0	0
IV	0	-1, 0, 1	1
V	1	0, 1	$\frac{1}{2}$
VI	2	1	0

We finally notice that the limitation to $u \leqslant 1$ would exclude all the presently unobserved particles (+).

There is at the moment no really compulsory theoretical reason for such a limitation, although it may simplify the theory [14, 20].

11·10. – Mathematical formulation of the Gell-Mann-Nishijima scheme.

The scheme of Gell-Mann and Nishijima may be simply expressed in lagrangian (or hamiltonian) form (†); to each of the particles with which we are dealing a field operator is associated: the task is to write down the interactions between such fields in such a way that they « contain » all the conservation properties postulated in the scheme of Gell Mann and Nishijima. Here we restrict ourselves to write down the strong interactions; how to introduce the electromagnetic interactions has been already mentioned, and the weak interactions will be discussed in Ch. **16**.

(+) There is always of course the possibility of an additional neutral boson with zero s, q and t (row III of the Table I); if such boson has the same mass as the usual π^0 and interacts with the nucleons, say, 5 or 10 times less than the usual π^0, it would be rather difficult to discover its existence. The only way would be through lifetime measurements; this new π^0 should have a longer lifetime than the old one

(†) This has been done first by D'ESPAGNAT and PRENTKY [14]; in this section we shall however follow the treatment by SALAM [15] which, though leading to the same lagrangian as that of [14] differs from the treatment of [14] in one respect (compare the next footnote (*)). A rather different treatment is instead that by UTIYAMA [16].

The most straightforward way to write down the lagrangian of the strong interactions is, following [15], simply to re-express step by step what has been said in Sect. **11·4** and **11·5**. The field associated to each strongly interacting particle is characterized (not only by certain space spin properties, but also) by its transformation properties under rotations in a threedimensional ortogonal space, the space of the isotopic spin.

These rotation properties are characterized by the eigenvalues of $\boldsymbol{t}$ and t_3 which are assigned to each particle in the way indicated in the Table I-**11·5**; thus the fields K, N and Ξ transform as two component (iso)spinors, π and Σ as (iso)vectors and Λ^0 as an (iso)scalar, under rotations in the isotopic spin space.

From now on in this section a (destruction) field operator $\psi_A(x)$ for the particle A will be simply indicated [15] by A; so instead of $\psi_{\Lambda^0}(x)$ we shall write Λ^0; instead of

$$\psi_{\mathrm{N}}(x) = \begin{pmatrix} \psi_{\mathrm{p}}(x) \\ \psi_{\mathrm{n}}(x) \end{pmatrix} \qquad \text{or of} \qquad \psi_{\Xi}(x) = \begin{pmatrix} \psi_{\Xi^0}(x) \\ \psi_{\Xi^-}(x) \end{pmatrix}$$

we shall write

$$\mathrm{N} = \begin{pmatrix} \mathrm{p} \\ \mathrm{n} \end{pmatrix} \qquad \text{or respectively} \qquad \Xi = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}.$$

Similarly:

$$\mathrm{K} = \begin{pmatrix} \mathrm{K}^+ \\ \mathrm{K}^0 \end{pmatrix}, \qquad \boldsymbol{\Sigma} = \begin{pmatrix} \Sigma_1 \\ \Sigma_2 \\ \Sigma_3 \end{pmatrix}, \qquad \boldsymbol{\pi} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix},$$

for the K particle spinor, for the $\boldsymbol{\Sigma}$ vector and for the $\boldsymbol{\pi}$ vector. In the last two cases the components of these vectors are related to the operators Σ^+, Σ^-, Σ^0 and respectively π^+, π^-, π^0 by the usual decomposition:

$$\text{(1-11·10)}_1 \qquad \begin{cases} \Sigma^+ = -\dfrac{1}{\sqrt{2}}(\Sigma_1 + i\Sigma_2)\,, \\[2ex] \Sigma^- = \dfrac{1}{\sqrt{2}}(\Sigma_1 - i\Sigma_2)\,, \\[2ex] \Sigma^0 = \Sigma_3\,, \end{cases}$$

$$\text{(1-11·10)}_2 \qquad \begin{cases} \pi^+ = -\dfrac{1}{\sqrt{2}}(\pi_1 + i\pi_2)\,, \\[2ex] \pi^- = \dfrac{1}{\sqrt{2}}(\pi_1 - i\pi_2)\,, \\[2ex] \pi^0 = \pi_3\,. \end{cases}$$

The (destruction) field of an antiparticle will be indicated by a bar; so, for instance,

$$\overline{\mathrm{K}} = \begin{pmatrix}\mathrm{K}^-\\ \overline{\mathrm{K}}^0\end{pmatrix}; \quad \overline{\mathrm{N}} = \begin{pmatrix}\overline{\mathrm{p}}\\ \overline{\mathrm{n}}\end{pmatrix} = \begin{pmatrix}\overline{\psi}_{\mathrm{p}}\\ \overline{\psi}_{\mathrm{n}}\end{pmatrix}, \quad \text{etc.}$$

Using the above fields one asks to construct the interactions between them in such a way that:

1) The total charge is conserved, that is the Lagrangian is invariant under the transformation $\psi_A \to \exp[i\alpha\varphi]\psi_A$ for any charged field operator ψ_A (*)

2) The baryonic number is conserved; that is the lagrangian is invariant under the transformation $\psi_B \to \exp[i\beta\varphi]\psi_B$, where ψ_B is a baryonic field.

3) The invariance under particle antiparticle conjugation is satisfied.

4) They are invariant under rotations in the above mentioned three-dimensional isotopic spin space (+).

Let us confine to threelinear interactions, although this is not necessary, and begin by satisfying points 1) and 4).

We remark first that, given a spinor $u = u_\lambda$ $(\lambda = 1, 2)$, the transformation properties under rotations of the spinor u and its complex conjugate u^* are not the same; more precisely u_1^* transforms as u_2 and u_2^* and $-u_1$; otherwise expressed u^* transforms as $i\tau_2 u$ under rotations.

With this in mind we may construct all the charge conserving, rotation invariant threelinear interactions, by the standard technique of the Clebsch-Gordon coefficients. Consider as an example the usual interaction between pions and nucleons. Through the spinor N and the complex conjugate spinor $\overline{\mathrm{N}}$ (to conserve nucleonic number) we may construct the three quantities:

$$(2\text{-}11{\cdot}10) \qquad -\overline{\mathrm{n}}\mathrm{p}\,, \qquad \frac{1}{\sqrt{2}}(-\overline{\mathrm{n}}\mathrm{n} + \overline{\mathrm{p}}\mathrm{p})\,, \qquad \overline{\mathrm{p}}\mathrm{n}\,,$$

which transform under rotations respectively as the $M = 1, 0, -1$ component

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(*) Here lies the difference between Salam's [15] and d'Espagnat-Prentky's [14] points of view; the last mentioned Authors in fact, instead of requiring from the start charge conservation, require the invariance of the strong interactions with respect to isotopic reflections (in addition, of course, to the invariance with respect to rotations, point 4) below); they then show that the conservation of charge follows automatically, provided that the isotopic reflection properties of the various particles are appropriately chosen, that the interactions are threelinear, and that particles with $u > 1$ do not exist; Racah [17] and Murai [18] have given a simpler proof of these points.

(+) Of course Lorentz invariance is assumed; in the following the attention will be focussed, however, on the problem of constructing isotopic rotation invariants.

of Y_1^M. A charge conserving invariant may then be constructed with the pion field:

$$-\frac{1}{\sqrt{3}}\,\bar{n}p\pi^- \,, \qquad -\frac{1}{\sqrt{6}}(-\bar{n}n+\bar{p}p)\pi^0 \,, \qquad +\frac{1}{\sqrt{3}}\,\bar{p}n\pi^+ \,,$$

where it is assumed that π^-, π^0, π^- transform again as Y_1^M respectively. Recalling the decomposition $(1\text{-}\mathbf{11}{\cdot}10)_2$ we have finally:

$$-\frac{1}{\sqrt{6}}(\bar{n}p+\bar{p}n)\pi_1-\frac{1}{\sqrt{6}}(\bar{p}p-\bar{n}n)\pi_3+\frac{i}{\sqrt{6}}(\bar{n}p-\bar{p}n)\pi_2 \,,$$

which is (more concisely) proportional to:

$$\overline{N}\boldsymbol{\tau}N\boldsymbol{\pi} \,,$$

where $\boldsymbol{\tau}\equiv(\tau_1,\tau_2,\tau_3)$ are the Pauli matrices.

As another example let us consider the interaction between Ξ, K and Σ; the construction of the invariant interaction is completely similar to that of the previous one, the $\boldsymbol{\Sigma}$ isovector replacing the pion. There is however a difference in combining the spinors Ξ and K because one has to do now with two spinors and not with a spinor and a complex conjugate spinor; therefore the three components of the spherical harmonic of order one which replace (2) are respectively:

$$\Xi^0K^+ \,, \qquad \frac{1}{\sqrt{2}}(\Xi^-K^+ + \Xi^0K^0) \,, \qquad \Xi^-K^0 \,.$$

Combining these with the $\overline{\Sigma}$ operators the charge conserving rotation invariant combination is:

$$\frac{1}{\sqrt{3}}\,\Xi^0K^+\overline{\Sigma}^+ - \frac{1}{\sqrt{6}}(\Xi^-K^+ + \Xi^0K^0)\overline{\Sigma}^0 + \frac{1}{\sqrt{3}}\,\Xi^-K^0\overline{\Sigma}^- \,.$$

Using the complex conjugate of equation $(1\text{-}\mathbf{11}{\cdot}10)_1$ we get:

$$-\frac{1}{\sqrt{6}}(\Xi^0K^+ - \Xi^-K^0)\overline{\Sigma}_1 - \frac{1}{\sqrt{6}}(\Xi^-K^+ + \Xi^0K^0)\overline{\Sigma}_3 + \frac{i}{\sqrt{6}}(\Xi^0K^+ + \Xi^-K^0)\overline{\Sigma}_2 \,,$$

which is equal, apart from a proportionality factor to:

$$i\overline{\boldsymbol{\Sigma}}\cdot\Xi\boldsymbol{\tau}\tau_2K \,.$$

In a similar way all the possible threelinear interactions may be easily constructed; hermiticity may be insured adding the hermitian conjugate to

each interaction and invariance under particle-antiparticle conjugation adding to each interaction its conjugated one; it may be shown (compare [15]) that the requirement of invariance under particle-antiparticle conjugation implies that the coupling constants must be real.

The interaction lagrangian turns out to be a sum of eight different terms, with eight different real coupling constants; four terms imply interactions between pairs of baryons and pions, four between pairs of baryons and K's. We shall not write here the lagrangian explicitly (one may find it in the references quoted [14, 15]) noting that what is interesting is not so much, at the moment, its explicit form, as the possibility which one has of giving a mathematical structure to the scheme of Gell-Mann and Nishijima.

Concerning the lagrangian itself there is not much, presently, to be done with it; one may attempt of course perturbation calculations (the scattering of K^+ by nucleons will be reported, as an example, in Sect. **19**.3); however in view of the uncertainties in the spin parity assignments to the various particles and of the complete indeterminacy of the eight constants involved (which may also be more than eight if, for instance, two kinds of K exist (compare Chapter **12**)) this is not a very brilliant program.

Some attempts are in course of evolution aiming to find relations between the eight (or more) coupling constants. The general idea is to require that the lagrangian must have a greater degree of symmetry; such symmetries as well as the various methods to obtain them have been examined in a forthcoming paper by D'ESPAGNAT, PRENTKY and SALAM [19] to which we refer.

Such paper discusses also the relation between the approaches based on considering isotopic rotations in three dimensions and those (for instance SCHWINGER [20], MATTHEWS and SALAM [21]) based on postulating some kind of invariance under four dimensional isotopic rotations, always in the frame of the Gell-Mann and Nishijima scheme.

It must be emphasized that in all these approaches for any new particle a new field is introduced and the lagrangian is written as a sum of lagrangians of the various particles plus their interactions; in other words each particle is accepted as it is and no attempt is made to predict in any way its properties, such as its mass and spin. This situation is not new, because the same holds already for the old particles, but it is clear that with the increasing number of particles it becomes more and more unsatisfactory; in this sense attempts of the kind of that of RAYSKI [22], who, attributing a structure to the bare particles and writing down wave equations for such structured particles tries to derive the mass values from an eigenvalue problem, depart from the conventional line.

To conclude we may say that it is possible to give a mathematical expression to the scheme of Gell-Mann and Nishijima; it does not seem however possible to learn at present much from such a mathematical formulation.

REFERENCES (Ch. 11)

[1] A. Pais: *Proceed. Nat. Ac. of Sciences* (*U.S.A.*), 184 (1954); *Proceed. of the Glasgow Conference* (1954), p. 350; *Proceed. of the Rochester Conference* (1955).
[2] M. Gell-Mann and A. Pais: *Proceed. of the Glasgow Conference* 1954 (London, 1955).
[3] A. Salam and J. C. Polkinghorne: *Nuovo Cimento*, **2**, 685 (1955).
[4] M. Gell-Mann: *Phys. Rev.*, **92**, 833 (1953).
[5] M. Gell-Mann: *Suppl. Nuovo Cimento*, **4**, 848 (1956).
[6] K. Nishijima and T. Nakano: *Progr. Theor. Phys.* (*Japan*), **10**, 581 (1953).
[7] K. Nishijima: *Progr. Theor. Phys.* (*Japan*), **12**, 107 (1954).
[8] K. Nishijima: *Progr. Theor. Phys.* (*Japan*), **13**, 285 (1954).
[9] A. Pais: *Physica*, **19**, 869 (1953).
[10] R. G. Sachs: *Phys. Rev.*, **99**, 1573 (1955).
[11] T. D. Lee: *Phys. Rev.*, **99**, 337 (1955).
[12] K. M. Case, R. Karplus and C. N. Yang: *Phys. Rev.*, **101**, 358 (1956).
[13] D. Feldman: *Phys. Rev.*, **103**, 254 (1956).
[14] B. D'Espagnat and J. Prentky: *Phys. Rev.*, **99**, 328 (1955); *Nucl. Phys.*, **1**, 33 (1956); *Phys. Rev.*, **102**, 1684 (1956).
[15] A. Salam: *Nucl. Phys.*, **2**, 173 (1956).
[16] R. Utiyama: *Phys. Rev.*, **100**, 248 (1955).
[17] G. Racah: *Nucl. Phys.*, **1**, 301 (1956).
[18] Y. Murai: *Nucl. Phys.*, **1**, 657 (1956).
[19] B. D'Espagnat, J. Prentky and A. Salam: to be published, CERN preprint.
[20] J. Schwinger: *Phys. Rev.*, **104**, 1164 (1956).
[21] P. Matthews and A. Salam: *Nucl. Phys.*, **2**, 582 (1956).
[22] J. Rayski: *Nuovo Cimento*, **4**, 1231 (1956); **5**, 872 (1957).

CHAPTER 12.

The Problem of the Bosons.

12·1. – Introduction.

We now come to a question which was left open in the past sections; we recall that, in developing the general theory we introduced a K^0, a K^+, a $\overline{K}^0$ and a K^-; we called K^0 the boson produced strongly with a Λ^0 and K^+ its positively charged partner; moreover two other bosons were introduced, $\overline{K}^0$ and K^- the charge conjugate of the previous ones; these four bosons have strangeness $S=\pm 1$ and isotopic spin $\frac{1}{2}$. Now we may ask if the above four bosons cover the totality of the observed heavy mesons which have been discussed in Chapter **2**, **3** and **4**; in other words we may ask whether or not all the observed mesons are simply different decay modes of the four bosons introduced above.

We shall fix our attention, in this Chapter, on the positively charged bosons, for which the experimental situation is clearer; however on account of what we have said in Sect. **11**·2 (compare also Sect. **12**·6) it is presumable that the same arguments are appropriate to the case of the negative bosons; for the neutral bosons the situation is more involved and the problem of the neutral bosons will be discussed separately (Chapter **14**).

As discussed in Chapter **2** we know presently five kinds of positive heavy mesons; they were denoted as $K^+_{\pi 2}$, $K^+_{\pi 3}$, $K^+_{\mu 2}$, $K^+_{\mu 3}$, K^+_{e3}.

The bosonic character of all these mesons appears to be established, from their decay modes. The question is if all these different kinds of decays are different decay modes of the same boson K^+ which was introduced, or if two or more different positive bosons are necessary to explain the different kinds of observed decays.

It was pointed in Sect. **2**·2 and **2**·5 that all the observed kinds of decay correspond, inside the present experimental errors, *to the same mass and the same lifetime*. Here we want first to take the point of view that the future experiments confirm the lifetime identity and only in Sect. **12**·4 discuss the situation if this will not be so.

If the mass and lifetime are equal, it appears at first presumable that the

different kinds of decay should be simply different decay modes of the same boson. This conclusion appears to receive support from two other facts:

1) If a beam which contains certain definite fractions of the different kinds of positive bosons is scattered from nuclei, the fractions are unchanged within the experimental error (presently 30%); the relevant experiments will be described in Sect. **19**·2.

2) The fractions of the different kinds of positive bosons in a beam appear to be independent of the energy and of the nature of the particle producing the beam. This conclusion appears from an inspection of the Table I-**2**·3.

At this point however a difficulty is encountered which is not easy to solve and has been the origin of much thinking; the difficulty lies in the fact that for concluding that different kinds of decays correspond to different decay modes of the same boson, it seems necessary that, besides satisfying all the previous conditions, they have the same spin and the same parity; but, as we shall see this does not seem to be the case in the present instance.

Let us consider just the $K^+_{\pi 2}$ and the $K^+_{\pi 3}$. For the $K^+_{\pi 2}$ the parity is directly determined by the orbital angular momentum of the two final pions, which, if the angular momentum is conserved in the decay, is equal to the spin S of the $K_{\pi 2}$; so only the following spin parity assignments are, in principle, possible for the $K_{\pi 2}$:

$$0+,\quad 1-,\quad 2+,\quad 3-,\quad 4+\quad \text{etc.} \tag{1-12·1}$$

In addition the 1 —, 3 — etc., assignments are impossible in view of the result of the Osher *et al.* [1] and Schwartz *et al.* [2] experiments, which indicate that the $K^0_{\pi 2}$ decay into two neutral pions exists.

For the $K^+_{\pi 3}$ the analysis of the angular and energy distribution of the decay pions which will be presented in Chapter **13**, shows that only the values:

$$0-,\quad 2- \tag{2-12·1}$$

are probable.

Therefore if we believe in the analysis mentioned above, leading to the result (2), the $K^+_{\pi 2}$ and the $K^+_{\pi 3}$ must differ in spin and/or parity and cannot be different decay modes of the same particle.

This being the situation two possibilities appear to exist:

1) The approximations which are necessarily introduced in the analysis leading to the result (2) for the $K^+_{\pi 3}$ do not represent correctly the physical situation: in other words the result (2) of such an analysis is not correct; in

such case $K^+_{\pi 2}$ and $K^+_{\pi 3}$ may have the same spin parity assignment and all the kinds of decays may be interpreted as different decay modes of the same boson.

2) The result (2) is correct; then some explanation must be found for the equality of the mass, of the lifetime, of the scattering cross-section and of the energy dependence of the production cross-section of the various kinds of bosons.

The preferred attitude has been generally the second, because no serious objection appears against the results of the analysis of the $K^+_{\pi 3}$ problem. Therefore several attempts have been made to explain the remarkable equalities in behaviour listed above. Such attempts will be described in the following sections and are essentially three (*):

a) Lee and Orear's cascade assumption.

b) Lee and Yang's parity doublet model.

c) Bludman's attempt.

In view of the fact that no one of these attempts seems to be really satisfactory, and each introduces some complications, the possibility 1) has also been considered, to see if there is some reason which may make unreliable the analysis of the $K^+_{\pi 3}$ problem leading to the assignments (2). This part of the discussion will be postponed after the presentation of the analysis of the $K^+_{\pi 3}$ problem (Chapter **13**). It may be however anticipated that up to now no serious argument against the assignments (2) has been found.

We shall proceed in the following sections to discuss the models, listed above as *a*), *b*), *c*), constructed to explain in some way the similarities in behaviour of the various kinds of bosons which appear very unexpected if the $K^+_{\pi 3}$ and the $K^+_{\pi 2}$ differ in spin and/or in parity.

Of course a possibility which may solve the difficulties, is to assume [3] that we have to do with one boson only and that the weak interactions, responsible for its decay do not conserve the parity; this possibility will be discussed in Sect. **12**·5, and might be the final explanation of the problem.

Apart from this possibility all the other attempts *a*), *b*), *c*), to explain the situation, assume the existence of at least two bosons, differing in spin and/or parity. We shall call these bosons K^+_θ and K^+_τ (+), meaning by K^+_θ that boson

(*) Also the possible existence of a π' with opposite parity as the normal one has been considered; it should be remarked that, if such π^0 has $t=0$, as one should believe, *and* is coupled to the nucleons with a strength *comparable* to that of the usual π^0, the result of Hildebrand's experiment would be affected by its presence. In fact such a π' might produce a $T=0$ contribution in the reaction $n+p \to d+\pi'$.

(+) The bosons indicated here as K^+_θ, K^+_τ are usually, in the theoretical papers simply indicated as θ^+ and τ^+. We have avoided this notation, because experimentalists use often the symbols τ and θ to indicate modes of decay ($\theta \equiv K_{\pi 2}$, $\tau \equiv K_{\pi 3}$).

which may decay with the 2π decay mode and possibly by other decay modes ($\mu\nu$, $\mu\nu\pi^0$, $e\nu\pi^0$) but not by the 3π decay mode; and by K_τ^+ that boson which may decay in the 3π decay mode and possibly by the other decay modes mentioned above, but not by the 2π mode. The word possibly has been used because the decay modes $\mu\nu$, $\mu\pi^0\nu$, $e\pi^0\nu$ may be attributed to the K_θ^+ and K_τ^+, or may be due to a third or fourth different boson. For the moment we shall however assume that the two bosons K_θ^+ and K_τ^+ are sufficient to explain all the decay modes.

The question which the models *a*), *b*), *c*) try to answer is then: why have K_θ^+ and K_τ^+ inside the experimental errors the same mass, lifetime, scattering cross-section and energy dependence of the production cross-section?

12·2. – The model of Lee and Orear.

The fact that two bosons having different spin and/or parity have the same lifetime is a very strange one; so LEE and OREAR [4] have tried to avoid this apparent consequence of the experiments (*). Lee and Orear's starting point is that the present experimental evidence does not imply that the two bosons K_θ^+ and K_τ^+ must have the same lifetime, if one of the two has a lifetime much less than 10^{-9} s; in fact the equality of the lifetimes of $K_{\pi 3}^+$ and $K_{\pi 2}^+$ has been proved until now only in so far times of flight longer than 10^{-8} s are concerned.

With this in mind they make the following three assumptions:

1) There is a small, at present undetectable, mass difference between K_θ^+ and K_τ^+ (the most recent determinations say that it cannot be larger than ~ 1 MeV).

2) The heavier of the two (we call it $K_>$ whatever it is) may either decay directly into pions (or muons or electrons), or may decay into the lighter of the two, $K_<$, by emission, for example, of γ-rays so as to carry away the difference of parity and/or spin: the branching ratio between the two possibilities being significant.

3) The lifetime of $K_>$ (to be denoted by $\tau_>$) is almost identical with the experimental value of the lifetime $1.1\cdot 10^{-8}$ s (as measured for times of flight longer than 10^{-8} s); on the contrary the lifetime of $K_<$ is much smaller; say less than 10^{-9} s.

If these conditions are fulfilled it is easy to realize that, though the life-

(*) This section was written at a time in which this model appeared as a possible explanation of the facts; now this is very improbable; still we have not modified this section, because the history of these attempts appears to us rather instructive.

We thank Prof. N. KROLL for several discussions on this model, during his stay in Rome (winter 1956).

times of $K_>$ and $K_<$ are different (point 3 above) they appear to decay with the same lifetime for times of flight larger than 10^{-8} s.

In fact for times of flight larger than 10^{-8} s all the $K_<$ directly produced, if there are, have decayed, and what one observes are either the direct decays of $K_>$ or the decays of those $K_<$ which have been produced, according to point 2) from the decay of $K_>$; but since the lifetime of $K_<$ is much smaller than that of $K_>$, the average time that the $K_<$ produced from the decay of $K_>$ spends before decaying is much smaller than the average travel time of the $K_>$, so that both appear to decay with the same lifetime.

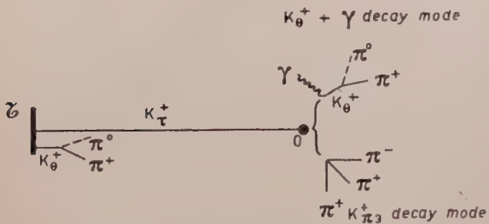

Fig. 1-12·2.

Fig. 1 illustrates the situation; here times are proportional to the length of the segments, and for definiteness we have assumed that $K_> \equiv K_\tau$ and $K_< \equiv K_\theta$; $\mathcal{T}$ is the production target in which both $K_>^+$ and $K_<^+$ may perhaps be directly created; however the $K_<$ live very little, decay in the vicinity of the target and are not detected; on the contrary the K_τ^+ travel much longer times ($\simeq 10^{-8}$ s) and when they decay may go either into K_θ^+ or into three pions with a reasonable branching ratio; this is represented by indicating in the figure at point O the two alternative ways of decaying. After the K_τ^+ has decayed electromagnetically into the K_θ^+, the K_θ^+ so produced travels a very short distance and then decays; and the total time of flight (from the target) of this K_θ^+ does not differ appreciably from the time of flight of the K_τ^+, as it is indicated by the figure. The same situation is described, on an energy level diagram in Fig. 2.

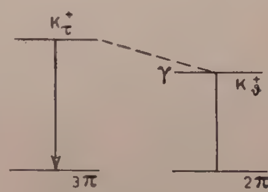

Fig. 2-12·2.

In order to discuss the model in more detail we must give estimates of the mass difference between $K_>$ and $K_<$ needed to give reasonable values for the branching ratio:

$$\varrho = \frac{\text{rate of } (K_> \to K_< + \gamma\text{-ray(s)})}{\text{rate of } (K_> \text{ decays directly})} . \tag{1-12·2}$$

The experimental value of ϱ, cannot be given exactly because we don't know how the decay modes other than the 2π and 3π have to be assigned to $K_>$ and $K_<$. Furthermore we do not know if $K_>$ is K_τ and $K_<$ is K_θ or vice-versa. However ϱ should be comprised between 10 and 10^{-1}, on the basis of the known values (Sect. 2·3) of the abundances of the various decays. In other

words the lifetime of the process in the denominator of (1) is 10^{-8} s and that in the numerator should be in between 10^{-7} and 10^{-9} s.

To give an estimate of the mass difference ΔM needed to give a value of ϱ of the order of magnitude just mentioned let's first assume that $K^+_>$ and $K^+_<$ have the same strangeness, in fact have strangeness 1. Of course the strangeness of $K_>$ *must* be 1 because this meson has to be identified with the one strongly produced, and for this the discussion in Sect. **11**·4 is valid.

Assuming then $K_>$ and $K_<$ to have the same strangeness, an estimate of the rate of the process in the numerator of (1) may be obtained [5] using the approximate formulas given by Weisskopf for the discussion of the electromagnetic transitions in the nuclei; one has to keep in mind, however, that these formulae disregard completely the detailed structure of the $K_>$ and $K_<$; moreover they depend on a «radius» R assumed for these particles which is not known; for all these reasons the estimate may be extremely crude.

Two cases have now to be distinguished:

1) That in which $K_>$ and $K_<$ are assumed to have both spin zero and opposite parity, so that two photons must be emitted in the transition.

2) All the other spin parity assignments, for which a photon is sufficient to produce the transition.

In the first case the lifetime for the decay $K_> \to K_< + 2\gamma$ is given by:

$$T \cong \frac{1}{\alpha^2}\,\frac{R}{c}\left(\frac{\Delta M c}{\hbar}\,R\right)^{-7}, \tag{2-12·2}$$

where ΔM is the mass difference and R is the radius of the two particles; α is the fine structure constant.

For a value of R as large as $0.7\cdot 10^{-13}$ cm one obtains in this case that the minimum value of ΔM capable to give rise to a sufficient rate for the process in question is ~ 5 MeV which is definitely excluded by the experiment. The lifetime T, as given by (2) for $\Delta M = 5$ MeV and $R = 0.7\cdot 10^{-13}$ cm is in fact $\sim 10^{-8}$ s and since (2) depends from $(\Delta M)^{-7}$ a value of 1 MeV would give rise to a value of T definitely too long.

Therefore the case $0+$, $0-$ is not acceptable, on this model. If on the contrary the spins of $K_>$ and $K_<$ are such that one 2^l pole photon may be emitted, the lifetime of the transition $K_> \to K_< + \gamma$ is:

$$T_l \cong \frac{1}{\alpha}\,\frac{R}{c}\,(2l+1)!!\,(2l-1)!!\left(\frac{\Delta M c}{\hbar}\,R\right)^{-(2l+1)}. \tag{3-12·2}$$

Here with a mass difference ΔM of the order 10 to 100 keV the values of T_l for $l = 1$, and 2 with the R value mentioned above result to be $\sim 10^{-8}$ s.

Such a small mass difference is inside the experimental error. Therefore such assignments as $K_\tau = 0-$, $K_\theta = 0+$ or $2-$, $2+$ or $2-$, $0+$ to remain in the field of the low spins would still be possible in principle.

In this discussion we did assume that the strangeness of $K_>$ and $K_<$ were both 1. If we now suppose the strangeness of $K_<$ to be different from that of $K_>$ the process in the numerator (1) becomes more slow, (ΔM remaining the same), by a factor say 10^{12}; therefore ΔM should be increased and, even in the second case, it becomes larger than the experimental error. So this assumption appears to be excluded.

We want to stress again that the above discussion is very crude; for instance one could not exclude at the moment a radius R larger than $0.7 \cdot 10^{-13}$ cm.

We believe indeed that, if independent evidence were available for the model, the above estimates could not be taken as absolutely excluding even the case $0+$, $0-$.

We want to examine now if, having disposed of the mass and lifetime equality in the way which we have seen, the Lee-Orear model is capable of explaining the equality in the scattering cross-section and the equality of the energy dependence of the production cross-section of the various kinds of bosons.

As far as the scattering cross-section is concerned the answer appears to be positive: in fact it is just $K_>$ which is scattered because $K_<$ decays near to the production place, so that the ratios between the different kinds of bosons should remain unaltered in the scattered beam. There is however a difficulty. If $K_>$ and $K_<$ do have the same strangeness the reaction

$$K_> + N \rightarrow K_< + N \tag{4-12.2}$$

being a strong reaction may take place with an appreciable cross-section; if this is the case one should notice an increase of the ratio (number of $K_<$)/(number of $K_>$) in the scattered beam with respect to the non-scattered one. However, the present errors are still large and if the matrix element for the process (4) is two or three times smaller than that of the process $K_> +$nucleon $\rightarrow K_> +$nucleon the variation of the above ratio may have escaped detection.

The same may be said for a shortening of the lifetime of the scattered K^+ with respect to the non scattered ones which should be present if the reaction (4) may take place; presently there is not evidence for it but the data are much too poor. In future, however such effects should be observed if the Lee-Orear model is valid.

The equality of the energy dependence of the production cross-section of the various kinds of bosons is explained by the Lee-Orear suggestion.

The bosons which we see (at reasonable distance from the target) are in fact only $K_>$ so that things go as if the various kinds of bosons were different decay modes of just one boson.

Finally we want to discuss other experimental possibilities to test Lee and Orear's suggestion. These are essentially three:

1) A direct determination of the masses of the decaying particles from measurements on the secondaries.

2) A direct proof of the existence of K^+ particles with a lifetime much shorter than 10^{-8} s.

3) Detection of the γ-rays emitted.

As far as point 1) is concerned the measurements presently indicate that the mass of the $K^{-}_{\pi 2}$ and of $K^{+}_{\pi 3}$ as determined from the ranges of the secondaries are respectively (962.8 ± 1.8) and (966.3 ± 0.7) electron masses (Table I-1·2), so nothing can be said. Considering point 2) according to the cosmic ray plate evidence there is no indication of a K^+ component of lifetime between $5 \cdot 10^{-11}$ and 10^{-9} s which, if present, would be detected, in the same way as $\Sigma^{\pm}$ may be detected and their lifetime measured; an indication from the Wilson chamber experiments for a K^- component with a lifetime less than 10^{-9} s does not rest on safe ground. Of course the possibility remains that $\tau_{<}$ is less than $5 \cdot 10^{11}$ s. For point 3) an experiment described by ALVAREZ [6] at the Rochester Conference 1956, shows that no γ-rays with an energy larger than 0.5 MeV are associated to the K_{θ} decays; so if K_{τ} is the heavier boson it should differ in mass from K_{θ} by less than 0.5 MeV; of course the possibility has also to be examined that K_{θ} is heavier than K_{τ}.

12·3. – The parity doublet model of Lee and Yang.

We have seen that the model by Lee and Orear tries to avoid the circumstance that two particles of different spin and/or parity decay with the same lifetime, a fact which would be very strange. LEE and YANG [7] instead fix their attention, in another paper, on the equality of the masses of the two bosons and, assuming that the two bosons differ just in their parity and not in their spins, they argue that such a coincidence of masses should be an indication of some more general invariance property of the theory; they investigate the consequences of such an invariance.

The invariance property postulated by LEE and YANG which, of course contains the mass coincidence as a particular case is to assume that the part of the hamiltonian including all the strong interactions is invariant when the operation is performed of exchanging the K_{τ} with the K_{θ}.

Such an operation is called « parity conjugation » and is denoted by C_p. The formal expression of the above invariance property is that the strong

interaction hamiltonian commutes with C_p. For this to be possible it is however necessary that, besides two kinds of K with opposite parities, K_τ^+ and K_θ^+, also two kinds of Λ^0 with opposite parity exist, say $\underset{+}{\Lambda^0}$, $\underset{-}{\Lambda^0}$ and two kinds of Σ's with opposite parity $\underset{+}{\Sigma}$, $\underset{-}{\Sigma}$. The operation C_p exchanges simultaneously K_θ^+ with K_τ^+, $\underset{+}{\Lambda^0}$ with $\underset{-}{\Lambda^0}$, $\underset{+}{\Sigma}$ with $\underset{-}{\Sigma}$; as we have said the strong interaction part of the hamiltonian commutes with C_p. It may be shown easily that the rates of production of K_θ^+ and K_τ^+ are in this scheme the same under all conditions; so are the rates of production of $\underset{+}{\Lambda^0}$ and $\underset{-}{\Lambda^0}$ or of $\underset{+}{\Sigma}$ and $\underset{-}{\Sigma}$, although it is by no means true that a K_θ has to be always produced together with a $\underset{+}{\Lambda^0}$ and a K_τ^0 together with a $\underset{-}{\Lambda^0}$ or viceversa.

The difficult point in this model is to explain the experimental equality of the lifetimes of the K_θ^+, K_τ^+; the model does not give any reason why they should be equal.

The first attempt of LEE and YANG was to try to reconcile the parity doublet idea with the Lee and Orear explanation of the lifetime equality.

This is however not possible for the following reason: the only mass difference between K_θ^+ and K_τ^+ which may be present in the model must be attributed to the weak interaction part of the hamiltonian, which cannot, of course, be invariant under parity conjugation. It should therefore be of the order $\sim 10^{-5}$ eV, which is not reconcilable with the Lee and Orear model.

If also the electromagnetic interactions were not invariant under C_p, one might obtain mass differences as high as a few MeV; but there is no reason for this; in particular [8] any minimal electromagnetic interaction (Sect. **11**·8) is invariant under parity conjugation.

Therefore no convincing reason is really given for the equality in the lifetimes: this is the weak point of the parity doublet model.

Of course if the lifetimes of $K_{\pi 2}^+$ and $K_{\pi 3}^+$ should turn out to be different, the model would acquire a particular interest (compare for this also the next section).

Notice also that the lifetimes of $\underset{+}{\Sigma^+}$ and $\underset{-}{\Sigma^+}$ (and the lifetimes of $\underset{+}{\Sigma^-}$, $\underset{-}{\Sigma^-}$) should in general be different, according to the model; the experimental evidence is against this; the results of the Berkeley experiment [9] are consistent with a unique lifetime for the Σ^+ (and also for the Σ^-).

Also in the neutral boson component, as we shall see (Sect. **14**·5) no evidence for a parity doublet structure appears [2].

A possible check of the model has been suggested by LEE and YANG [10] (compare also [11]); it is based on the analysis of certain angular distributions in the decays of polarized hyperons and will be discussed in Sect. **15**·5; at present it does not furnish any support to the model.

12·4. – Bludman's remarks.

Though all the experiments show that the lifetimes of the $K^+_{\pi2}$, $K^+_{\pi3}$ and $K^+_{\mu3}$ are equal within say 20%, one may still believe that differences of this order of magnitude exist. Of course if the K^+_τ and K^+_θ lifetimes should finally turn out to be different the strong objection against the parity doublet model—that it does not explain the equality of the lifetimes—should not hold any more; then, one should be sure that two particles are necessary to construct the theory (*).

We want now to show, following BLUDMAN, that in a two boson model a difference in the lifetimes would be reflected [12] *a*) in the fact that the $K_{\mu2}$ decay curve would be not a pure exponential but a composite decay curve, *b*) in the fact that the branching ratios into the various modes would depend on the time of observation.

If only two heavy bosons are assumed to exist the total rate of decay $R_\tau = 1/\tau_\tau$ of K^+_τ is the sum of the rate of decay $R_{\tau3}$ into the $K_{\pi3}$ mode plus the rate of decay $R_{\mu2}$ in the $K_{\mu2}$ mode; similarly the total decay rate $R_\theta = 1/\tau_\theta$ of K^+_θ is the sum of the rate of decay $R_{\theta2}$ in the $K_{\pi2}$ mode plus the rate of decay $R_{\theta\mu}$ in the $K_{\mu2}$ mode; we disregard here the other decay modes

Thus:

$$(1\text{-}12{\cdot}4)\qquad \left\{\begin{aligned} R_\tau &= R_{\tau3} + R_{\tau\mu}\,,\\ R_\theta &= R_{\theta2} + R_{\theta\mu}\,. \end{aligned}\right.$$

Now from the decay equations of K^+_θ and K^+_τ it is possible to prove the following :

1) The decay modes $K^+_{\pi2}$ and $K^+_{\pi3}$ have lifetimes T_2 and respectively T_3 given by

$$(2\text{-}12{\cdot}4)\qquad T_2 = R_\theta^{-1}\,,\qquad T_3 = R_\tau^{-1}\,.$$

The lifetimes of such modes are just the lifetimes of K^+_θ and K^+_τ.

2) The decay mode $K^+_{\mu2}$ is not formed with an exponential law. If we call f the (percentual) number of K_τ originally created and $1-f$ the fractional number of K_θ, then the number of $K_{\mu2}$ decays in the $K_{\mu2}$ mode per unit time at time t is:

$$(3\text{-}12{\cdot}4)\qquad \nu_{\mu2}(t) = fR_{\tau\mu}\exp[-R_\tau t] + (1-f)R_{\theta\mu}\exp[-R_\theta t]\,.$$

(*) The time behaviour of a beam of K^+ according to the parity doublet model has been studied in detail by ARNOWITT and TEUTSCH: *Phys. Rev.*, **105**, 285 (1957).

If then an experiment is made with the usual techniques to measure the apparent « lifetime » of $K_{\mu 2}$ between the times t_1 and t_2, the « lifetime » in question $T_\mu(t_1 t_2)$ depends on the individual values of t_1 and t_2 which are chosen for the experiment.

3) With the same meaning of f as above the branching ratios between the numbers of $K_{\pi 2}$, $K_{\pi 3}$ and $K_{\mu 2}$ modes which are observed if an experiment is performed which is able to discriminate all those decays which take place between the times, say t_1', t_2', depend on these times (*).

The foregoing formulae prove the assertions at the beginning of this section. Careful experiments might perhaps try to test the possibility of the

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(*) These statements may be derived in this way: call $N_\tau(t)$ and $N_\theta(t)$ the percentages of K_τ and K_θ present at time t if f and $1-f$ are their initial percentages. They satisfy the equations:

$$-\dot{N}_\tau = R_\tau N_\tau\,, \qquad -\dot{N}_\theta = R_\theta N_\theta\,.$$

Call then the numbers of decays $K_{\pi 2}$, $K_{\pi 3}$, $K_{\mu 2}$ up to time t respectively $N_2(t)$, $N_3(t)$, $N_\mu(t)$. These satisfy the equations:

$$\dot{N}_2(t) = R_{\theta 2} N_\theta(t)\,; \qquad \dot{N}_3(t) = R_{\tau 3} N_\tau(t)\,; \qquad \dot{N}_\mu(t) = R_{\tau\mu} N_\tau(t) + R_{\theta\mu} N_\theta(t)\,.$$

From these equations we may derive the expressions for the numbers of decays $K_{\pi 2}$, $K_{\pi 3}$, $K_{\mu 2}$, per unit time at time t. These are:

$$(\alpha)\qquad \left\{\begin{array}{l} \nu_2(t) = \dot{N}_2(t) = R_{\theta 2}(1-f)\exp[-R_\theta t]; \qquad \nu_3(t) = \dot{N}_3(t) = R_{\tau 3} f \exp[-R_\tau t]; \\ \nu_\mu(t) = \dot{N}_\mu(t) = f R_{\tau\mu}\exp[-R_\tau t] + (1-f) R_{\theta\mu}\exp[-R_\theta t]\,. \end{array}\right.$$

These expressions contain all the statements below. In particular the total branching ratios for the decay into the $K_{\pi 2}$, $K_{\pi 3}$, and $K_{\mu 2}$ modes may be derived from them, integrating (α) from $t=0$ to $t=\infty$. These branching ratios are given by:

$$N_2 = (1-f)\left(1-\frac{R_{\theta\mu}}{R_\theta}\right); \qquad N_3 = f\left(1-\frac{R_{\tau\mu}}{R_\tau}\right); \qquad N_\mu = 1 - N_2 - N_3\,.$$

One may then ask: 1) assume $f=\frac{1}{2}$ as given by the parity doublet model and assume further $R_{\theta\mu}=R_{\tau\mu}$ (for this equality there is not much reason). Which value does one obtain for the ratio $R_\tau/R_\theta = T_2/T_3$ using the experimental values $N_2=0.28$, $N_3=0.09$? The answer is:

$$R_\tau/R_\theta = (1-2N_2)/(1-2N_3) = 0.55\,.$$

(Using $N_2=0.28$ and $N_3=0.16$, that is attributing to N_3 all the $K_{\mu 3}$ and $K_{3\beta}$ decay modes, one gets instead ~ 0.7). 2) assume again: $R_{\tau\mu}=R_{\theta\mu}$; what must be f so that $R_\tau=R_\theta$? The answer is: $f\simeq 0.25$.

Therefore if we assume $f=0.25$ and $R_{\tau\mu}=R_{\theta\mu}$, we can say to have « explained » the equality of the lifetimes; it is unnecessary to point out that one would have then to « explain » the value of f.

variation of the $K_{\mu 2}$ lifetime and of the branching ratios; of course, since the directly determined difference in lifetimes between $K^+_{\pi 2}$ and $K_{\pi 3}$ is, if it exists, very small, this is not an easy task.

12·5. – The assumption of non conservation of parity in the weak interactions.

The fact that no one of the solutions presented up to now is entirely satisfactory has recently led [3] to consider another possibility: it is the assumption that parity may not be conserved in the weak interactions. Of course in principle, this may solve all the difficulties. If the same boson, decaying, may give rise both to states with parity —, and states with parity +, we may assume that there is only one boson present with spin e.g. 0 and that this boson may give rise when it decays both to a state 0 — with three pions ($K_{\pi 3}$) and to a state 0+ with two pions ($K_{\pi 2}$) (*). Furthermore, writing the weak interaction producing the decay of the boson as

$$aH_1 + bH_2\,,$$

where H_1 gives rise to the — parity state and H_2 to the + parity state, the coefficients a and b may be chosen so that the ratio between the number of $K_{\pi 3}$ and $K_{\pi 2}$ is equal to the observed one (compare, on this point also Sect. **16·11**).

Of course, in order that the idea is acceptable it must be shown first that it does not contradict any known fact and second that it does explain other independent facts. The conservation of parity is in fact the consequence of the invariance of the physical laws with respect to a reflection of the co-ordinate axis and one has to proceed with great caution before giving up such a fundamental invariance property even if only for the weak interactions.

Practically every phenomenon of atomic or nuclear physics shows that the parity is a good quantum number; thus the strong and electromagnetic interactions should conserve the parity; LEE and YANG have however observed that an eventual lack of parity conservation confined to the weak (β like) interactions should not have been revealed by the experiments performed up to now.

The reasons for this are two:

1) In some experiments the accuracy was not such as to allow to determine a lack of parity conservation of the weak interactions.

(*) As far as the parity of such bosons is concerned this may be fixed conventionally. Only the relative parity of the K and of a hyperon is physically significant if parity is not conserved in the decay.

For example consider the electrical dipole moment of a nucleon which, if parity is not conserved, should be different from zero.

This electrical dipole moment is expected to be of the order eg^2 times the dimensions of the nucleon, where g is the coupling strength of the interaction which does not conserve the parity. If such interactions are beta-like interactions, $g^2 \leqslant 10^{-12}$ (*), so that the electrical dipole moment in question should be less than or equal to $e\cdot(10^{-25}$ cm). The most accurate measurement performed up to now of the electric dipole moment of the neutron only shows that it is less than $e\cdot(5\cdot 10^{-21}$ cm).

2) In other experiments one has measured up to now only quantities which would not have revealed a lack of parity conservation, even if present. Consider, e.g., the β decay of an unpolarized neutron (or more realistically nucleus) at rest; the quantities which one can measure in such an experiment are the distribution of the energies of the outgoing particles and, say, the distribution in angle between the momentum of the electron and the momentum of the neutrino. Such quantities are all invariant with respect to spatial reflections (in fact they are quadratic expressions in the momenta); this implies that the distributions in the above quantities can say nothing about parity conservation or non conservation of the β decay interaction.

In order to reveal such conservation or non conservation, one has to measure quantities which are odd with respect to reflections. For instance if one might polarize the neutron or the nucleus of the above example and measure the angular distribution of the emitted electron with respect to the direction of the spin of the decaying neutron (or nucleus), this distribution might reveal if parity is conserved or not, in the decay in question. In fact the angle between the momentum $\boldsymbol{p}$ of the electron and the spin of the neutron (or nucleus) is proportional to $\boldsymbol{\sigma}\cdot\boldsymbol{p}$, which quantity changes sign under a reflection.

All the experiments reported below, to test parity conservation, are in fact based on this idea, of being able of forming a quantity which is odd with respect to reflections, in terms of experimentally measured quantities. In particular the experiment number two listed below is simply the one described just now, the nucleus in question being the ^{60}Co.

Summing up, if one therefore introduces the assumption that the parity is not conserved in the weak interactions (we repeat that by weak interactions we mean those interactions which have the strength of the β interactions) one does not contradict any known experimental fact. However, as we have mentioned, to accept such an assumption as correct, independent evidence is

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(*) The value 10^{-12} for g^2 is the one corresponding to $g=3\cdot 10^{-49}$ erg·cm³, when this value is multiplied by $(\hbar c)^{-1}R^{-2}$ to make it adimensional, R being taken equal to the Compton length of a pion.

necessary. LEE and YANG, in their paper have suggested the following three experiments:

1) Consider the $\pi \to \mu \to e$ decay from a pion at rest; it may be shown that, if the parity is not conserved in the decay $\pi \to \mu + \nu$, the μ, in general, is expected to be polarized in its direction of flight. The distribution of the angle θ between the line of flight of the electron e (from the successive decay: $\mu \to e + \nu + \nu$) and that of the μ (in the rest system of the latter) should be asymmetrical with respect to $\theta = 90°$ if the parity is also not conserved by the interaction producing the decay of the muon.

2) Consider the angular distribution of the electrons from the decay of an oriented nucleus (for instance ^{60}Co). Then a proof of parity non conservation would be furnished by an asymmetry with respect to 90° of the θ distribution, θ being the angle between the direction of the electron and that of the spin orientation of the parent nucleus (this is the experiment already discussed above).

3) The third experiment refers to the study of the possible asymmetries in certain angular distributions from the decay of hyperons. This will be discussed in Sect. **15·5**.

The experiment 1) has been performed by the Rome emulsion group [13] with a set of 400 pions. The result is that larger statistics are needed to detect an asymmetry, if it exists.

From the third experiment (to be discussed in Sect. **15·5**) no conclusion is presently possible.

It may then be said that the preliminary observations do not confirm the assumption, but it may be advisable to perform all the suggested experiments with increased statistics.

12·6. – Added note on the non-conservation of parity (*February 28, 1957*).

The experiments 1 and 2 suggested by LEE and YANG have now been performed providing evidence that parity is not conserved in the processes in question. In view of the importance of such results we want to discuss briefly their implications, notwistanding the dead line fixed in closing this review. Our discussion will be very short also because, at the time of writing, there is, strictly speaking, still no evidence, concerning parity conservation or non-conservation in processes involving the new particles.

a) *β decay of* ^{60}Co. – This experiment has been performed by WU and coll. [14] polarizing ^{60}Co nuclei and measuring the degree of polarization of the ^{60}Co through the anisotropy of the γ-rays subsequent to the β decay.

The experiment has shown that the angular distribution of the decay electrons is strongly asymmetrical when the anisotropy of the γ radiation is strong (which means that the nucleus is strongly polarized); the asymmetry disappears when the ^{60}Co (which was polarized by the Röse-Gorter method) warms up; at the same time also the γ-anisotropy disappears. It is also observed that the preferential direction of emission of the electrons is opposite to the direction of the nuclear spin. On writing the angular distribution of the electrons as:

$$I(\theta)\,\mathrm{d}\cos\theta = (1+\alpha\cos\theta)\,\mathrm{d}\cos\theta\,, \tag{1-12·6}$$

where θ is the angle between the direction of orientation of the nuclear spin and that of emission of the β-ray, α may be measured by detecting the intensity at 0° and 180° with respect to the direction of the nuclear spin. In the conditions in which the experiment was made and applying all the necessary corrections to the data, α resulted around -0.4 for a value $v/c \simeq 0.6$ of the emitted electrons. According to [14] the polarization of the ^{60}Co was about $\langle J_z\rangle/J = 0.6$ in the conditions of the experiment.

The expected asymmetry α may be calculated, for an allowed Gamow-Teller transition such as the one in play here according to the formula:

$$\alpha = \beta\,\frac{\langle J_z\rangle}{J}\,,$$

with

$$\beta = \frac{2\,\mathrm{Re}\,[g_T g_T'^* - g_A g_A'^* + i(Ze^2/\hbar c p)(g_A g_T'^* + g_A' g_T^*)](v/c)}{|g_T|^2+|g_T'|^2+|g_A|^2+|g_A'|^2}\,. \tag{2-12·6}$$

In the above formula g_A, g_A', g_T, g_T' are the coefficients of the axial and tensor part of the interaction; if parity is not conserved this is now a mixture of couplings and pseudocouplings; g_A, g_T refer to the couplings and g_A', g_T' to the pseudocouplings; v is the velocity of the electron, p its momentum. The other notations are explained in [3] (*).

Since $|\alpha| \geqslant 0.4$ and $\langle J_z\rangle/J \simeq 0.6$, $|\beta|$ turns out to be, in this experiment $\geqslant 0.7$. This is a very high value (the maximum admissible value being of course unity) and implies the following important consequence: besides not conserving the parity, the interaction which produces the β decay must also be non-invariant with respect to charge conjugation. This may be seen as follows: the first term in the numerator of (2) may be shown to be absent if the β interaction is invariant with respect to charge conjugation; the second term is absent if the theory is invariant with respect to time reversal; but, from the

(*) Only c's are substituted by g's.

value of $\beta \simeq 0.7$, mentioned above it results that the first term must be present; the second term alone, in fact, would not be capable to account for such a large value of β due to the factor $Ze^2/\hbar cp$, a rather small ($\simeq \frac{1}{5}$) quantity.

Some implications of the fact that, together with parity, also the charge conjugation invariance is destroyed, will be discussed in a moment; we prefer first to give the results of the:

b) $\pi \to \mu \to e$ *decay experiment.* – This experiment has now been performed, both using the $\pi \to \mu \to e$ decays from pions stopped in nuclear emulsions, and using artificially produced muons and counter detectors [15]. The results from the plates have still somewhat large statistical fluctuations, although they confirm definitely the effect; here we shall describe only the results with the delayed coincidence technique.

The muons in this experiment [15] are from the decay in flight of artificially produced 85 MeV positive pions and travel in the same direction as the pions; the pions are eliminated using an absorber which stops the pions but not the muons. The muons which, if parity is not conserved in the process $\pi \to \mu + \nu$, should be, as we have said, polarized along their direction of motion, are finally stopped in a carbon absorber and the electrons from their decay, at a fixed angle, are measured; this means, if one assumes that the spin orientation of the muons is not changed during the time which they spend in carbon, before decaying, that one measures the intensity of the electrons emitted at a certain angle with respect to the spin direction in question. To get the complete distribution, one now might change the position of the electron counter; however the method used in the experiment is to keep fixed the position of the counter and to make the spin orientation of the muon to precess, before its decay, by using a convenient magnetic field. This method has the double advantage that it allows to measure the complete angular distribution of the decay electrons with respect to the spin orientation of the decaying muon and, at the same time, gives a measure of the magnetic moment of the muon.

The main results of this experiment are the following:

1) The magnetic moment of the muon is « normal » (g factor $= 2.00 \pm 0.10$).

2) The asymmetry in the angular distribution of the electrons, defined as the coefficient a in the distribution

$$(1 + a\cos\theta)\,\mathrm{d}\cos\theta$$

where θ is the angle between the direction of emission of the electron and the velocity of the incident muon, is $a = -0.33 \pm 0.05$. Other results are:

3) The absolute value of the asimmetry increases slightly with increasing energy of the selected electrons.

4) An asimmetry (15% of the above one and with the same sign) is also found for μ^- stopping in carbon.

It is therefore apparent that also in the processes $\pi \to \mu + \nu$, $\mu \to e + \nu + \nu$ the parity is not conserved.

c) Some theoretical implications of the above results. – First of all it has to be pointed out that if a lagrangian which is invariant with respect to the proper Lorentz group, does not conserve the parity, it must also destroy time reversal and/or charge conjugation invariance. More generally the following theorem is valid: calling respectively P, C, T the parity, charge conjugation, and time reversal operations, any local theory invariant with respect to the proper Lorentz transformations must be invariant when the product operation PCT is effected (the order in which the above three operations are effected is irrelevant) [16, 17, 18].

Therefore the interpretation of the WU *et al.* result, as showing that the theory destroys also the invariance with respect to charge conjugation, is not unexpected on the basis of the above theorem.

One may ask whether also the experimental results on the $\pi \to \mu \to e$ experiment do show the non-invariance of the weak interactions producing such decays with respect to charge conjugation. The answer is yes if one takes into account the following general theorem by Lee, Oehme and Yang [19]: If a particle A (in our case the μ) decays through a weak interaction H_{weak} which is *invariant* under charge conjugation, and if one may disregard the interaction between the final decay products (in our case $e\nu\nu$), then, to the lowest order in H_{weak} there is no interference, between the parity conserving and the parity non-conserving parts of H_{weak} in the decay of A (hence, there cannot be asymmetry in the angular distribution). On the basis of the above theorem one also understands that, if one may disregard the Coulomb deformation of the final electron wave function in the β decay experiment by WU *et al.* the β interaction *must* be non-invariant with respect to C to give an asymmetry.

One might finally ask whether, if the charge conjugation invariance of the weak interactions is violated one should find different decay lifetimes for a particle A and its antiparticle $\bar{A}$; this is however not so [19], at least to the first order in H_{weak}, if the decay products of A and $\bar{A}$ are different (e.g. when A is charged). The same may be said for the mass.

Finally we shall shortly refer on some attempts [20-23] which have been made to *explain* the non-conservation of parity. The basis of all these attempts, presently, lies in the fact that the neutrino has mass zero, which implies that the Dirac hamiltonian for the free neutrino commutes with γ_5. In other words the free neutrino hamiltonian is invariant with respect to the substitution $\psi_\nu \to \gamma_5 \psi_\nu$. Requiring that the same property must be valid also

for the interactions in which the neutrino intervenes, one is led naturally to interactions which destroy parity and also charge conjugation invariance.

A detailed scheme along these lines has been developed by LEE and YANG[20]. Due to the commutativity of γ_5 with the free neutrino hamiltonian, the solutions of the neutrino Dirac equation may be taken as eigenfunctions of γ_5; more definitely positive energy neutrinos are restricted to satisfy the equation $\gamma_5\psi_\nu = -\psi_\nu$ and negative energy neutrinos the equation $\gamma_5\psi'_\nu = \psi'_\nu$. It follows easily from this that a neutrino has always its spin parallel to its momentum and an antineutrino has its spin always antiparallel to its momentum; the neutrino is a right handed screw; the anti-neutrino a left handed one.

The fact that the neutrinos of this theory must satisfy the equation $(\gamma_5+1)\psi_\nu = 0$ is reflected in the fact that in all the interactions involving neutrinos written down in the usual way the destruction operator of a neutrino is multiplied by $1-\gamma_5$ and the destruction operator of an anti-neutrino is multiplied by $1+\gamma_5$; the theory thus predicts conserving and non conserving parts for the interactions involving one neutrino, which have the same magnitude. Several specific consequences follow from this theory; they are very interesting but would lead us too far from our present subject; the experimental evidence is still too preliminary to confirm or disprove the theory; it may be mentioned that the theory is invariant under time reversal, hence non-invariant with respect to both parity and charge conjugation.

Concerning the neutrino « philosophy » it must be said that, if the neutrino is the only agent through which the parity is destroyed, it appears difficult to explain, by means of the parity non-conservation the problem of the bosons, which was the source of all these developments. In fact in the $K_{\pi 2}$ and $K_{\pi 3}$ decays, neutrinos are not involved at the first order in H_{weak} and their possible partecipation at higher orders should be irrelevant.

It would be extremely interesting to know if the parity conservation is violated also in processes not involving neutrinos; in particular in the decay processes of the hyperons. There is not yet evidence for such processes (compare for their discussion Sect. **15**.5) and it will be difficult to find it because we do not know the polarization of the hyperons at production.

REFERENCES (Ch. 12)

[1] J. E. OSHER, B. J. MOYER and S. J. PARKER: *U.C.R.L.* 3300.
[2] M. SCHWARTZ, J. STEINBERGER, R. PLANO and N. SAMIOS: to be published.
[3] T. D. LEE and C. N. YANG: *Phys. Rev.*, **104**, 254 (1956).
[4] T. D. LEE and J. OREAR: *Phys. Rev.*, **100**, 932 (1955).
[5] S. B. TREIMAN, H. W. WYLD jr.: *Phys. Rev.*, **99**, 1039 (1955).
[6] L. ALVAREZ: *Proc. of the Rochester Conf.* (1956).

[7] T. D. Lee and C. N. Yang: *Phys. Rev.*, **102**, 290 (1956).
[8] M. Gell-Mann: *Proceed. of the Rochester Conf.* (1956).
[9] L. Alvarez, H. Bradner, P. Falk Vairant, J. Gow, A. Rosenfeld, F. Solmitz and R. Tripp: UCRL-3583, and *Nuovo Cimento*, **5**, 1026 (1957).
[10] T. D. Lee and C. N. Yang: *Phys. Rev.*, **104**, 822 (1956).
[11] G. Morpurgo: *Nuovo Cimento*, **4**, 1222 (1956).
[12] S. Bludman: *Phys. Rev.*, **102**, 1420 (1956).
[13] C. Castagnoli, C. Franzinetti and A. Manfredini: *Proceed. of the Turin Conf.* (Sept. 1956).
[14] C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes and R. P. Hudson: *Phys. Rev.* (to be published).
[15] R. L. Garwin, L. M. Lederman and M. Weinrich: to be published.
[16] B. Zumino: quoted in [17].
[17] G. Lüders: *Det. Kgl. V. Selsk.*, **28**, no. 5 (1954).
[18] W. Pauli: *Niels Bohr and the Development of Physics* (London, 1955).
[19] T. D. Lee, R. Oehme and C. N. Yang: *Phys. Rev.* (to be published).
[20] T. D. Lee and C. N. Yang: *Phys. Rev.* (to be published).
[21] L. Landau: to be published.
[22] A. Salam: *Nuovo Cimento*, **5**, 299 (1957).
[23] B. F. Touschek: *Nuovo Cimento*, **5**, 754 (1957).

Chapter 13.

The Determination of the Spin of the $K^+_{\pi 3}$.

13·1. – Introduction.

The whole discussion of Chapter **12** was based on the fact that the spin and or the parity of the $K^+_{\pi 2}$ and $K^+_{\pi 3}$ are different. Whereas for the $K^+_{\pi 2}$ the derivation of the spin parity assignments listed in (1-**12**·1) has been outlined, we have not yet given an account of the analysis on which the spin parity assignments (2-**12**·1) for the $K^+_{\pi 3}$ are based. This will be the purpose of the present chapter.

In developing such an analysis we shall try to state as clearly as possible the approximations which have been made. As already said in Sect. **12**·1 it appears now extremely unlikely that, on account of such approximations, the assignments (2-12.1) for the spin and parity of the $K^+_{\pi 3}$ are not correct; in any case we shall first develop the whole analysis arriving to the assignments (2-**12**·1) and only at the end discuss the possible criticisms.

When speaking of the $K^+_{\pi 3}$ decays we will always refer to the $K^+_{\pi 3}$ decaying at rest in the emulsions, unless otherwise stated; the discussion of Sect. **2**·1 shows that practically no $K^-_{\pi 3}$ decays at rest in emulsion. Also we shall refer to the $\pi^+\ \pi^+\ \pi^-$ mode of decay; the $\pi^+\pi^0\pi^0$ will be shortly considered in Sect. **16**·11.

The analysis to be developed is essentially due to Dalitz [1, 2]; a paper by Fabri [3] also contains a very clear derivation of many results. The analysis is based on the study of the angular and energy distributions of the decay pions. The $K^+_{\pi 3}$ constitutes an unique case, among the charged unstable particles, because it decays into three charged spinless particles and so we may interpret and measure such distributions rather easily.

13·2. – Decomposition of the outgoing wave.

For any assumed spin J and parity P of the decaying $K^+_{\pi 3}$ the possible distributions are either determined, or in some way restricted; the problem of determining the $K^+_{\pi 3}$ spin may then be divided into two parts: the first [1-3]

consists in determining for any assumed spin and parity certain angle and energy distributions — for example the distribution in the cosine of the angle θ between the direction of flight of the negative pion and the relative direction of the two positive pions and the distribution in energy of the negative pion. The second part consists in obtaining sufficient experimental information to compare with the predicted distributions.

To solve the first part of the problem we introduce the (normalized) relative co-ordinates of the three pions and their conjugated momenta [3]:

$$(1\text{-}13.2)\quad \begin{cases} \boldsymbol{r} = \dfrac{1}{\sqrt{2}}(\boldsymbol{r}_2 - \boldsymbol{r}_1)\,, & \boldsymbol{p} = \dfrac{1}{\sqrt{2}}(\boldsymbol{p}_2 - \boldsymbol{p}_1) = \boldsymbol{u}p\,, \\[2ex] \boldsymbol{r}' = \sqrt{\dfrac{2}{3}}\left(\boldsymbol{r}_3 - \dfrac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2)\right), & \boldsymbol{p}' = \sqrt{\dfrac{2}{3}}\left(\boldsymbol{p}_3 - \dfrac{1}{2}(\boldsymbol{p}_1 + \boldsymbol{p}_2)\right) = \boldsymbol{u}'p'\,. \end{cases}$$

Here 1, 2 refer to the positive pions and 3 to the negative one; p and p' are the moduli of $\boldsymbol{p}$ and $\boldsymbol{p}'$; $\boldsymbol{u}$ and $\boldsymbol{u}'$ their versors.

Defining $\boldsymbol{l} = \boldsymbol{r} \wedge \boldsymbol{p}$ and $\boldsymbol{l}' = \boldsymbol{r}' \wedge \boldsymbol{p}'$, where $\boldsymbol{l}$ and $\boldsymbol{l}'$ are respectively the angular momentum of the two positive pions and the angular momentum of the negative one, the total angular momentum of the three pions is:

$$(2\text{-}13.2)\qquad \boldsymbol{J} = \boldsymbol{l} + \boldsymbol{l}'$$

and such angular momentum is conserved in the process of decay (see Fig. 1-13.2).

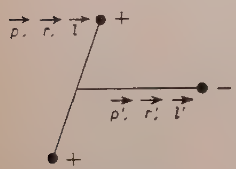

Fig. 1-13.2.

The decomposition (2) of the total angular momentum suggests to write the angular part of the outgoing three pion wave in the center of mass system, as a sum of partial waves; each partial wave being characterized by given values of l and l', the quantum numbers corresponding to $|\boldsymbol{l}|^2$ and $|\boldsymbol{l}'|^2$. Such partial waves, which will be called $Z_{ll'}^{J,m_J}(\boldsymbol{u}, \boldsymbol{u}')$ are essentially combinations of spherical harmonics of the directions characterized by $\boldsymbol{u}$ and $\boldsymbol{u}'$.

On calling $\psi_J^{m_J}(p\,|\,\boldsymbol{u}, \boldsymbol{u}')$ the angular part of the outgoing three pion wave, which, as indicated by the notation, is characterized by the total angular momentum J and its z component m_J and by the modulus p of $\boldsymbol{p}$ (p' is then determined by the conservation of the energy), we may then write:

$$(3\text{-}13.2)\qquad \psi_J^{m_J}(p\,|\,\boldsymbol{u}, \boldsymbol{u}') = \sum_{l,l'} c_{ll'}(p)\, Z_{ll'}^{J,m_J}(\boldsymbol{u}, \boldsymbol{u}')\,.$$

For a given J only terms such that:

$$|l - l'| \leqslant J \leqslant |l + l'|$$

are of course contained in the above expansion; moreover, on account of the identity of the two positive pions l has to be even and, on account of the parity conservation l' has to be even or odd according if the parity of the $K^+_{\pi 3}$ is odd or even; the coefficients $c_{ll'}(p)$ in (3) depend on the mechanism of the decay and we will come back to them later. It may be useful to give the explicit expression of $Z^{J,m_J}_{ll'}(\boldsymbol{u}, \boldsymbol{u}')$ which is

$$Z^{(Jm_J)}_{ll'} = \sum_{m_l, m_{l'}} C_{ll'}(Jm_J; m_l m_{l'}) Y^{m_l}_l(\boldsymbol{u}) Y^{m_{l'}}_{l'}(\boldsymbol{u}') ; \tag{4-13.2}$$

the $C_{ll'}(Jm_J; m_l m_{l'})$ being the appropriate Clebsch-Gordon coefficients.

The configuration of the decay products in the plane of decay is completely specified when the angle θ between $\boldsymbol{u}$ and $\boldsymbol{u}'$, and the modulus of momentum p are given; as already observed p' is then determined by the conservation of the energy; if the distribution density of the above quantities is known, any other distribution of interest may be derived from it.

From (3) and (4) the density distribution function $F_J(p|\theta)$ in the above quantities may be readily calculated for each spin and parity assignment; an average over m_J has to be performed if the $K^+_{\pi 3}$ is initially unpolarized, as it is normally; the function $F_J(p|\theta)$ has the general expression:

$$F_J(p|\theta) = \sum_{m_J} \left| \sum_{ll'} c_{ll'}(p) C_{ll'}(Jm_J; 0m_J) \cdot \left[(2l+1)(2l'+1)\frac{(l'-|m_J|)!}{(l'+|m_J|)!}\right]^{\frac{1}{2}} P^{|m_J|}_{l'}(\cos\theta)\right|^2 , \tag{5-13.2}$$

where $P^m_l(\cos\theta)$ is as usual defined as:

$$P^m_l(\cos\theta) = Y^m_l \exp[-im\varphi]\left[\frac{2l+1}{4\pi}\frac{(l+m)!}{(l-m)!}\right]^{-\frac{1}{2}} .$$

So far the treatment is exact; from this expression of the density distribution function it is possible to calculate the distributions of interest by multiplying it with the density of the final states, provided the $c_{ll'}(p)$ are known; this is however not the case; in the next section we shall see how one may overcome this point, by making a suitable approximation.

13·3. – The small radius approximation.

The approximation which we shall make [1-3] in order to determine effectively the distributions is based on the fact that the de Broglie wave lengths of the outgoing pions are large with respect to the radius of the $K^+_{3\pi}$

meson, if we assume that this last quantity is much smaller than the Compton wave length of the pion.

This has two related consequences; one is that the centrifugal barrier will prevent waves with a high angular momentum to escape, favouring the partial waves with the smallest l and l' values; therefore the coefficients $c_{ll'}(p)$ will decrease with increasing l, l' and, referring to the infinite set of l and l', compatible with a given assignment of spin and parity only the coefficients $c_{ll'}(p)$ having values of l and l' with the minimum value of $l+l'$ will be important in determining the distributions; the second, and related consequence is that the p dependence of a given $c_{ll'}(p)$ coefficient (with given l and l') may be predicted to be of the form $(p/M)^l(p'/M)^{l'}$, apart from terms of higher order in $(p/M)^2$; here $\hbar/Mc$ is the assumed radius of the $K^+_{\pi 3}$.

These two consequences are summarized in the following formula (1) which gives just the term of lowest order in p/M, p'/M for any l and l' value:

$$c_{ll'}(p) = \frac{A_{l+l'}}{(l+l+4)!!}\,\frac{(l+1)!!}{(2l+1)!!}\,\frac{(l'+1)!!}{(2l+1)!!}\left(\frac{p}{M}\right)^l\left(\frac{p'}{M}\right)^l . \tag{1-13·3}$$

In (1) $A_{l+l'}$ is a coefficient which decreases with decreasing $l+l'$. Formula (1) is essentially obtained calculating the amplitude of the emitted three pion wave (corresponding to the minimum value of $l+l'$) at the « $K^+_{\pi 3}$ surface » for the values p and p' of the relative momenta; as already mentioned such amplitude decreases, for given values of p and p', with increasing $l+l'$ due to the centrifugal barrier effect. It may be shown from formula (1) that the ratio between a $c_{ll'}(p)$ and the next one (obtained increasing l or l' by 2) is about 10^{-2} for a radius of the $K^+_{\pi 3}$ equal to its Compton length.

We proceed now, on the basis of the above approximation, to study the distributions for several assignments of the spin and of the parity.

The pairs of l and l', denoted by (l, l'), compatible with the spin and parity assignment and corresponding to the lowest value of $l+l'$ are reported, for several spin parity assignments, in the second column of the following Table I; in the third column are reported also the (l, l') pairs giving rise to the next higher value of $l+l'$.

It is seen from the table that for the five assignments marked with an asterisk (henceforth called « simple ») there is just one such choice of l and l' which gives rise to the lowest $l+l'$ value; in all the other cases there is more than one choice, *except in the case of spin zero and parity* + (0+) *in which case the decay into three pions is absolutely forbidden* (*). Therefore we conclude

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(*) That the decay, in the 0^+ case, is absolutely forbidden may be understood remarking that the final three pion state should have l' odd and l even to conserve the parity; it is then impossible to obtain a resultant $J=0$.

TABLE I-13·3.

J	Parity	(l, l')	
0	+	—	—
	− (*)	(0.0)	(2.2); (4.4); .....
1	+ (*)	(0.1)	(2.1); (4.3); .....
	− (*)	(2.2)	(4.4); (6.6); .....
2	+ (*)	(2.1)	(2.3); (4.3); .....
	−	(0.2); (2.0)	(2.4); (4.2); .....
3	+	(0.3); (2.1)	(2.3); (4.1); .....
	− (*)	(2.2)	(2.4); (4.2); .....
4	+	(2.3); (4.1)	(2.5); (4.3); .....
	−	(0.4); (2.2); (4.0)	(2.4); (4.2); .....
.....		.....	.....

that in the five simple cases the distributions, under the hypothesis made in this section are completely determined; they may easily be calculated from the general expression (5-**13**·2). The distribution densities for the five simple cases 0 —, 1+, 1 —, 2+, 3 — are reported in the Table II.

TABLE II-13·3.

[0 −]	1
[1 +]	p'^2
[1 −]	$p^4p'^4 \sin^2\vartheta \cos^2\vartheta$
[2 +]	$p^4p'^2 \sin^2\vartheta$
[3 −]	$p^4p'^4 \sin^2\vartheta(5+3\cos^2\vartheta)$

For the non simple cases the energy and angular distribution densities have instead to be calculated combining two or more amplitudes in (5-**13**·2) with the appropriate $c_{ll'}(p)$ coefficients, and squaring; if one could take seriously the expression (1) for $c_{ll'}(p)$ then, of course, one might calculate completely also for these non simple cases the distributions; but the value of formula (1) is of course just to give an order of magnitude and the p dependence of the $c_{ll'}(p)$, not their precise values. The distribution densities in the non simple cases will then be a combination of two (or more) elementary distributions with arbitrary coefficients plus one (or more) interference term and for some cases they are calculated in [4].

As already mentioned, to obtain the distributions from the function

$F_J(p|\theta)$ we have still to multiply by the density of the final states. In the non relativistic case, to which we shall refer in the following discussion, the density of the final states, expressed through p and $\cos\theta$ is proportional to

$$\text{(2-13·3)} \qquad \mathrm{d}\varrho = p'p^2\,\mathrm{d}p\,\mathrm{d}\cos\theta\,,$$

where p and p' are related (non relativistically) by:

$$\text{(3-13·3)} \qquad \frac{p^2}{2m}+\frac{p'^2}{2m}=Q\,, \qquad Q = 75\,\text{MeV}\,,\quad m = \text{pion mass}\,.$$

It is perhaps more convenient to introduce instead of p the variable $\varepsilon = E_3/E_{\max}$ defined as the ratio between the laboratory energy E_3 of the negative meson and the maximum value which it may have $E_{\max} = \frac{2}{3}Q$.

In terms of ε we have:

$$\text{(4-13·3)} \qquad \mathrm{d}\varrho = \varepsilon^{\frac{1}{2}}(1-\varepsilon)^{\frac{1}{2}}\,\mathrm{d}\varepsilon\,\mathrm{d}\cos\theta\,.$$

For the five simple cases the distributions in ε and in $\cos\theta$, defined as

$$\frac{\mathrm{d}N(\varepsilon)}{N} = I(\varepsilon)\varepsilon^{\frac{1}{2}}(1-\varepsilon)^{\frac{1}{2}}\,\mathrm{d}\varepsilon \qquad \text{and} \qquad \frac{\mathrm{d}N(\theta)}{N} = I(\theta)\,\mathrm{d}\cos\theta\,,$$

respectively, may be calculated readily from expressions in the Table II and are reported in Table III below; $I(\varepsilon)$ and $I(\theta)$ will be also reported graphically in Fig. 2-13·6 and Fig. 3-13·6 of Sect. **13·6** where they will be compared with the experiment.

TABLE III-13·3.

	$I(\varepsilon)$	$I(\theta)$
0^-	1	1
1^+	ε	1
1^-	$\varepsilon^2(1-\varepsilon)^2$	$\sin^2\theta\cos^2\theta$
2^+	$\varepsilon\,(1-\varepsilon)^2$	$\sin^2\theta$
3^-	$\varepsilon^2(1-\varepsilon)^2$	$\sin^2\theta(5+3\cos^2\theta)$

It appears from the tables and the figures that the shapes of both the distributions are simply a constant in the $0-$ case; for the other cases they are usually more complicated; in the most part of the cases their behaviour near $\varepsilon = 0$ or 1 and near $\cos\theta = 0$ or 1 may be understood by simple arguments (compare the Dalitz arguments at the end of the next section).

13·4. – Distributions of interest.

Besides the distributions $I(\varepsilon)$ and $I(\theta)$ which will be referred to in the following as the *Dalitz distributions*, there are also other distributions which one may wish to consider; they give different ways of comparing the theory with the experiment; they are however related through the fact that they may be all calculated starting from the same expression (5-**13**·2) of the distribution density. We shall here mention only the so called « Dalitz triangular » plot [1], the Fabri [5] energy distribution and the Costa and Taffara [6] angular distributions.

a) The Dalitz triangular plot (*). – While $I(\varepsilon)$ and $I(\theta)$ are derived from the general expression (5-**13**·2) by integrating respectively over θ or over ε, the Dalitz triangular plot is simply a two dimensional graphical representation of the general expression (5-**13**·2) itself.

It is therefore the most general representation and all the possible distributions (like the previous ones $I(\varepsilon)$ and $I(\theta)$ and the ones which will follow) may be derived from it by convenient integrations. One arrives at this representation by observing that, in an equilateral triangle, the sum of the distances of any point inside the triangle from the three sides is equal to the height of the triangle. If we now interprete the above distances as proportional to the kinetic energies of the three pions emitted in the disintegration of the $K^+_{\pi 3}$ and if the height of the triangle is equal to the available energy, then the relation $E_1+E_2+E_3=Q$ for any point internal to the triangle is automatically satisfied; that is, to any decay event some point P internal to the triangle is unambiguously associated; the conservation of the momentum implies further that not all the points inside the triangle may represent decay events, but only those points which are contained in a circle inscribed in the triangle (relativistically it is a somewhat more complicated figure but it does not deviate much from a circle). The situation is represented in Fig. 1; the distance PU of P from the side AB represents there the energy of the negative meson, and the distances from the sides CA and CB the energies of the two positive pions; in view of the fact that the two pions are identical, the distribution of the representative points in the triangle must be simmetrical with respect to the height CO of the triangle; therefore all the points may be reported in a half of the triangle, say COB and the distance PL represents

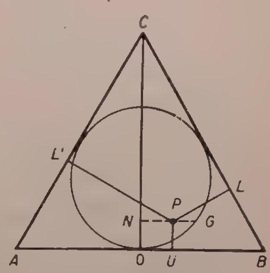

Fig. 1-13·4.

(*) This has already been mentioned in Sect. 1·4.

then the energy of the less energetic positive pion. One can show also that for any point P the ratio PN/NG is equal to $\cos\theta$.

From the expressions of Table II-**13**.3 and from the expression (2-**13**.3) for the density of the final states it is easy to derive the predicted densities of the representative decay points inside the half-triangle corresponding to various spin parity assignments.

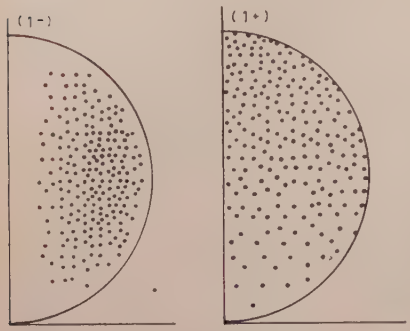

Fig. 2-**13**.4. – (From [9, 16]). Distribution of 215 $K^{+}_{\pi 3}$ decays if the spin and parity are the ones indicated.

We will only say here that the density of the final states (2-**13**.3) is, when the energies are expressed in terms of the cartesian co-ordinates in the plane of the triangle, directly proportional to the elementary area $dx\,dy$, so that in the 0 — case the distribution of the points inside the circle is isotropical; in the other cases the density of the points is not isotropical and for two typical cases it is reported in Fig. 2.

b) The Fabri energy distribution. – From the Dalitz triangular diagram it is possible, for any given spin and parity assignment, to calculate the fraction of events in which the energy of the negative pion is *a*) less then the energy of either of the two positive pions; or *b*) in between the two energies of the positive pions; or *c*) larger than the energy of either of the two positive pions. Such fractions have been calculated by FABRI for each of the simple cases and are reported in Fig. 4 of Sect. **13**.6 in which they are also compared with the experimental data; according to FABRI in the distributions in question the relativistic corrections are not negligible.

c) The Costa and Taffara angular distribution. – In a similar way COSTA and TAFFARA [6] have calculated for several spin parity assignments the fraction of events to be expected for which the angle between the two like mesons is comprised in the intervals 0⊢60°, 60°⊢120°, 120°⊢180°; a similar calculation they have done for the angle between the unlike and one of the like mesons; the calculations are non relativistic; the fractions in question are reported graphically respectively in Fig. 5 and 6 of Sect. **13**.6 where they are also compared with the experiment.

13.5. – Dalitz's arguments.

Before comparing the various theoretical distributions with the experiment, we want to add four arguments, due to DALITZ [2, 7] which give the intuitive reason for the behaviour at certain limiting values of the energy and of the

angles of the distributions corresponding to the various spin-parity assignments. Of course, what these arguments say is already contained in the explicit expressions for the distributions, which we have already given, but, as we shall see, the arguments are very interesting, showing clearly which are the most indicative features of the distributions.

Moreover one may expect, on the basis of the DALITZ arguments that such features will be unchanged also in a more exact theory, which does not make use of the approximations of the previous theory (compare also Sect. **13**·7).

The Dalitz arguments simply express how some fundamental facts, such as the impossibility for a particle with vanishing momentum to cross a centrifugal repulsive barrier, must be reflected in the distributions.

The three first Dalitz arguments, reported below, refer to a $K_{\pi 3}$ having either odd spin and odd parity or even spin and even parity. The fourth refers to a $K_{\pi 3}$ having just even parity. They are as follows:

1) For a $K_{\pi 3}$ having either odd spin and odd parity or even spin and even parity, the probability of a decay with the emission of a positive or negative π with vanishing momentum vanishes; in fact the π with vanishing momentum must be produced in an S state because a wave with vanishing momentum has zero probability of passing through the centrifugal barrier which is otherwise present; the two other pions have then to be produced in a state of relative motion with a parity opposite to the one of the original $K_{\pi 3}$; but this is not possible because the parity is determined by the angular momentum which has to be conserved. Therefore, the distributions previously calculated must show in these cases a strong decrease when the energy of the emitted (positive or negative) pion decreases; this is in fact the case (compare Table II-**13**·3 and Fig. 2-**13**·6 for the $1-$, $2+$, $3-$ distributions; compare also Fig. 2-**13**·4 showing the calculated Dalitz plot for the $1-$ case; the density of representative points decreases near the AB and CB sides of the triangle).

2) For a $K_{\pi 3}$ having either odd spin and odd parity or even spin and even parity the amplitude for the emission of a negative pion with its maximum possible energy must vanish, in other words the emission of a fast negative pion is improbable; this is proved by considerations similar to the above one, noting that the two positive pions must then have zero relative momentum.

3) For a $K_{\pi 3}$ having either odd spin and odd parity, or even spin and even parity a disintegration in an collinear configuration, that is with a $\cos\theta$ value equal to 1 cannot take place; in other words disintegrations with $\cos\theta$ near to 1 are improbable; this may be understood, considering the properties of the spherical harmonics.

The fourth Dalitz argument refers instead to $K_{\pi 3}$ having any spin but even parity; then it may be shown that:

4) For a $K_{\pi 3}$ having even parity the distributions tend to zero when the energy of the negative pion tends to zero; in other words the emission of a slow negative pion is improbable.

In all these arguments the word « improbable » means improbable with respect to the probability given by the phase space alone, which is the same, as mentioned already, as the probability for the $0-$ case.

It has to be recalled that, according to what has been said in Sect. **12·1** the $K_{\pi 3}$ must have spin and parity both odd or both even, if we want that $K_{\pi 2}^{+}$ and $K_{\pi 3}^{+}$ have the same parity and spin; so Dalitz' arguments have the advantage of clearly showing which are the particular features of the distributions which should be examined in order to see if the $K_{\pi 2}$ and $K_{\pi 3}$ may have the same spin parity assignment. As already mentioned such arguments, depending on very general reasons, have also the advantage that they do not depend on the approximations made in developing the previous theory *and apply also to the non simple cases*.

13·6. – Comparison with the experiment.

In comparing the previous theory with the experiment it is convenient to discuss two points separately:

1) Whether the experimental distributions are compatible with a spin parity assignment for the $K_{\pi 3}^{+}$ which is possible also for the $K_{\pi 2}^{+}$.

2) Whether the experimental distributions allow us to establish in a definite way the spin and parity of the $K_{\pi 3}^{+}$.

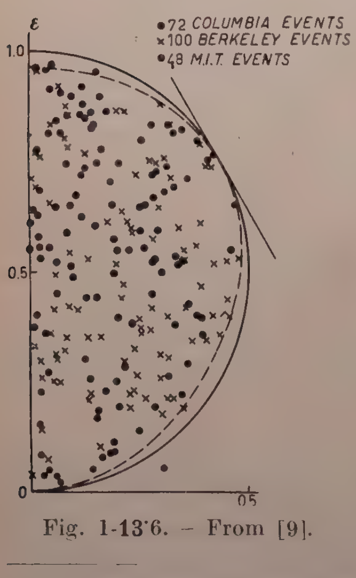

Fig. 1-13·6. – From [9].

The answer to the first question appears to be almost certainly negative; the answer to the second is perhaps not so definite but it appears that the extremely most probable assignment is $0-$.

To see this let us consider the experimental distributions (*).

They are reported in the following figures; most of this work is due to the Brussels Milan-Padua (B.M.P.) group which, alone, has analyzed a collection of 402 $K_{\pi 3}^{+}$ decays

(*) The possible biasses have been discussed in Sect. 1·4.

in homogeneous conditions and trying to avoid carefully observational biasses.

The results of the other laboratories [8-14], and of cosmic rays [15] are usually consistent, within the fluctuations, with those of the Brussels-Milan-Padua group.

In Fig. 1 we have reported a Dalitz plot compiled on 220 events presented by Gottstein [16] at the Rochester Conference 1956.

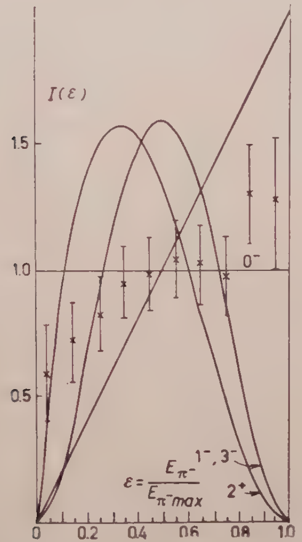

Fig. 2-13·6. – The $I(\varepsilon)$ distributions. The experimental points are those of the B.M.P. group [17].

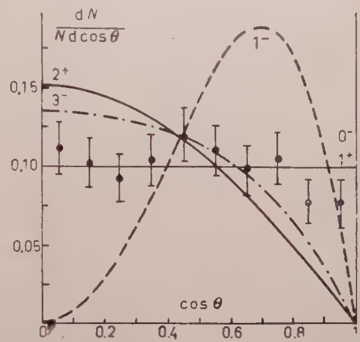

Fig. 3-13·6. – (From [17]). The $I(\theta)$ distribution together with the theoretical curves for the « simple » cases.

Fig. 2 and 3 contain the Dalitz distributions $I(\varepsilon)$ and $I(\cos\theta)$; Fig. 4 to 6 report respectively the Fabri and the Costa and Taffara distributions.

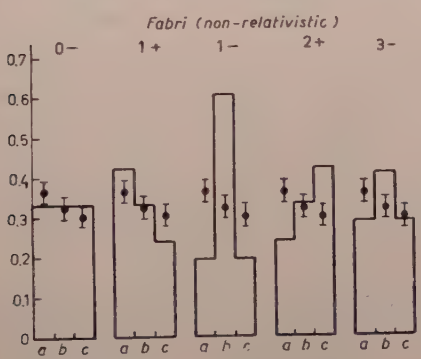

Fig. 4-13·6. – (From [17]).

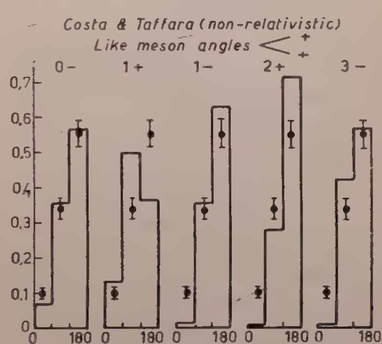

Fig. 5-13·6. – (From [17]).

Finally Fig. 7 reports the $I(\varepsilon)$ distribution multiplied by the phase space factor $\varepsilon^{\frac{1}{2}}(1-\varepsilon)^{\frac{1}{2}}$. Fig. 3 to 7 are from the B.M.P. group [17].

The theoretical curves which appear in the various figures have been

already discussed. We shall now concentrate on the experimental distributions (+).

To answer the question n. 1 let us observe that the $I(\varepsilon)$ distribution (Fig. 3) does not tend to zero when $\varepsilon \to 0$ or when $\varepsilon \to 1$; and the $I(\cos\theta)$ distribution does not tend to zero when $\cos\theta \to 1$; the number of events per unit phase space observed say from $\varepsilon = 0$ to $\varepsilon = 0.05$ or from $\varepsilon = 0.95$ to $\varepsilon = 1$ is not very different from that observed in any other similar energy interval. Due to the rather large number of events the probability that this is due to fluctuations is now extremely small. The same may be said for the events in the $\cos\theta$ interval from $\cos\theta = 0.95$ to $\cos\theta = 1$; their number shows that there is not a strong decrease of $I(\cos\theta)$ near $\cos\theta = 1$ as should be the case if $\lim I(\cos\theta) = 0$.

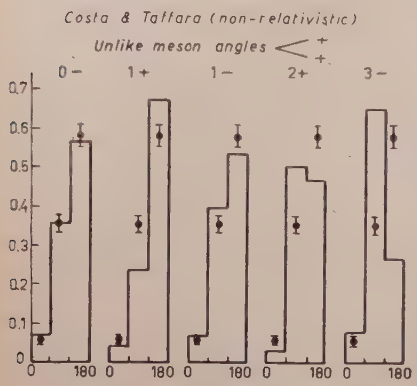

Fig. 6-13.6. – (From [17]).

Recalling the Dalitz arguments reported in the last section we thus see that the $K_{\pi3}$ cannot have even spin and even parity or odd spin and odd parity. But, as explained in Sect. 12·1 the $K_{\pi2}$ must have (if the angular momentum and the parity are conserved in the decay) even spin and even parity, or much less probably odd spin and odd parity. It follows that no one of the assignments possible for the $K^+_{\pi2}$ is possible for the $K^+_{\pi3}$ or viceversa. It was this conclusion which has been the starting point of all the questions discussed in Chapter 12.

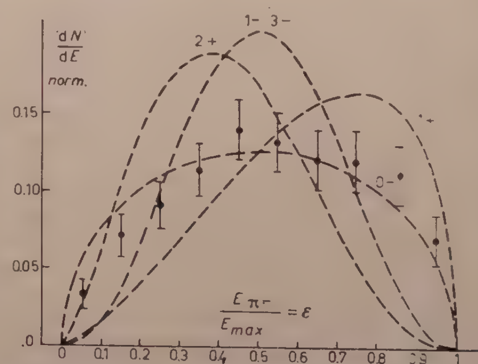

Fig. 7-13·6. – (From [17]). Graphs of $dN/d\varepsilon = I(\varepsilon)\sqrt{\varepsilon(1-\varepsilon)}$ and experimental points.

We now discuss the second point, that is we try now to establish, which is the value of the spin and parity of the $K_{\pi3}$. The above discussion shows that assignments with odd spin and odd parity or even spin and even parity are excluded. We may now see that also an assignment in which the $K_{\pi3}$ has

(+) The probability that the observed distributions represent a statistical fluctuation from the curves corresponding to any given spin-parity assignment, may be calculated by standard probability methods; we shall not however report the results of these calculations, because, in our opinion, they do not tell much more than the direct observation of the graphs.

even parity (and any spin) is very improbable since the experimental $I(\varepsilon)$, then, would contradict the fourth Dalitz assignment; at least if we interpret the experimental curve (Fig. 2) as showing that $I(\varepsilon)$ does not tend to zero when $\varepsilon \to 0$.

Therefore it appears that the only possible assignments are those with even spin and odd parity. Among these, of course, the $0-$ assignment is the simplest one and the theoretical curves for this case (which do not include either the Coulomb or the relativistic corrections) are reported in Fig. 2 to 7 together with the experimental distributions. It is apparent from the figures that a rather good fit is obtained; for completeness, in the figures the theoretical distributions for the other simple cases are also reported: no one provides a good fit to all the distributions. This is simply a repetition of the arguments given above since the cases ($1-$, $1+$, $2+$, $3-$) have either odd spin and odd parity or even spin and even parity, or even parity.

So we may conclude that $0-$ provides a rather good fit to all the distributions. It is however probably not possible to exclude presently, at least in principle, a $2-$ assignment or a $4-$ assignment (and so on), since the distributions corresponding to such cases contain an adjustable parameter which may be varied so as to give the best fit. However, rather strong independent arguments may be found against such assignments (compare the end of the next section).

Coming back to the fit of the predicted $0-$ distributions to the experimental curves we may notice that there is a slight tendency of the experimental points in the direction corresponding to the $1+$ assignment.

This tendency is shown both by the $I(\varepsilon)$ distribution (the experimental points are somewhat under the $0-$ line for small ε and somewhat over for large ε) and by the Dalitz triangle (the two things are in fact equivalent); the density of the points, which should be uniform for a $0-$ assignment, according to the theory developed in the past sections, somewhat increases from the low to the high π energies. A Dalitz plot described (but not given) by DALITZ [7] at the Rochester Conference 1956, containing 600 events is remarkably uniform. No apparent correlations are seen, though the density (per unit phase space) of fast π, going up to the maximum energy of 48 MeV is about 30% higher than that of slow π.

These slight deviations from the $0-$ curves however do not certainly point towards a different assignment; they may be due to a number of reasons as it will become apparent from the next section.

To conclude the answer to the question number 2 at the beginning is: the most probable assignment for the $K^+_{\pi 3}$ is $0-$; $2-$ cannot be excluded but is (compare the end of the next Sect. 13'7) improbable.

13·7. – A discussion of the approximations.

The conclusion reached with regard to the question number 1 of the last section is such as to stimulate a reexamination of the theory leading to it; we ask: is it possible, with a treatment which does not make use of the approximations of the theory previously developed, to modify the $K^+_{\pi 3}$ decay spectra in such a way as to make them compatible with an assignment possible for the $K^+_{\pi 2}$?

Notice by the way that the lowest assignment simultaneously possible for $K^+_{\pi 2}$ and $K^+_{\pi 3}$ is $2+$, if the $K^0_{\pi 2}$ may decay in two neutral pions as it seems indicated from the experiments; otherwise it is $1-$.

The approximations which we did introduce either explicitly or not in the theory of the $K^+_{\pi 3}$ decay are the following:

1) Neglect of relativistic effects.

2) Small « radius » approximation.

3) Neglect of interactions between the final pions.

As far as 1) is concerned the situation is this. There are mainly two effects which come into play: *a*) The modification of the equation of energy conservation and of the expression for the phase space (4-**13**·3); both these effects imply generally corrections, which we may call « kinematical » corrections to the theoretical distributions, which are in some cases of the order of 20% and have been explicitly calculated, for instance, in [18]. *b*) The modification of the decay matrix element due to relativistic effects. To understand these modifications consider for instance a $(0-)$ $K_{\pi 3}$ and assume that the interaction responsible for its decay may be written as:

$$H' = g \int \psi_\tau(\boldsymbol{x})\Phi^+(\boldsymbol{x})\Phi^+(\boldsymbol{x})\Phi(\boldsymbol{x})\,\mathrm{d}^3x + \text{h.c.}\,. \tag{1-13·7}$$

This is equivalent to consider a $K_{\pi 3}$ having a vanishing radius (see later). Here $\psi_\tau(\boldsymbol{x})$ is destruction operator of a $K_{\pi 3}$ at the position $\boldsymbol{x}$ and $\Phi^+(\boldsymbol{x})$, $\Phi(\boldsymbol{x})$ are the creation operators of positive and respectively negative pions. The expression (1) is a relativistic scalar if the $K_{\pi 3}$ has a $0-$ assignment. Then, on evaluating the square of the matrix element of (1) one will not get simply a constant, as was the non relativistic result (compare the first line of the Table III-**13**·3) where this constant was normalized to 1), but one will get an energy dependent normalization factor of the kind:

$$(\omega_{p_1}\omega_{p_2}\omega_{p_3})^{-1}\,,$$

where ω_{p_1}, ω_{p_2}, ω_{p_3} are the total energies of the three emitted pions. The corrections in the energy distribution which this factor brings are a few percent but should be included.

Summing up, the relativistic corrections may change appreciably the shapes of the spectra but should not change the conclusions of the last section on the spin-parity assignment.

Let us now examine the corrections 2) and 3) namely the «small radius» approximation and the final state interactions.

To understand their meaning and, at the same time, to understand the meaning of the expression (1) previously written down, we may argue as follows: let us consider, for simplicity, again a $(0\,-)$ $K^+_{\pi 3}$; then, whatever the mechanism of the decay is, the effective operator producing the decay of such a $K_{\pi 3}$ may be written, in the rest system of the decaying $K_{\pi 3}$:

$$(2\text{-}13{\cdot}7)\qquad H' = g\int \psi_\tau(\boldsymbol{x})G(\boldsymbol{x}-\boldsymbol{x}', \boldsymbol{x}-\boldsymbol{x}'', \boldsymbol{x}-\boldsymbol{x}''')\cdot \cdot\Phi^+(\boldsymbol{x}')\Phi^+(\boldsymbol{x}'')\Phi(\boldsymbol{x}''')\,\mathrm{d}^3\boldsymbol{x}\,\mathrm{d}^3\boldsymbol{x}'\,\mathrm{d}^3\boldsymbol{x}''\,\mathrm{d}\boldsymbol{x}''' + \text{h.c.}\,,$$

with the meaning of the operators ψ_τ, $\Phi^+(\boldsymbol{x})$, $\Phi(\boldsymbol{x})$, specified in connection with (1), and where G is an arbitrary rotation invariant function of the arguments indicated. Of course the expression (2) is the most general one since it contains the arbitrary function G of the three relative co-ordinates; the arbitrariness of G implies that the matrix element of H' (2) between the initial and final state is an arbitrary rotation invariant function of the momenta of the emitted pions; since the only variables upon which the matrix element for the $K_{\pi 3}$ decay depend are the momenta, the expression (2) is the most general form of effective decay operator for a $K_{\pi 3}$.

In terms of the expression (2) the two kinds of approximations which we want to discuss may be very easily expressed: *the small radius approximation* amounts to say that the function G is a product of δ functions of the three arguments $\boldsymbol{x}-\boldsymbol{x}'$, $\boldsymbol{x}-\boldsymbol{x}''$, $\boldsymbol{x}-\boldsymbol{x}'''$, or, more generally, is a function which goes rapidly to zero when $|\boldsymbol{x}-\boldsymbol{x}'|$, $|\boldsymbol{x}-\boldsymbol{x}''|$, $|\boldsymbol{x}-\boldsymbol{x}'''|$ are larger than some value R which is much smaller than the wave length of the faster emitted pion. This approximation of course implies that, in a power expansion of the Fourier transform of G in series of the momenta, only the terms of the lowest order are important, which is just what we did assume in writing down the distributions in the table.

On the other hand the neglect of the final state interactions between the pions means that one evaluates the matrix element of (2) inserting for the final pion wave, pure plane waves.

If one takes into account these final state interaction one should insert in (2) the distorted (ingoing!) waves, the kind and nature of the distortion depending on the nature of the, almost unknown, pion interaction.

What we did say in the case of a (0) $K_{\pi 3}$ meson may be easily extended to any spin parity assignment, considering instead of a rotation invariant G, a function G having the appropriate transformation character.

We then ask: is it possible for a $K_{\pi 3}$ with a spin parity assignment compatible with that for the $K_{\pi 2}$ (for instance $2+$), to obtain decay distributions which simulate the ones from a (0 −) meson (which agree with the experiments) if we renounce to the small radius approximation and assume convenient interactions between the final pions?

Two attempts have been made, one by MARSHAK and SUDARSHAN [19], the other by MORPURGO and TOUSCHEK [20] assuming a $(2+)$ $K_{\pi 3}$ meson.

The attempt by MARSHAK and SUDARSHAN assumes that the radius of the $K^+_{\pi 3}$ is so large that in addition to the final state with $l = 2$, $l' = 1$ also the final state with $l = 2$, $l' = 3$ intervenes with comparable probability. The mixture parameter s of the two states is varied and the $I(\varepsilon)$ and $I(\cos\theta)$ curves are presented for various values of such parameter in the Fig. 1-13·7.

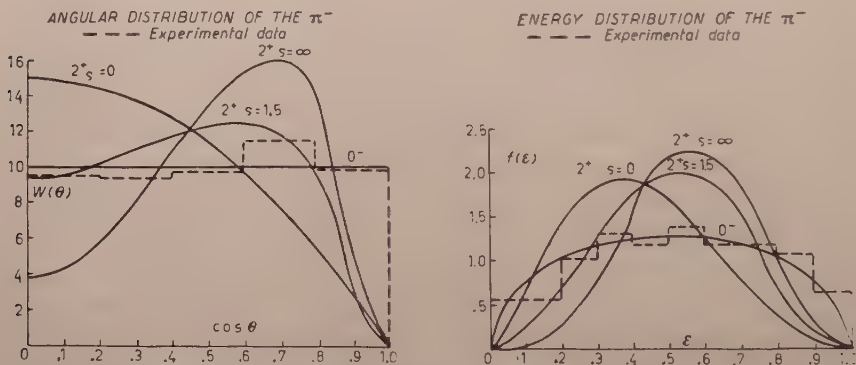

Fig. 1-13·7. – Marshak and Sudarshan spectra (the experimental data are not the most recent ones, though they do not differ much from them).

The attempt by MORPURGO and TOUSCHEK instead considers a $K^+_{\pi 3}$ with a small radius, but with a strong interaction between two pions in a final D state with $T = 0$. The distributions are then calculated assuming that a resonance takes place in that state at a convenient energy. The most convenient value of the kinetic relative resonance energy of the two pions (defined as the sum of the kinetic energies of the two pions) appears to be ~ 100 MeV and the spectra for such choice are presented in Fig. 2-13·7.

It is apparent that neither of the attempts is successful since the predicted spectra near $\varepsilon = 0$ and 1 and near $\cos\theta = 1$ are still too poor. This was indeed expected since in these regions the distributions are governed by the Dalitz arguments. The hope was however to obtain curves which, though satisfying the Dalitz rules, would fall down very steeply near the points

indicated above, being similar to the (0 —) ones for all the remaining regions.

Therefore we may say that the conclusion reached in the section is still valid and it appears improbable that mechanisms of the kind outlined in this section may change it.

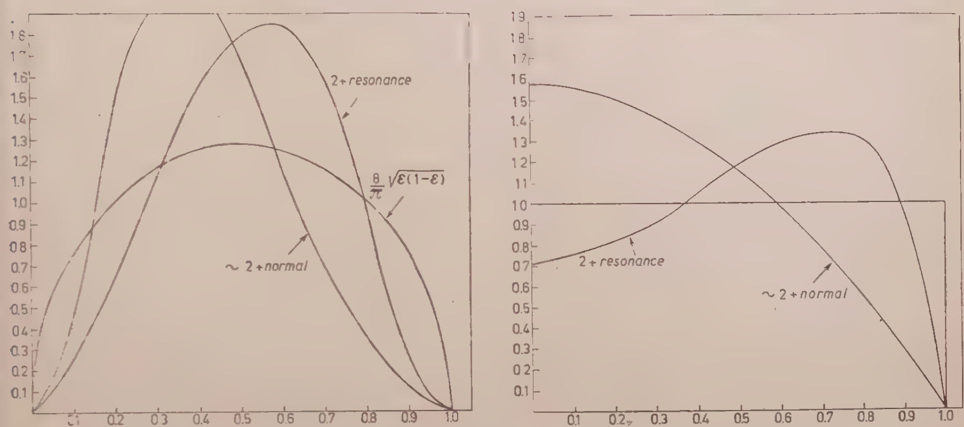

Fig. 2-13'7. – Morpurgo and Touschek spectra.

Of course both the relativistic corrections, the presence of a form factor and interactions between the final pions, in particular the *Coulomb ones* (*), may produce changes in the distributions, as calculated without taking them into account: it is possible, if the conclusion (0 —) is accepted that these effects are responsible for the (rather small) differences between the experimental and theoretical distributions in such case.

We may finally notice that a high value for the $K^+_{\pi 3}$ spin, say larger than 3, is very improbable also because then its decay should be much too slow on account of the slowing down effect of the centrifugal barrier; at least if the $K_{\pi 3}$ radius is assumed less than the wave length of the pion.

Another problem, which arises already for a spin 2 is that such a $K^+_{\pi 3}$ should possibly prefer to decay through the reaction $K^+_{\pi 3} \to \pi^+ + \gamma$ and not through the emission of three pions. This is not the case if the $K_{\pi 3}$ (is 0 —) because then it must radiate at least two photons. Some estimates of the rates of these electromagnetic processes may be found in [21] (compare also the Sect. **16**'10).

(*) The M.I.T. group (unpublished) has remarked that there is some neutralization in the 0 — case between the Coulomb corrections and the relativistic kinematical ones.

13·8. – Another possible way for getting information on the spin of the $K^+_{\pi3}$ (and of other K's).

We end this discussion of the $K^+_{\pi3}$ spin by reporting a method proposed by TEUCHER, THIRRING and WINZELER [22]. Following a method proposed by WENTZEL [23] in 1949 for determining the spin of the pion, these Authors have pointed out that if the $K^+_{\pi3}$ has a spin different from zero it might come out polarized from the process in which it is produced; if this is the case, due to the fact that the polarization should not be appreciably lost in the process of slowing down [24], the distribution in the cosine of the angle θ_n between the normal to the plane of decay and the direction of emission from the star in which it has been produced, might be anisotropical

Neither in the events considered by TEUCHER *et al.* nor in the B.M.P. events, nor in the reported cloud chamber events decaying in flight (DALITZ: private communication to Prof. AMALDI), this seems to be the case. Of course this does not prove that the $K^+_{\pi3}$ has spin zero because *a*) it might have spin different from zero but come out unpolarized in the reaction in which it is produced; *b*) it might have spin different from zero and be polarized, but the polarization state be such as to give rise to a more or less isotropical distribution in the particular angle considered.

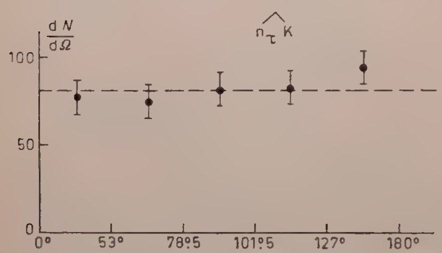

Fig. 1-13·8. – (From [17]).

Anyway in the Fig. 1-13·8 we have reported the distribution in θ_n for the 402 Padua events. Of course a similar method may be applied to determine the spin of the $K_{\pi2}$ or $K_{\mu2}$. The diagram [12] analogous to Fig. 1-13·8 for 701 K_L gives also an isotropical distribution; as far as we know no such diagram in which $K_{\pi2}$ are separated from the other K_L has been constructed.

A general observation is that in using a method like the above one it should be better in spite of the smaller statistics, to collect $K_{\pi3}$ produced in similar conditions (same angle and energy of production; same material of the production target) if one wants to avoid that a possible polarization is averaged to zero by the mixture of many inhomogeneous cases.

We finally mention here that the decay spectra for the $K_{\beta3}$ and $K_{\mu3}$ have also been calculated [25] in order to determine their spins and to find the deviations which specific decay interactions might produce from the statistical spectrum. Presently the data are much too poor and too biassed to make a discussion useful. (See Sect. 1·4).

REFERENCES (CH. 13)

[1] R. DALITZ: *Phil. Mag.*, **44**, 1068 (1953).
[2] R. DALITZ: *Phys. Rev.*, **94**, 1046 (1954).
[3] E. FABRI: *Nuovo Cimento*, **11**, 479 (1954).
[4] B. T. FELD: unpublished.
[5] E. FABRI: *Suppl. Nuovo Cimento*, **12**, 205 (1954).
[6] G. COSTA and L. TAFFARA: *Nuovo Cimento*, **4**, 169 (1956).
[7] R. DALITZ: *Proceed. of the Rochester Conf.* (1956).
[8] N. BREENE, K. H. HANSEN, J. E. HOOPER and M. SCHARFF: *Nuovo Cimento*, **4**, 1059 (1956).
[9] J. OREAR, G. HARRIS and S. TAYLOR: *Phys. Rev.*, **102**, 1676 (1956).
[10] B. T. FELD, A. C. ODIAN, D. M. RITSON and A. WATTENBERG: *Phys. Rev.*, **100**, 1539 (1955).
[11] R. P. HADDOCK: *Nuovo Cimento*, **4**, 240 (1956).
[12] N. N. BISWAS, L. CECCARELLI-FABBRICHESI, M. CECCARELLI, K. GOTTSTEIN, N. C. VARSHNEYA and P. WALOSCHEK: *Nuovo Cimento*, **3**, 825 (1956).
[13] B. BHOWMICK, D. EVANS, I. J. VAN HEERDEN and D. J. PROWSE: *Nuovo Cimento*, **3**, 574 (1956).
[14] H. HECKMAN, F. M. SMITH and W. H. BARKAS: *Nuovo Cimento*, **4**, 51 (1956).
[15] E. AMALDI: *Suppl. Nuovo Cimento*, **4**, 179 (1956).
[16] K. GOTTSTEIN: *Proceed. of the Rochester Conf.* (1956).
[17] M. BALDO-CEOLIN, A. BONETTI, W. D. B. GREENING, S. LIMENTANI, M. MERLIN and G. VANDERHAEGHE (B.M.P. group): *Nuovo Cimento*, 1957 (to be published).
[18] Y. EISENBERG, E. LOMON and S. ROSENDORFF: *Nuovo Cimento*, **4**, 610 (1956).
[19] R. E. MARSHAK and G. SUDARSHAN: unpublished.
[20] G. MORPURGO and B. F. TOUSCHEK: unpublished.
[21] R. DALITZ: *Phys. Rev.*, **99**, 915 (1955).
[22] M. TEUCHER, W. THIRRING and H. WINZELER: *Nuovo Cimento*, **1**, 733 (1955); H. WINZELER: *Suppl. Nuovo Cimento*, **4**, 216 (1956).
[23] G. WENTZEL: *Helv. Phys. Acta*, **22**, 101 (1949).
[24] W. THIRRING: *Nuovo Cimento*, **4**, 666 (1956).
[25] S. FURUICHI, T. KODAMA, Y. SUGAHARA and M. YONEZAWA (to be published).

Chapter **14.**

The situation of the neutral bosons.

14·1. – Introduction.

The purpose of this chapter will be to discuss the situation which the scheme of Gell-Mann and Nishijima, on one side, and our knowledge of the charged bosons, on the other, imply for the neutral bosons.

We refer to Chapter **3** for a discussion of the present experimental knowledge on the neutral bosons; we only recall that, contrarily to the charged case, the existence of two neutral boson components has been established, having lifetimes differing by a large factor ($\gtrsim 100$). The short lived component decays mainly in the 2π mode, the long lived one decays mainly by the $e\pi\nu$, and $\mu\pi\nu$ modes. Our first point will be to show [1] that the existence of two such components for the neutral bosons is quite natural, and indeed a necessity in the scheme of Gell-Mann and Nishijima. We shall assume throughout this chapter that the weak interactions producing the decays are invariant with respect to charge conjugation, and only at the end (Sect. **14·8**) discuss the new features introduced by the non conservation of parity and charge conjugation and/or time reversal.

14·2. – The bosons K_1^0 and K_2^0.

It has been already mentioned in Sect. **10·3** that the two bosons K^0 and $\overline{K}{}^0$ which we did introduce must have a different value for the third component of t, or equivalently for s, as a natural consequence of the scheme of Gell-Mann and Nishijima; here we want to explore the implications of the existence of these two different bosons for the decay modes of the neutral bosons.

It has to be remarked immediately that, if two kinds of charged bosons exist, the K_θ^+ and the K_τ^+, as discussed in the last chapter, then altogether four different neutral bosons would exist; the K_θ^0, $\overline{K}{}_\theta^0$ and the K_τ^0, $\overline{K}{}_\tau^0$. Of course K_θ^0, $\overline{K}{}_\theta^0$ would have the same spin and parity as K_θ^+; and K_τ^0, $\overline{K}{}_\tau^0$ the same spin

and parity as K^+_τ while the spin and/or parity of the K^0_θ would be different from the one of the K^0_τ in the same way as the spin and/or parity of the K^+_θ differs from that of the K^+_τ. Each of these four bosons would have, besides the decay modes specified by the lower suffix, other possibilities of decay and, of course, some decay modes might be common to the K^0_θ, $\overline{K}^0_\theta$ pair and to the K^0_τ, $\overline{K}^0_\tau$.

To simplify the situation we shall first assume here that K^+_θ and K^+_τ are the same particle, so that only two neutral bosons exist, which we simply call K^0 and $\overline{K}^0$; the considerations which we shall develop extend, in an obvious manner, to the case of the four neutral bosons.

The production, and, in general, the strong reactions of the neutral bosons, are simply described in terms of K^0 and $\overline{K}^0$; this is because K^0 and $\overline{K}^0$ have a definite strangeness; they behave, as far as the strong reactions are concerned in exactly the same way as K^+ and K^- respectively. However K^0 and $\overline{K}^0$ have not simple decay properties; in particular it may be shown that neither the decay time dependence of K^0 nor the one of $\overline{K}^0$ can be a pure exponential; it is therefore not possible to speak of lifetime of K^0 (or of $\overline{K}^0$) so that, as far as their decay properties are concerned K^0 and $\overline{K}^0$ do not deserve the name of « particles ».

The reason for the non exponential decay of K^0 and of $\overline{K}^0$ is, crudely, the following: the weak interactions, which produce the decay of the K^0 and $\overline{K}^0$, may also produce a substantial mixing of the states K^0 and $\overline{K}^0$. Consider some state j (for instance the state $\pi^+ + \pi^-$) into which both K^0 and $\overline{K}^0$ may decay; then the possibility of the process:

$$K^0 \rightleftarrows j \rightleftarrows \overline{K}^0 \tag{1-14·2}$$

exists, which means that K^0 may be transformed into $\overline{K}^0$ and viceversa; therefore the variation of the amplitude of the $\overline{K}^0$ state with the time is determined also by the amplitude of the $\overline{K}^0$ state which is being formed and viceversa; this produces a non exponential decay (*).

We may however ask whether there exist states representing neutral bosons having a purely exponential decay. It is clear that the decay properties of

(*) The situation is completely similar to that of an atom which happens to have a degenerate excited level with two wave functions u and v corresponding to the same energy and angular momentum. Then instead of u and v, two independent combinations, say $au+bv$ and $a'u+b'v$, may be taken to describe the state in question; the problem then arises of knowing which are the « correct » linear combinations which correspond to an exponential decay of the excited level in question. Such problem may be solved using the Weisskopf-Wigner treatment. The statements below may also be obtained using such treatment, as has been done in ref. [11].

the neutral bosons will be best discussed in terms of such states. Now we may show that the states:

(2-14·2) $$K_1^0 = \frac{K^0 + \overline{K}^0}{\sqrt{2}},$$

and

(3-14·2) $$K_2^0 = \frac{K^0 - \overline{K}^0}{i\sqrt{2}},$$

do have this property, at least if the charge conjugation is conserved by the weak interactions producing the decay. Let us remark that, in the above way of writing, K^0 has to be understood as the wave function of a K^0 in a certain state and $\overline{K}^0$ as the wave function of a $\overline{K}^0$ in the same state; what (2) and (3) say is that, instead of considering the wave functions K^0 and $\overline{K}^0$ we want to consider the two linear combinations which we have called K_1^0 and K_2^0; K^0 and $\overline{K}^0$ may be of course expressed in terms of K_1^0 and K_2^0:

(4-14·2) $$K^0 = \frac{K_1^0 + iK_2^0}{\sqrt{2}},$$

(5-14·2) $$\overline{K}^0 = \frac{K_1^0 - iK_2^0}{\sqrt{2}};$$

so that once the properties of K_1^0 and K_2^0 are known the same may be said for K^0 and $\overline{K}^0$.

Now, as we have said above, we claim that K_1^0 and K_2^0 *do* have a purely exponential decay. Notice first that K_1^0 and K_2^0, for the way in which they have been constructed, are eigenstates of the charge conjugation, belonging respectively to the eigenvalues $+1$ and -1. If the charge conjugation is conserved in the weak interaction, K_1^0 may give rise only to decay states belonging to the eigenvalue $+1$ of the charge conjugation, and K_2^0 only to decay states belonging to the eigenvalue -1. Therefore all the decay states possible for K_1^0 are impossible for K_2^0 and viceversa, so that no transformation of K_1^0 into K_2^0 may take place in contrast to the situation (1) for K^0 and $\overline{K}^0$.

We may therefore conclude that K_1^0 and K_2^0, and not K^0 and $\overline{K}^0$, are the « particles », characterized by an exponential decay with definite lifetimes; *moreover the lifetimes of* K_1^0 *and* K_2^0 *are expected to be different* (and correspondingly a small difference in mass is also expected) since the decay states possible to K_1^0 are not possible to K_2^0 and viceversa. On the other hand the bosons K^0 and $\overline{K}^0$ behave, as far as their decay properties are concerned, as « particle mixtures »; their amplitudes vary with the time as superpositions of exponentials with different decay constants; in the following sections we shall examine this behaviour more in detail. Here we say only that, since a beam

of produced K^0 contains initially (at the time of production) K_1^0 and K_2^0 particles with equal weight, it may be, and it has been, possible to check experimentally the difference in lifetime by simple measurements.

We finally remark that the above considerations are valid separately for K_θ^0, $\overline{K}_\theta^0$ and K_τ^0, K_τ^0 if the existence of such bosons has to be assumed. In such case we shall have to introduce a $K_{1\theta}^0$, $K_{2\theta}^0$ and a $K_{1\tau}^0$, $K_{2\tau}^0$. Here $K_{1\theta}^0$, $K_{2\theta}^0$ and $K_{1\tau}^0$, $K_{2\tau}^0$ are the particles with charge conjugation number $+1$ and respectively -1 arising from the decomposition, similar to (4) (5) of the K_θ^0, $\overline{K}_\theta$ and K_τ^0, $\overline{K}_\tau^0$ particles.

In the following, if the distinction between a K_θ and a K_τ either is not accepted, or is irrelevant to the argument in question, we shall speak only of K_1^0 and K_2^0.

14·3. – The decay modes of K_1^0 and K_2^0.

The purpose of this section will be to discuss the states into which K_1^0 and K_2^0 may decay; it was pointed out in the past section that the lifetimes of K_1^0 and K_2^0 are expected to be different just on account of the fact that these states are different and exclusive.

First of all it may be predicted [1] that the decay into $\pi^+ + \pi^-$ will be possible only for K_1^0 and not for K_2^0 or viceversa. Infact a state of two pions with definite angular momentum l is a state belonging to the eigenvalue $+1$ of the charge conjugation, if l is even, or -1, if l is odd. Since the angular momentum l of the two pions is equal to the spin S of the initial K^0 the decay into 2π will be a decay mode possible for K_1^0 but not for K_2^0 is S if even; viceversa if S is odd. *From now on, to simplify the discussion, we shall take as established S to be even.* In such case also the decay mode $\pi^+ + \pi^0$ is possible and is again a possible decay mode also for K_1^0.

Concerning the other expected decay modes of a neutral boson, we may illustrate the situation as follows: consider, as an example, a decay into an electron, a pion and a neutrino, a decay which, by the way, has been observed; corresponding to a given angular momentum and parity we may here construct both states belonging to the eigenvalue $+1$ of the charge conjugation, and states belonging to the eigenvalue -1; symbolically:

$$(e^-\pi^+\nu) + (e^+\pi^-\tilde{\nu}) \tag{1-14·3}$$

will be a state belonging to the eigenvalue $+1$, and

$$(e^+\pi^+\nu) - (e^-\pi^-\tilde{\nu}) \tag{2-14·3}$$

to the eigenvalue -1.

It may therefore be expected that *both* K_1^0 and K_2^0 may decay into an electron a pion and a neutrino, although the wave functions of the final products will be different. The same of course may be said for the decay into π, μ and neutrino. The decay mode into three pions will be discussed in a moment; also other decay modes are possible, in principle (*).

Here we shall not discuss all the possible decay modes but simply summarize the situation as follows: the particle K_1^0 is expected to decay into the modes $\pi^+ + \pi^-$ and $\pi^0 + \pi^0$ plus « other modes » (as e.g. $\pi e \nu$ states of the kind (1) etc.); the particle K_2^0 will only decay in to the « other modes » (e.g. (2)). If the probability for decay into two pions is dominant with respect to the probabilities for decay into the other modes the lifetime of K_1^0 is expected to be much shorter than that of K_2^0.

This is experimentally the case (compare Ch. **3**); what in fact one observes in a beam of produced K_0's are *mainly* $K_{2\pi}^0$ at small distances from the production target, and *only* other decay modes at longer distances.

14·4. – The three pion decays.

Consider now the three pion decays, that is the decays into:

a) $$\pi^0 + \pi^0 + \pi^0 \,,$$

b) $$\pi^+ + \pi^- + \pi^0 \,.$$

The mode *a*) certainly has charge conjugation number $+1$; the mode *b*) may contain both states of charge conjugation $+1$ and -1.

The above decays may be examined under two different assumptions:

1) K_τ and K_θ are the same particle so that the three pion decay modes are simply additional decay modes of K_1^0 and K_2^0; the possibility of detecting in an experiment such decays depends on the branching ratio which they have with other decay modes; a short discussion of such branching ratios, under the assumption that parity and charge conjugation are not conserved in the decay of K_1^0, K_2^0 (but time reversal is), will be given at the end of Sect. **14·7**.

2) K_τ and K_θ are different so that the three pion decay modes belong to $K_{1\tau}^0$, $K_{2\tau}^0$; parity and charge conjugation are both conserved in the decay; then *a*) is a mode of decay only of $K_{1\tau}^0$ and *b*) may be a mode of decay of both.

(*) E.g. the decay into $\pi^+\pi^-\gamma$ (compare for this decay ref. [1]).

Here the possibility of detecting the three pion decays will depend on the ratio between the rate of production of K_{τ}^0 and K_{θ}^0, on the lifetime of $K_{1\tau}^0$ and of $K_{2\tau}^0$ (a short lifetime makes the detection difficult) on the branching ratio (which will be different for $K_{1\tau}^0$ and $K_{2\tau}^0$) between the three pion decay and the other possible decay modes of $K_{1\tau}^0$ and $K_{2\tau}^0$.

Of course here a more detailed discussion might be made if more specific assumptions (such as the Lee-Orear suggestion, or the parity doublet model) are introduced. We shall not go into further details observing only that if K_{τ} is different from K_{θ} four different lifetimes have to be expected in principle, unless the lifetime of $K_{1\tau}^0$ is equal to that of $K_{1\theta}^0$ and that of $K_{2\tau}^0$ equal to the one of $K_{2\theta}^0$. For some considerations on the relative rates to be expected for the three pion decays if a K_{τ} and K_{θ} exist, the references [2, 3] may be consulted.

14·5. – Experimental verification of the existence of two neutral heavy components.

The experiments, as discussed in Chapter **3**, entirely confirm the predictions of the last sections. Here we want to summarize shortly the main points of evidence.

1) The existence of two boson components of which one decays predominantly in the $K_{\pi 2}^0$ mode and has a life of $0.95 \cdot 10^{-10}$ s, and the other decays mainly by the $e\pi\nu$ and $\mu\pi\nu$ modes, and also perhaps by the 3π mode with a lifetime [4-6] in between $3 \cdot 10^{-8}$ and 10^{-7} s has been established; although, as it appears from the Chapter **3**, the study of the properties of the longlived component, in particular the mass, lifetime and branching ratios are just at the beginning.

2) The experiment by the Columbia-Brookhaven group [6] (Sect. **3**·5) shows that when the reaction $\pi^- + p \rightarrow \Lambda^0 + K^0$ is produced in a bubble chamber of such dimensions that the shortlived boson component is expected to decay almost completely inside the chamber (but not of course the longlived component), 50% of the produced K^0 escape the chamber in agreement with the theory; in fact (compare (4-**14**·2)) according to the theory a produced K^0 is a 1 to 1 superposition of shortlived and longlived bosons. In addition we may say that the same experiment would be difficult to interpret if there are four neutral bosons each having a different lifetime; the fact that 50% of the produced neutral bosons decay in the chamber would not, in general, be expected to be true if K_{τ} and K_{θ} exist as different particles, unless special and unlikely assumptions are introduced (compare for this the discussion in the ref. [6]).

3) The Pais-Piccioni experiment [7] has not yet been made but an experiment similar in character has (point 4 below) given a positive result.

Consider a thin target A in which K^0's are produced by protons or by pions (Fig. 1-**14**.5). At the time of production the state of the produced K^0 may be decomposed according to (4-**14**.2):

$$K^0 = \frac{K_1^0 + iK_2^0}{\sqrt{2}}.$$

The lifetime of the K_2^0 is much longer than that of the K_1^0; then, at a sufficient distance d from the target A all the K_1^0 will have disappeared, having decayed mainly as $K_{\pi 2}^0$ and we shall have to do with a beam of pure K_2^0 with an amplitude $1/\sqrt{2}$ of the initial beam of K^0; but (compare (3-**14**.2)) a K_2^0 state may be written as:

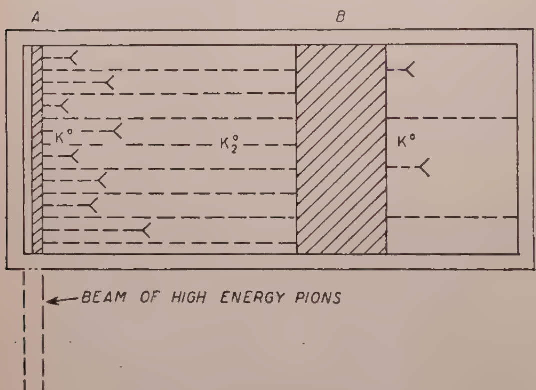

Fig. 1-14.5. – (From [7]).

$$\text{(1-14.5)} \quad K_2^0 = \frac{1}{i\sqrt{2}}(K^0 - \overline{K}^0).$$

We now recall that the interaction properties of K^0 and $\overline{K}^0$ with the nucleons are completely different, because they have opposite strangeness; the $\overline{K}^0$ behaves as a K^- and the K^0 as a K^+ (Chapters **19** and **20**); $\overline{K}^0$ may be absorbed, for example, through the strong reaction:

$$\text{(2-14.5)} \qquad \overline{K}^0 + N \to Y + \pi,$$

while the K^0 may only be scattered (with, or without, charge exchange).

Therefore, if at the distance d from the thin target A we insert a thick absorber B, it will remove from the K_2^0 beam (1) all its $\overline{K}^0$ part; the beam which has traversed the absorber will therefore consist of K^0 only and will have ah amplitude $\frac{1}{2}$ as that of the K^0 beam originally produced at A; as a consequence one will observe again near B, on its right hand side, the $K_{\pi 2}^0$ decay mode, the number of $K_{\pi 2}^0$'s expected being $\frac{1}{4}$ of that near the target A.

Also on the right hand side of B one should observe Y's, produced through the process (2) inside B. Some difficulties for the realization of this experiment have been pointed out by CASE [8].

4) As we did say the experiment No. 3 has not yet been performed; however the following experiment gives evidence along the same line [9]. Suppose to substitute to the thick absorber B of the previous experiment, a stack of nuclear emulsions. Then one should observe there reactions of the kind (2); in particular one should observe stars in which the emission of an hyperon is observed and which may be attributed to the absorption of a neutral heavy boson. This is in fact the case [9].

14·6. – Interferences between the short lived and long lived component.

We shall in this section assume the existence of a K_1^0 and K_2^0, not of a $K_{1\theta}^0$, $K_{2\theta}^0$, $K_{1\tau}^0$, $K_{2\tau}^0$. This is done for simplicity, but of course similar considerations are valid for the more general case, although the experimental verification of the effects which we shall consider may then become more involved. Also we shall assume that K_1^0 has the mode of decay $K_{\pi 2}$ and only another mode of decay (say $e\pi\nu$) and that K_2^0 has the mode of decay $(e\pi\nu)$ and just another mode of decay (say $\pi^+\pi^-\gamma$). We want to study, following TREIMAN and SACHS [10], the time dependence of the rates of the various decay processes in a beam of K^0's; peculiar effects appear which in principle may be experimentally checked.

Again the state vector of a K^0 produced at the time $t = 0$ has to be decomposed according to (4-**14**·2):

$$K^0 = \frac{1}{\sqrt{2}}(K_1^0 + iK_2^0)\,. \tag{1-14·6}$$

The amplitudes of the states K_1^0 and K_2^0 will then vary with the time and the amplitude of the state which develops from (1) may be written at a time t:

$$\psi(t) = \frac{1}{\sqrt{2}}\left\{K_1^0 \exp\left[-\frac{\lambda_1 t}{2} - i\omega_1 t\right] + iK_2^0 \exp\left[-\frac{\lambda_2 t}{2} - i\omega_2 t\right]\right\}. \tag{2-14·6}$$

Here λ_1 and λ_2 are the decay constants of K_1^0 and K_2^0; $\hbar\omega_1$, $\hbar\omega_2$ are the total energies of K_1^0 and K_2^0:

$$\hbar\omega_i = c\sqrt{p_i^2 + M_i^2 c^4}\,.$$

The energy difference $\hbar(\omega_1 - \omega_2)$ is proportional to the mass difference between the two bosons, which has been mentioned in Sect. **14**·2

$$\hbar(\omega_1 - \omega_2) \simeq c^2 \Delta M\,.$$

We want to find the rates of the various decays as functions of t. The decay of K_1^0 is characterized by two constants a_1 and b which specify the branching ratio between the $(e\pi\nu)$ and $K^0_{\pi 2}$ modes of decay,

$$K_1^0 \to a_1(e^+\pi^-\nu + e^-\pi^+\tilde{\nu}) + b(\pi^+\pi^-) . \tag{3-14.6}$$

Similarly the K_2^0 decay is characterized by a_2 and c according to:

$$K_2^0 \to - ia_2(e^+\pi^-\nu - e^-\pi^+\tilde{\nu}) + c(\pi^+\pi^-\gamma) . \tag{4-14.6}$$

Here a_1 and a_2 may be chosen to be real; this follows from the invariance with respect to time reversal.

From (2) and (3) the rate of decay at the time t into two pions is:

$$R(\pi^+\pi^- | t) = \tfrac{1}{2} |b|^2 \exp[-\lambda_1 t] . \tag{5-14.6}$$

Similarly the rate of decay into $\pi^+\pi^-\gamma$ will be:

$$R(\pi^+\pi^-\gamma | t) = \tfrac{1}{2} |c|^2 \exp[-\lambda_2 t] . \tag{6-14.6}$$

The interesting point is now that the time behaviour of the rate of decay into the state $(e^+\pi^-\nu)$, and similarly that into the state $(e^-\pi^+\tilde{\nu})$ will be more complicated than a simple combination of exponentials and will contain some interference terms between the K_1^0 and K_2^0 waves; terms which may be subjected possibly to an experimental verification.

The rate of decay into $(e^+\pi^-\nu)$ may be written:

$$R(e^+\pi^-\nu | t) = \frac{1}{2}\left| a_1 \exp\left[-\frac{\lambda_1 t}{2}\right] + a_2 \exp\left[-\frac{\lambda_2 t}{2}\right] \exp[i\Delta M t]\right|^2 =$$
$$= \frac{1}{2}|a_1|^2 \exp[-\lambda_1 t] + \frac{1}{2}|a_2|^2 \exp[-\lambda_2 t] + a_1 a_2 \cos(\Delta M t) \exp - \left[\frac{\lambda_1 + \lambda_2}{2} t\right] . \tag{7-14.6}$$

Similarly the negative electron rate is:

$$R(e^-\pi^+\tilde{\nu} | t) = \frac{1}{2}\left| a_1 \exp\left[-\frac{\lambda_1 t}{2}\right] - a_2 \exp\left[-\frac{\lambda_2 t}{2}\right] \exp[i\Delta M t]\right|^2 =$$
$$= \frac{1}{2}|a_1|^2 \exp[-\lambda_1 t] + \frac{1}{2}|a_2|^2 \exp[-\lambda_2 t] - a_1 a_2 \cos[\Delta M t] \exp - \left[\frac{\lambda_1 + \lambda_2}{2} t\right] . \tag{8-14.6}$$

Both in (7) and (8), aside from the two exponential terms corresponding to the lifetimes λ_1^{-1}, λ_2^{-1}, there is a term characterized by a decay constant $\frac{1}{2}(\lambda_1 + \lambda_2)$. Such term has opposite sign for the $(e^+\pi^-\nu)$ and the $(e^-\pi^+\tilde{\nu})$ decay *so that if just the sum of the two is considered, that is, if one does not distinguish*

between the signs of the charges, the interference term does not appear. On the contrary if one fixes the attention separately on the two decay processes the term in question may be important; $R(e^+\pi^-\nu|t)$ may then be very different from $R(e^-\pi^+\tilde{\nu}|t)$ (*).

More precisely the difference between the two rates may be important if t is not appreciably larger than λ^{-1} and will be significant if two conditions are satisfied:

1) a_1 and a_2 do have comparable magnitudes.

2) ΔM is much smaller than λ_1; if it is not so the oscillations of the cos factor in (7) and (8) are too rapid and cancel the effect.

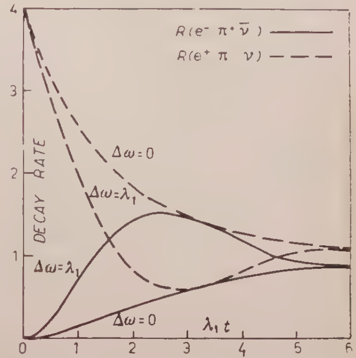

Fig. 1-14·6. – (From [10]). Decay rates of the K^0 into the modes $(e^+\pi^-\nu)$ (dashed curves) and $(e^-\pi^+\tilde{\nu})$ (solid curves) under the assumption $a_1/a_2=1$. The abscissa is the time in units of the K_1^0 mean life. The two curves in each case correspond to different choices for the mass difference.

If the two conditions above are satisfied, the decay rates of a K^0 originally produced, into the $e^+\pi^-\tilde{\nu}$ or $e^-\pi^+\nu$ states are very different functions of the time, as illustrated in Fig. 1 which refers to $a_1 = a_2$, $\lambda_1 \gg \lambda_2$ and where the curves are given both for $\Delta M = 0$ and $\Delta M = \lambda_1$.

It appears that the curves depend very strongly on ΔM so that this effect could perhaps serve to measure ΔM.

14·7. – Consistency between the numbers of « anomalous » decay modes so far observed in different experiments.

Let us consider finally (following [10]) if the number of the « anomalous » decay events so far observed in cloud chamber experiments by several people (compare Sect. 3·7) is consistent with the order of magnitude which one would expect in the K_1^0, K_2^0 picture. It is clear that the « anomalous modes » have to be identified with all the neutral modes which are not $K_{\pi 2}$. The rate of the anomalous component may be then written:

$$R\,(\text{anom.}) = A_1 \exp[-\lambda_1 t] + \lambda_2 \exp[-\lambda_2 t]\,, \tag{1-14·7}$$

where A_1 ($A_1 < \lambda_1$) is the partial rate of decay of K_1^0 via anomalous modes; λ_1 and λ_2 are instead the total rates of decay of K_1^0 and K_2^0. We may calculate

(*) Notice that the two rates become equal for large t, that is in the long lived component.

from (1) in terms of Λ_1, λ_1, λ_2 the fraction f of the anomalous decay modes which one may expect in a « typical cloud chamber experiment ». Here by « typical cloud chamber experiment » we mean an experiment which can observe decay processes over a time interval T of the order of $5\cdot 10^{-10}$ s starting from the time of production (measured in the meson rest frame).

The fraction f, the observed value of which is around 0.1, is given by:

$$f = \frac{\int_0^T R(\text{anom.})\,\mathrm{d}t}{\int_0^T R(\text{total})\,\mathrm{d}t} = \frac{\Lambda_1\lambda_1^{-1}(1-\exp[-\lambda_1 T]) + (1-\exp[-\lambda_2 T])}{(1-\exp[-\lambda_1 T]) + (1-\exp[-\lambda_2 T])}\,. \tag{2-14·7}$$

Inserting in (2) $f = 0.1$, $\lambda_1^{-1} = 10^{10}$ s hence $\lambda_1 T \simeq 5$ one may get from (2)

$$\lambda_2 T \lesssim 0.1 \qquad \text{that is:} \qquad \lambda_2^{-1} \geqslant 5\cdot 10^{-9}\ \text{s}\,. \tag{3-14·7}$$

That is the lifetime to be attributed to the long lived component to be consistent with the number of anomalous decays in the « typical cloud chamber experiments » must be longer than $5\cdot 10^{-9}$ s. This is consistent with the result of [9, 10]. Another consequence which may be derived from (2) is that the branching ratio Λ_1/λ_1 between anomalous and normal ($K_{\pi 2}$) decays of K_1^0 should be less than 0.1.

Of course the whole argument is weakened by the fact that the « typical cloud chamber experiment » introduced so far is an approximation of the conditions in the real cloud chamber experiments, and by the fact that the value 0.1 taken for f is not certain (compare the discussion in Sect. **3**·7).

14·8. – The main consequences of the non conservation of parity, charge conjugation and time reversal for the decays of the neutral bosons.

If the charge conjugation invariance of the decay interactions of the heavy neutral bosons does not hold it looks at first as if the theory developed in the past sections should not hold anymore. This is however not true [11]. Consider first the case in which the charge conjugation and the parity are not conserved, but the time reversal invariance still holds. Then it may be shown again that, while K^0 and $\overline{K}^0$ do not have an exponential decay, still the combinations K_1^0 and K_2^0 do have such a decay; infact it may be shown easily that in such case, the combinations (2-**14**·2) (3-**14**·2) are not only eigenfunctions of the charge conjugation operator, but also eigenfunctions of the operator CP (product of the charge conjugation times parity) belonging to the eigenvalues of CP respectively $+1$ and -1; now (Sect. **12**·6) if the invariance with respect to time reversal of the decay interactions still holds, the product CP

is still a constant of the motion. Therefore the decay states originating from K_1^0 belong to the eigenvalue $+1$ of CP and the ones from K_2^0 belong to the eigenvalue -1; the two sets of states are again independent and K_1^0, K_2^0 still do have an exponential decay. Also, for an even (odd) spin the 2π states still have eigenvalue $+1$ (-1) of CP so the 2π decay mode is a possible decay mode for K_1^0 (K_2^0) but not for K_2^0 (K_1^0). Therefore a strong difference in lifetime between K_1^0 and K_2^0 is still expected and the same experimental consequences as before may be expected. It may be shown also that, notwithstanding that charge conjugation invariance is violated, still the ratio between the number of decays of the longlived component into $e^+\pi^-\nu$ and into $e^-\pi^+\tilde{\nu}$ is again one [11, 12] (compare the footnote of Sect. **14**·6); the same is of course true for the $\pi\mu\nu$ mode.

If neither the parity, nor the charge conjugation, nor the time reversal invariance are preserved, on the other hand, it has been shown in [11] that, in general the states K_1^0 and K_2^0 defined by (2-**14**·2) and (3-**14**·2) are not any more purely exponentially decaying states. In this case the situation is more complicated; one may still construct states having a purely exponential decay but one may expect in principle that both may decay by the 2π mode; although the rates of decay may well be different by a large factor. Also one may expect a deviation from unity of the ratio between the $e^+\pi^-\tilde{\nu}$ and $e^-\pi^+\nu$ decay modes in the longlived component.

We finally discuss here [13], as mentioned in Sect. **14**·4, the expected branching ratio for the three pion decays of K_1^0, K_2^0 under the assumption that just one K_1^0 and one K_2^0 exist and parity is not conserved in their decay. We shall however assume that CP is conserved; we shall also assume spin zero for the K. The situation is then the following:

1) The decay into $\pi^0+\pi^0+\pi^0$ is possible only for the longlived component (K_1^0), not for the shortlived one (K_2^0). This is simply due to the fact that the eigenvalue of CP for the $\pi^0+\pi^0+\pi^0$ state is certainly -1 (the opposite as for the $\pi^++\pi^-$ state).

2) The decay into $\pi^++\pi^-+\pi^0$ is possible in principle for both the components but is likely to be unimportant (probability $\lesssim 10^{-5}$) for the short lived one. This may be seen as follows: *a*) even if the rate of decay of the K_1^0 into $\pi^++\pi^-+\pi^0$ were as large as the rate of decay of the K^+ into $\pi^++\pi^-+\pi^+$, the decay $\pi^+ + \pi^- + \pi^0$ would be unimportant having a branching ratio $\sim 10^{-3}$ with respect to the dominant 2π decay (this of course assumes that in passing from the charged to the neutral three pion decays, no similar enhancement of the rates of decay takes place as is the case of the $K_{2\pi}$; notice that while a reason can be given for the enhancement in the 2π decays (Sect. **16**·10) no similar reason applies to the three pion decays); *b*) there is a further factor of the order 100 in disfavour of the $\pi^++\pi^-+\pi^0$ decay, due

to the fact that l and l' (l = relative orbital momentum of $\pi^+ + \pi^-$, l' = relative angular momentum of the π^0 with respect to the π^+, π^- centre of mass) must be at least both 1 since the three pion state originating from the K_1^0 decay has $CP = +1$.

3) The branching ratio for decay of the longlived component into $\pi^+ + \pi^- + \pi^0$ with respect to the other modes, as well as the lifetime of the longlived component may be estimated under three assumptions: *a*) that the rates of decay into the $\pi\mu\nu$ and $\pi e\nu$ modes are the same for the K^+ and for the K^0; *b*) that transitions with $|\Delta T| = \frac{1}{2}$ are dominant and that decays into three pion states non-symmetrical in the momenta of the three pions are negligible with respect to decays into symmetrical states; these two assumptions (compare for their proper meaning Sect. **16**·10, 11) imply [3] that the total rate of decay into three pions is the same for the K_2^0 as for the K^+; *c*) that no other decay mode contributes appreciably to the decay of the K_2^0 (for instance $\nu + \nu$ or $\pi^0 + \nu + \nu$, etc.).

If *a*), *b*) and *c*) are satisfied it is straightforward to show that the life time of the K_2^0 should be about $7 \cdot 10^{-8}$ s and that the branching ratio for decay into $\pi^+ + \pi^- + \pi^0$ of K_2^0 should be $\frac{1}{5}$; the branching ratio for decay into $\pi^0 + \pi^0 + \pi^0$ should be [3] 1.5 times as large as this.

It appears that the above figures are not inconsistent with the present facts.

REFERENCES (Ch. 14)

[1] M. Gell-Mann and A. Pais: *Phys. Rev.*, **97**, 1387 (1955).
[2] G. Snow: *Phys. Rev.*, **103**, 1111 (1956).
[3] R. Dalitz: *Proceed. Phys. Soc.*, A **69**, 527 (1956).
[4] K. Lande, E. T. Booth, J. Impeduglia, L. M. Lederman and W. Chinowsky: *Phys. Rev.*, **103**, 1901 (1956).
[5] K. Lande, L. M. Lederman and W. Chinowsky: to be published.
[6] M. Schwartz, J. Steinberger, N. P. Samios and R. Plano: to be published.
[7] A. Pais and O. Piccioni: *Phys. Rev.*, **100**, 1487 (1955).
[8] K. M. Case: *Phys. Rev.*, **103**, 1449 (1956).
[9] M. Baldo-Ceolin, C. C. Dilworth, W. Fry, W. D. B. Greening, H. Huzita S. Limentani and A. Sichirollo: *Nuovo Cimento*: to be published.
[10] R. G. Sachs and S. B. Treiman: *Phys. Rev.*, **103**, 1545 (1956).
[11] T. D. Lee, R. Oehme and C. N. Yang: to be published.
[12] W. Wyld jr. and S. Treiman: to be published.
[13] G. Morpurgo: *Colloquium given in Rome*, March 27 (1957).

Note added in proof. – A letter by Gatto in the *Phys. Rev.* and a preprint by Pais and Treiman have now appeared on this same subject.

CHAPTER 15.

The Determination of the Spins of the Hyperons.

15·1. – Introduction.

The $K^{+}_{\pi 3}$ is the only particle which, undergoing a decay into three charged spinless particles may be analyzed in greater detail for determining its spin and parity. For the particles which undergo a two body decay, and in particular for the hyperons, it is much harder to derive information on the spin. In this chapter we want to summarize the methods which have been suggested to get such information. Our description will deal with the methods based on analyzing the angular distributions of the decay products of the hyperons and with the less direct considerations of RUDERMAN and KARPLUS [1] for determining the spin of the Λ^0 from the ratio between the mesonic and non-mesonic decay of the hyperfragments.

15·2. – Some experimental data.

In this section we collect some experimental data on Λ^0, Σ's angular decay distributions of which use will be made in the following sections. Consider the production and the subsequent decay of any of the above hyperons in a Wilson or bubble chamber (for an extensive discussion of these experiments, compare Chapter **17** and Sect. **20**·6); for any observed production event one may determine some angles which will be important later:

a) The angle φ between the plane of production and the plane of decay of the hyperon; the plane of production is the plane containing the line of flight of the hyperon and the line of flight of the producing particle; the plane of decay is the plane containing the lines of flight of the decay products of the hyperon, or (it is equivalent) the line of flight of the hyperon and the line of flight of one of its decay products; the quantities quoted above are equally well measurable if the hyperon is a charged or a neutral one. The angle φ may vary between 0 and 2π.

b) The angle θ between the line of flight of the hyperon and the line of flight of the decay products of the hyperon in the rest system of the same: θ varies from 0 to π. The angles θ and φ are reproduced in the Fig. 1-**15**'2 below; there the z-axis is the line of flight of the decaying hyperon; the (zx) plane is the plane of production of the hyperon and the plane D is the decay plane (*).

It may be useful sometimes to distinguish between the complete distributions which are plotted in the whole intervals $0<\varphi<2\pi$ and $0<\theta<\pi$ and the « folded distributions » in which all events are reported in the intervals $0<\varphi<\pi/2$ and $0<\theta<\pi/2$ respectively. For the θ distribution the « folding » process consists simply in a reflection through $\theta=90°$, for the φ distribution it consists in summing the events produced in planes forming the angles φ, $\varphi+\pi$, $2\pi-\varphi$, $\pi-\varphi$.

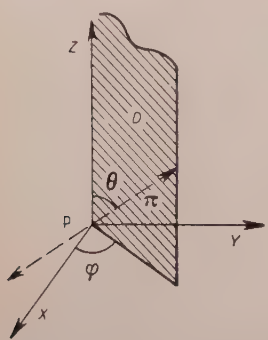

Fig. 1-**15**'2. – (From [8]). Meaning of the angles.

The knowledge of the distributions in the above angles for each kind of hyperon may give an information on the spin of the hyperon in question, as we shall discuss more in detail in the next section. Therefore it is important to plot these distributions for the events which are known; this will be done here. In this section we shall consider only the events in which the hyperons are produced in hydrogen; the ones in which they are produced in a dense material will be mentioned separately (Sect. **18**'2.5); a very rapid increase in the number of H events may be expected mainly through the bubble chamber techniques [2].

The most extensive collections of events produced in H are the ones from the Columbia group experiments [2-4] and the ones from the Berkeley group experiment [5]. The producing reaction is, in the Columbia experiment (Ch. **17**):

$$\pi^- + p \rightarrow K + Y \tag{1-15'2}$$

and in the Berkeley experiment (Sect. **20**'6)

$$K^- + p \rightarrow \pi + Y . \tag{2-15'2}$$

To the reaction (1) belong also the events reported by SHUTT and coll., which first suggested the study of the distribution in question, and the ones reported by WALKER and SHEPHARD respectively at pion energies of 1.37 and 1 GeV; a detailed description of all these important experiments is postponed to Chapter **17** and Sect. **20**'6.

(*) Our φ differs by $\pi/2$ from the Φ of [4] and of the Figs. 2 and 3.

We shall first consider the events from the reaction (1); WALKER and SHEPHARD [6] first reported in a diagram the distributions in $\cos\theta$ and in φ for 12 Λ^0 events observed by them and by SHUTT and coll.; such distributions showed strong anisotropy (+), most probably due to statistical fluctuations. It has to be mentioned that the events assembled by them were really a mixture of two kinds of events: events in which a Λ^0 is directly produced in the reaction (1) and events in which the observed Λ^0 is the son of an originally produced Σ^0 which has decayed presumably according to the decay reaction $\Sigma^0 \to \Lambda^0 + \gamma$.

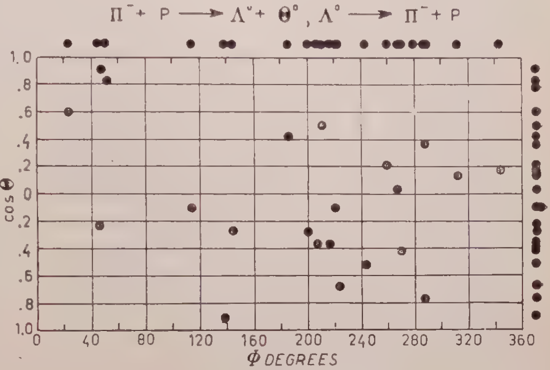

Fig. 2-15˙2. – (From [4]). Distributions in θ and Φ in the decays of Λ^0 produced in hydrogen.

This should not be the case for the Λ^0 events of [4], for which the angular distribution in θ and Φ $(=\varphi+\pi/2)$ are reported in Fig. 2-15˙2; according to the Columbia group, these are almost certainly Λ^0 directly produced in H. In Fig. 3-15˙2 we have also presented the angular distribution in θ and Φ the for Σ^- from the same experiment.

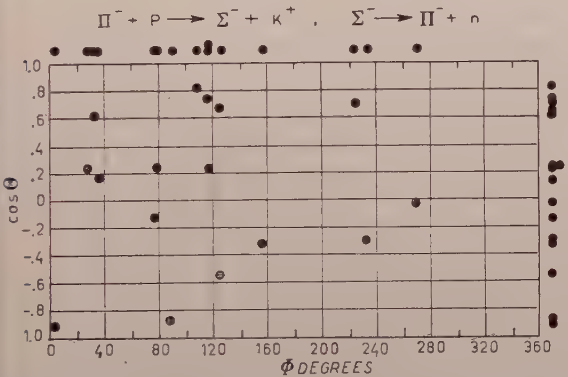

Fig. 3-15˙2. – (From [4]). Distributions in θ and Φ in the decays of Σ^- produced in hydrogen.

The Columbia distributions in θ and Φ do not seem to deviate essentially from isotropy, though the *statistics are still much too poor*; better statistics will be soon available from the examination (now in progress) of the much larger set of events obtained in [2] (*).

Concerning the evidence from reaction (2-15˙2) a distribution in θ is available [5] based on 155 Σ decays ($\sim 100\ \Sigma^-$, $55\ \Sigma^+$), where the Σ are produced from the absorption of K^- at rest. θ is here again the angle between the line

(+) No event had $\varphi > 45°$ in the folded distribution in φ of the Λ^0 events.

(*) This is being done at Bologna (PUPPI and coll.) and at Pisa (CONVERSI and coll.).

of flight of the Σ and the line of flight of its decay products (in the centre of mass of the Σ). The distribution is consistent with isotropy.

15·3. – General information obtainable from the distributions.

The distributions in φ and θ may be generally written, of course, in the form [7]

$$(1\text{-}15{\cdot}3)\qquad \left\{\begin{aligned} I_S(\varphi) &= \frac{1}{2\pi}\left(1 + \sum_M A_M^{(S)} \cos M\varphi + \sum_M B_M^{(S)} \sin M\varphi\right), \\ I_S(\theta) &= \frac{1}{2} + \sum_M \mathcal{A}_M^{(S)} P_M(\cos\theta)\,. \end{aligned}\right.$$

For a spin S the sums over M extend in general from $M=0$ to $M=2S$; here, as in the following, we shall call « anisotropy terms » in the distributions those with M even and « asymmetry terms » those with M odd.

From the values of the coefficients $A_M^{(S)}$, $B_M^{(S)}$ and $\mathcal{A}_M^{(S)}$ one may hope to derive some information on the spin of the decaying hyperon. The method to derive such information which we shall follow [8] will consist in computing, assuming a given spin S for the hyperon, the coefficients A, B and $\mathcal{A}$ in (1) in terms of the parameters (to be called in the following λ_M) describing the polarization state of the hyperon at the moment of the decay.

We shall first *assume that these parameters λ_M are essentially unknown*; though in such case the information derivable from the analysis of the distributions may give only a lower limit to the spin of the hyperon [8, 9], the results, to be summarized below, are very general; the points which may be stated knowing nothing of the polarization parameters are:

1) No conclusion is possible if all the distributions are isotropic.

2) If no parity doublet model (Sect. **12·3**) is assumed and if the parity is always conserved, no asymmetry term may be present in the distributions. Anisotropy terms may be present if the hyperon is partially polarized at the moment of the decay and its spin larger than $\frac{1}{2}$; any anisotropy indicates a spin of the hyperon larger than $\frac{1}{2}$; better lower limits may be then given by a more detailed study of the anisotropies [7-9].

3) If either a parity doublet model or the non conservation of parity in the weak interactions are assumed and if the hyperon is partially polarized there may be, whatever the spin of the hyperon is, asymmetrical terms in the distributions [8, 10]. For the anysotropical terms the statement under 1) continues to be valid.

4) The asymmetrical terms mentioned under 3) are different if the assumption of the parity doublets model or if the assumption of the parity non conservation in the weak interactions is made. In the first case the θ distribution may be asymmetrical and the φ distributions may contain asymmetrical terms of the kind $\cos(2M+1)\varphi$, in the second case the θ distribution is symmetrical and the φ distribution may contain terms of the kind $\sin(2M+1)\varphi$.

Later we shall consider the cases in which more is known on the parameters λ_M describing the polarization state of the hyperon at the moment of the decay; then of course, more precise statements may be made and sometimes also « the value » of the spin and not only a lower limit to it, may be given. Such knowledge of the λ_M may be obtained considering particularly simple production processes of the hyperon or putting oneself in particularly simple conditions of observation.

In the following two sections we shall first consider the general case in which the λ_M are essentially unknown and we shall justify the statements 1), 2), 3), 4) above; in Sect. **15·6** we shall consider in detail those processes for which we have more information on the λ_M from the production process.

15·4. – Angular distributions in terms of the polarization parameters.

The purpose of this section is to express A, B, $\mathcal{A}$, in (1-**15·3**) in terms of the polarization coefficients λ_M which are supposed unknown.

To solve this problem [8] we fix our ideas on a Y produced in the reaction (1-**15·2**) (the treatment for the reaction (1-**15·3**) is very similar however); we assume that the K produced together with the Y has spin zero but the statements 1), 2), 3), 4) of the last section and the majority of the results to be given in this section, hold independently of this assumption.

We consider in this section the simplest case in which no parity doublet assumption is made and it is assumed that parity is *always* conserved.

We call χ_M^S the spin functions of the Y with spin S and spin component M in the direction of flight of the Y ($M = -S \dots +S$) and introduce $2S+1$ complex coefficients λ_M depending on the details of the production process, which specify the state of polarization of the decaying Y.

The normalized ${}_\alpha\chi_S$ wave function of the Y may then be generally written as:

$$ {}_\alpha\chi_S = \Lambda^{-\frac{1}{2}} \sum_M \lambda_M \chi_S^M , \tag{1-15·4}$$

if the Y is produced in a collision of the pion with a proton with spin up and as

$$ {}_\beta\chi_S = \Lambda^{-\frac{1}{2}} \sum_M (-1)^{|M-\frac{1}{2}|} \lambda_M \chi_S^{-M} , \tag{2-15·4}$$

if it is produced in the collision with a proton with spin down. Expression (2) is obtained [8] applying to (1) the operator PR where P is the parity operator and R is an appropriate rotation by 180°. For an unpolarized proton gas the distributions have to be calculated with the two wave functions (1) and (2) and the average has to be taken. When the Y decays, each function χ_S^M gives rise to an outgoing pion-proton wave, the amplitude of which is:

$$\text{(3-15·4)}\qquad \underset{+}{\psi}{}_S^M = \left(\frac{S+M}{2S}\right)^{\frac{1}{2}} \alpha Y_{S-\frac{1}{2}}^{M-\frac{1}{2}}(\theta,\varphi) + \left(\frac{S-M}{2S}\right)^{\frac{1}{2}} \beta Y_{S-\frac{1}{2}}^{M+\frac{1}{2}}(\theta,\varphi)\,,$$

or

$$\text{(4-15·4)}\qquad \underset{-}{\psi}{}_S^M = -\left(\frac{S-M+1}{2S+2}\right)^{\frac{1}{2}} \alpha Y_{S+\frac{1}{2}}^{M-\frac{1}{2}}(\theta,\varphi) + \left(\frac{S+M+1}{2S+2}\right)^{\frac{1}{2}} \beta Y_{S+\frac{1}{2}}^{M+\frac{1}{2}}(\theta,\varphi),$$

according to the parity of the Y. Here φ and θ are just the angles considered so far. The final wave functions of the decay products are now obtained inserting (3) or (4) into (1) and (2); and the distributions in φ and θ are immediately obtained as the averaged squares of the final wave functions. It may be shown that they are independent of the parity of the decaying Y so that the same results are obtained using $\underset{+}{\psi}{}_S^M$ or $\underset{-}{\psi}{}_S^M$ in the following formulas (5) and (6). To be definite we shall insert $\underset{+}{\psi}{}_S^M$. One has

$$\text{(5-15·4)}\qquad I_S(\varphi) = \frac{1}{2}\Lambda^{-1}\sum_{\substack{\text{spin}\\ \text{final}\\ \text{nucleon}}} \int \mathrm{d}\cos\theta \left\{\sum_M |\lambda_M \underset{+}{\psi}{}_S^M|^2 + |\sum_M (-1)^{|M-\frac{1}{2}|}\lambda_M \underset{+}{\psi}{}_S^{-M}|^2\right\},$$

and

$$\text{(6-15·4)}\qquad I_S(\theta) = \frac{1}{2}\Lambda^{-1}\sum_{\substack{\text{spin}\\ \text{final}\\ \text{nucleon}}} \int \mathrm{d}\varphi \left\{|\sum_M \lambda_M \underset{+}{\psi}{}_S^M|^2 + |\sum_M (-1)^{|M-\frac{1}{2}|}\lambda_M \underset{+}{\psi}{}_S^{-M}|^2\right\}.$$

Performing the explicit calculations and comparing (5) and (6) with (1-**15**·3) one obtains the coefficients $A_M^{(S)}$, $B_M^{(S)}$ expressed through the λ_M's which define the polarization state. One may check that all the coefficients $B_M^{(S)}$ vanish identically as well as the coefficients $A_M^{(S)}$ and $\mathcal{A}_M^{(S)}$ with M odd. The other coefficients do not generally vanish if the hyperon is polarized. From the above statements it follows that the sums in (1-**15**·3) extend up to $M = 2S-1$ so that in particular for $S=\frac{1}{2}$ the distributions are always isotropical independently from the polarization. This result has been already mentioned.

By varying the polarization state of the Y, that is the λ_M, one gets, from (5) and (6) for any spin S the set of simultaneous admissible distributions in φ

and in $\cos\theta$. Some of them are reported in the figures 1, 2 (taken from [8]) for the cases of spin $\frac{3}{2}$ and $\frac{5}{2}$; one may see, for instance, that for a spin $\frac{3}{2}$ to the most peaked distribution in φ corresponds necessarily an isotropical distribution in $\cos\theta$; similar features are apparent in the other figures.

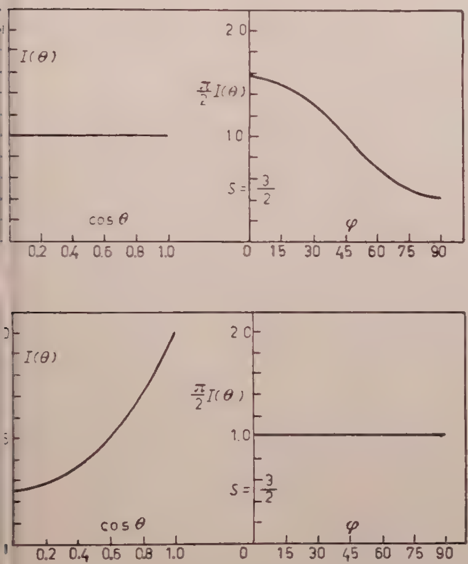

Fig. 1-15'4. – (From [8]). The distributions in θ and φ which correspond tot he maximum φ peaking (upper curves), or to the maximum value of $I(\theta=180^\circ)/I(\theta=90^\circ)$ (lower curves) (assumed spin of the hyperon $=\frac{3}{2}$).

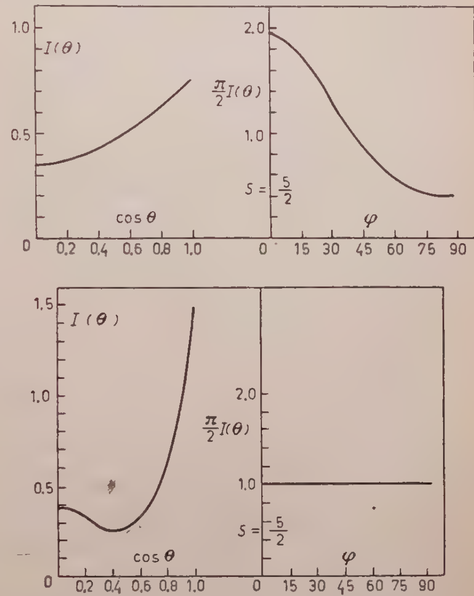

Fig. 2-15'4. – (From [8]). The distributions in θ and φ which correspond to the maximum φ peaking (upper curves), or to the maximum value of the intensity at 180° (lower curves) (assumed spin of the hyperon $=\frac{5}{2}$).

One may also see that an isotropic φ distribution does not imply necessarily an isotropic $\cos\theta$ distribution. We do not insist on other peculiarities of the distributions waiting for other experimental results; if all the distributions turn finally out to be isotropical no conclusion is possible, as already pointed out.

15'5. – The distributions under the assumption of the parity doublet model or of non conservation of parity in the weak interactions.

Always in the line of leaving the λ_M unrestricted we now study the distributions under the two alternative assumptions mentioned in the title [10, 11].

Assume first that parity is not conserved in the weak interactions (com-

pare Sect. **12**·5). The spin of the K is assumed to be zero. The reaction considered is always (1-**15**·2).

Then each outgoing wave will be a superposition, with coefficients a and b of the two waves $\underset{+}{\psi}{}^S_M$ and $\underset{-}{\psi}{}^S_M$ with opposite parity. The expression (5-**15**·4) of the φ distribution is therefore changed into:

$$(1\text{-}\mathbf{15}\cdot 5)\qquad I_S(\varphi)=\frac{1}{2}\Lambda^{-1}N^{-1}\sum_{\substack{\text{spin}\\ \text{final}\\ \text{nucleon}}}\int \mathrm{d}\cos\theta\,\cdot$$

$$\cdot\Big\{\Big|\sum_M \lambda_M(a\underset{+}{\psi}{}^S_M+b\underset{-}{\psi}{}^S_M)\Big|^2+\Big|\sum_M(-1)^{|M-\frac{1}{2}|}\lambda_M(a\underset{+}{\psi}{}^M_S+b\underset{-}{\psi}{}^M_S)\Big|^2\Big\},$$

where N is a normalization factor $=|a|^2+|b|^2$.

The θ distribution is of course obtained performing the integration in (1) over φ instead of over $\cos\theta$.

The change from (5-**15**·4) to (1-**15**·5) implies the presence of interference terms between waves with opposite parity. In the φ distribution they give rise to asymmetric terms of the kind $\sin(2M+1)\varphi$ $(M=0, 1 \ldots, S-\frac{1}{2})$; in the θ distribution they cancel so that no asymmetry is introduced. We may give [11] the explicite expression of $B_1^{(\frac{1}{2})}$, that is the coefficient of $\sin\varphi$ in the distribution (1) when the hyperon spin is $\frac{1}{2}$.

$$B_1^{(\frac{1}{2})}=\frac{\pi}{4}\,\frac{ab^*+ba^*}{|a|^2+|b|^2}\,(-i)\,\frac{\lambda_{\frac{1}{2}}\lambda^*_{-\frac{1}{2}}-\lambda^*_{\frac{1}{2}}\lambda_{-\frac{1}{2}}}{|\lambda_{\frac{1}{2}}|^2+|\lambda_{-\frac{1}{2}}|^2}\,.$$

For spins larger than $\frac{1}{2}$ the coefficients have more complicated expressions but they all share with $B_1^{(\frac{1}{2})}$ the two properties: 1) of being proportional to ab^*+ba^*; 2) of vanishing if the Y is unpolarized.

We may notice that the maximum value of $B_1^{(\frac{1}{2})}$ is $\pi/4$; this corresponds to a ratio $\int_0^\pi I(\varphi)\mathrm{d}(\varphi)/\int_\pi^{2\pi} I(\varphi)\mathrm{d}\varphi=3$, a quite large asymmetry (*).

We now examine the asymmetries which may be present if the assumption of the parity doublet model is made; instead, parity is assumed to be always conserved.

(*) However one cannot really hope to detect such large asymmetries even if $(ab^*+ba^*)/(|a|^2+|b|^2)=1$, after one integrates to all the production angles of the Y. In fact the polarization $(-i)[\lambda_{\frac{1}{2}}\lambda^*_{-\frac{1}{2}}-\lambda^*_{\frac{1}{2}}\lambda_{-\frac{1}{2}}]/(|\lambda_{\frac{1}{2}}|^2+|\lambda_{-\frac{1}{2}}|^2)$ depends on the angle of production, and even if it is 1 at some angle it will be less than one at the others; the expression of the polarization for the case in which only outgoing s and p waves intervene is given in [12]; it appears that one has to be rather lucky to detect an asymmetry at all.

It must be emphasized that such asymmetries will be present only if the mass and lifetime differences between the two members of a parity doublet are very small. We have already pointed out that the equality of the lifetimes is not understood in the parity doublet model.

In such case the Y, K wave produced is a combination of four waves; considering first the proton spin up, we have to write for this wave instead of (1-15·4):

$$\text{(2-15·5)} \qquad \sum_M [\lambda_M^{++} \underset{+}{\chi_S^M} \underset{+}{\theta} + \lambda_M^{+-} \underset{+}{\chi_S^M} \underset{-}{\theta} + \lambda_M^{-+} \underset{-}{\chi_S^M} \underset{+}{\theta} + \lambda_M^{--} \underset{-}{\chi_S^M} \underset{-}{\theta}] \, ,$$

where $\underset{+}{\chi^M}$, $\underset{-}{\chi^M}$ are the spin functions of the two Y's and $\underset{+}{\theta}$, $\underset{-}{\theta}$ are the functions of the two K's (which are assumed to have spin zero); the λ_M^{++}, λ_M^{+-}, λ_M^{-+}, λ_M^{--} are the amplitudes of these waves and specify the polarization state; the fact that the state in question is an eigenstate of the parity conjugation belonging to the eigenvalues $+1$ is expressed by the equalitie:

$$\text{(3-15·5)} \qquad \lambda_M^{++} = \lambda_M^{--} \, , \qquad \lambda_M^{+-} = \lambda_M^{-+} \, .$$

The four waves included in (2) may interfere at the decay only if the lifetimes and masses of the $\underset{+}{Y}$, $\underset{-}{Y}$ do not appreciably differ, as we have said.

Making use of (3) the wave (2) may be rewritten (normalized):

$$\Lambda'^{-\frac{1}{2}} [\sum_M (\lambda_M^{++} \underset{+}{\chi_S^M} + \lambda_M^{-+} \underset{-}{\chi_S^M}) \underset{+}{\theta} + (\lambda_M^{-+} \underset{+}{\chi_S^M} + \lambda_M^{++} \underset{-}{\chi_S^M}) \underset{-}{\theta}]$$

where

$$\Lambda' = 2 \sum_M (|\lambda_M^{++}|^2 + |\lambda_M^{-+}|^2) \, .$$

The wave produced when the photon spin is initially down is obtained, as (2-15·4) from (1-15·4), by application of the operator PR. It is:

$$\Lambda'^{-\frac{1}{2}} \sum_M [(\lambda_M^{++} \underset{+}{\chi_S^{-M}} - \lambda_M^{-+} \underset{-}{\chi_S^{-M}}) \underset{+}{\theta} - (\lambda_M^{-+} \underset{+}{\chi_S^{-M}} - \lambda_M^{++} \underset{-}{\chi_S^{-M}}) \underset{-}{\theta}] (-1)^{|M-\frac{1}{2}|} \, .$$

On decay each $\underset{+}{\chi}$ goes into $\underset{+}{\psi}$ and each $\underset{-}{\chi}$ goes into $\underset{-}{\psi}$, where the $\underset{+}{\psi}$ and $\underset{-}{\psi}$ are given by (3-15·4) (4-15·4).

The φ distribution is then obtained on writing:

$$\text{(4-15·5)} \qquad I_S(\varphi) = \frac{1}{2} \Lambda'^{-1} \left\{ \int |\sum_M (\lambda_M^{++} \underset{+}{\psi_S^M} + \lambda_M^{-+} \underset{-}{\psi_S^M})|^2 \, \mathrm{d} \cos \theta + \right.$$

$$\left. + \int |\sum_M (-1)^{|M-\frac{1}{2}|} (\lambda_M^{++} \underset{+}{\psi_S^M} - \lambda_M^{-+} \underset{-}{\psi_S^{-M}})|^2 \, \mathrm{d} \cos \theta + \text{same expression with } \lambda_M^{++} \rightleftarrows \lambda_M^{-+} \right\} .$$

The θ distribution has the same expression with the integration over φ instead of over $\cos\theta$.

We consider first the θ distribution; contrary to the case treated previously the distribution in θ may contain now asymmetrical terms; that is terms with odd M may be present in $I_S(\theta)$. For $S=\frac{1}{2}$ only $\mathcal{A}_1^{(\frac{1}{2})}$ is different from zero; its expression is:

$$\mathcal{A}_1^{(\frac{1}{2})} = \Lambda'^{-1}(-\lambda_{\frac{1}{2}}^{++}\lambda_{\frac{1}{2}}^{*-+} + \lambda_{-\frac{1}{2}}^{++}\lambda_{-\frac{1}{2}}^{*-+} + \text{c. c.})\,.$$

Its maximum value is $\frac{1}{2}$ and the corresponding maximum asymmetry is 3 (*).

Also the φ distribution, calculated from (4), has now asymmetrical terms; they have now the form $\cos(2M+1)\varphi$, no term in $\sin M\varphi$ being present. For $S=\frac{1}{2}$ the coefficient $A_1^{(\frac{1}{2})}$ is reported below:

$$A_1^{(\frac{1}{2})} = \frac{\pi}{2}\,\Lambda'^{-1}[\lambda_{\frac{1}{2}}^{++}\lambda_{-\frac{1}{2}}^{*-+} + \lambda_{-\frac{1}{2}}^{++}\lambda_{\frac{1}{2}}^{*-+} + \text{c.c.}]\,.$$

Both $A_1^{(\frac{1}{2})}$ and $\mathcal{A}_1^{(\frac{1}{2})}$ vanish for an unpolarized hyperon. The experimental data are, as yet, insufficient to make detailed comparisons.

15·6. – Specialization of the general formulae to particular processes.

The discussion in Sect. **15**·3 to **15**·5 has been very general, since it does not assume any knowledge of the parameters λ_M specifying the state of the hyperon at the moment of the decay. Correspondingly the information on the spin which one may derive gives just, as already pointed out, lower limits to the value of the spin.

It has been remarked however that, if from the study of the production process of the hyperon the λ_M are either known, or in some way subjected to some conditions, more information on its spin may be derived and, in favourable conditions, also « the » value of the spin. Three processes will be considered in the following in which this situation presents itself; they are:

a) the production of a hyperon in the reaction (1-**15**·2) when one considers only the production events at particular angles [13];

b) the production of a hyperon from the absorption of a bound K^- from a proton (reaction (2-**15**·2)) [14];

c) the decay of a Ξ^- particle [15].

(*) Compare the preceding footnote.

The Y which is formed in these three cases (Λ^0 or Σ in *a*), *b*); Λ^0 in *c*)) will be seen to be in a state in which the λ_M are either determined or in some way restricted.

We shall consider closely in the following just the anisotropies in the distributions; as far as the asymmetries are concerned (which may be present either if the parity is not conserved, or if a parity doublet model is assumed) we shall report only the results.

Consider first the process *a*). Assume the K produced with the Y to have spin zero and suppose to detect only those hyperons which are produced in the forward or backward direction with respect to the direction of the incoming pion (in the centre of mass system of the colliding pion and proton). We want to show that in this case the distribution in θ (the angle between the line of flight of the Y and the line of flight of its decay products in the centre of mass system of the Y) is completely determined by the spin S of the Y. This may be seen as follows: consider the line of flight of the incident π which is the same as that of the outgoing Y; with respect to such line the component of the orbital angular momentum both of the incoming pion and of the outgoing Y vanishes. Since the K has spin zero, the component of the spin of the Y with respect to such line must be equal to the component of the spin of the collided proton, which is $\frac{1}{2}$ or $-\frac{1}{2}$. Therefore the produced Y wave is simply described by $\chi_S^{\frac{1}{2}}$ if the proton had spin up, and $\chi_S^{-\frac{1}{2}}$ if it had spin down, before the collision. The θ distribution is obtained from (6-15.4) putting all the $\lambda_M = 0$ except $\lambda_{\frac{1}{2}}$ and $\lambda_{-\frac{1}{2}} = 1$; the distributions for various spins S are reported in the following Table I-15.6.

TABLE I-15.6.

Spin value of the Y	Distribution
1/2	1
3/2	$1 + 3\cos^2\theta$
5/2	$9/4(5\cos^4\theta - 2\cos^2\theta + 1)$

As ADAIR notices it is not necessary that the produced Y goes absolutely forward or backward; for an energy of 950 MeV of the pion a cone of 20° half opening is admissible.

Notice [16, 8] that the above argument is also valid in another case: namely if one may be sure that the Y is produced in an *s* state, whatever its direction is. Then the same formulae hold for the angular distribution of the line of flight of its decay product (in its rest system) with respect to its direction of flight.

As far as the asymmetries in the θ distribution are concerned it is clear from the general treatment that no asymmetry may be present if the parity

is not conserved: if the doublet parity model is assumed to hold instead, there may be an asymmetry.

Coming now to the process b) (absorption of a K^- bound in a Coulomb orbit by a proton), and always assuming the K^- to have spin zero let us first suppose the absorption to take place from an s state [7]. Then, if we consider the line of flight of the produced hyperon and the angle θ between such line of flight and the line of flight of the decay products of the hyperon in the rest frame of the same, the distribution in θ is completely determined by the spin S of the hyperon. The argument is in fact identical to the one given above, for the s state production of a hyperon in a π^--p collision. The θ distributions are also the same as before and are therefore to be found in the same Table I-**15**·6. If the absorption of the K does not take place exclusively from an s state, as it may be the case [17] (*), the distributions of the decay products are no longer completely determined from the spin S of the hyperon; for instance if the absorption of the K takes place from a p state, partially or entirely, the expected distribution for the case of a $\frac{3}{2}$ spin hyperon, has the form $1+A\cos^2\theta$ where A may have any value from -1 to 3.

The experimental θ distribution for the decay of Σ's from K^- capture are presently consistent with isotropy (Sect. **15**·2). On account of what we have said this would decide spin $\frac{1}{2}$ for the Σ, if the K absorption took place from an s state; it is no longer strictly a proof if the absorption may take place also from a p state.

Finally we may say that no asymmetry is expected in the θ distributions of the decay products of hyperons produced by K^- capture at rest, if parity is not conserved; asymmetries may be present if the parity doublet model holds; there is no evidence presently for them.

The process c) which will be finally discussed consists in studying [15] the distribution of the angle θ between the direction of emission of a Λ^0 originating in the decay of a Ξ^- and the line of flight of the Λ^0 decay products in the rest frame of the Λ^0. Also this method gives definite angular distributions for any assumed spin and parity of the Ξ^- and Λ^0. The distributions however for a spin of the Ξ^- larger than $\frac{1}{2}$ contain a parameter ξ which is uncertain and depends on the radius of the Ξ^-.

The distributions are reported in a table of [15]; one gets an isotropic distribution if the Λ^0 has spin $\frac{1}{2}$ whatever the spin and parity of the Ξ^- are. Considering the other cases one gets a definite distribution $1+3\cos^2\theta$ if the Ξ^- has spin $\frac{1}{2}$ and any parity, and the Λ^0 has spin $\frac{1}{2}$ and any parity; the remaining distributions all contain the parameter ξ and are plotted in several figures in the reference quoted, under the assumption of a vanishing ξ (which

(*) Detection of γ-rays as in the π-mesic atoms might decide this point.

means radius of the Ξ^- small with respect to the wavelength of the emitted $\Lambda^0\pi$ wave).

On account of the few cases of Ξ^- so far observed, a profitable comparison with the experiment cannot yet be made (*).

15·7. – The spin of the Λ^0 from the ratio between mesonic and non-mesonic decay of the hyperfragments.

A quite different and less direct method for getting information on the spin of the Λ^0 (this method may be of use only for the Λ^0) is the one proposed by RUDERMAN and KARPLUS [1], based on the fact that the ratio between the mesonic and non mesonic decay of the Λ^0-hyperfragments depends on the assumed spin of the Λ^0.

As we shall explain in detail in Chapter **21** a Λ^0 hyperfragment is a nuclear fragment ejected from a star and containing inside a bound Λ^0; after an average time of 10^{-12} to 10^{-10} s such hyperfragments decay as a result of the instability of the Λ^0 which they contain; and may decay either with or without the emission of a pion, that is respectively, by mesonic or non-mesonic decay. On examining the detailed mechanism through which the two modes of decay take place, KARPLUS and RUDERMAN have shown that the ratio between the rates of mesonic and non-mesonic decay of a given kind of hyperfragments, depends rather strongly on the assumed spin for the Λ^0, so that one may be able to deduce from the observed ratio the value of the spin in question; it must be said that the method is rather indirect and involves the knowledge of the wave function (or at least of the binding energy) of the Λ^0 in the nuclear fragment; however it is perhaps the only method which presently gives some indication on the spin of the Λ^0 and we shall describe it here in detail.

For semplicity we shall, unless otherwise stated, disregard the $(\Lambda^0 \to n\pi^0)$ decay and fix our attention only on the π^- p decay mode of the Λ^0.

The starting point of the Ruderman and Karplus' method (a point which is due to PRIMAKOFF and CHESTON [20]) is that one may consider the non mesonic decay of the excited fragments as a kind of internal conversion process, that is as a virtual mesonic decay of the Λ^0 followed by the absorption of the

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(*) We may add that another, completely different, method for determining the K, Λ, Σ spins consists in considering [18], in analogy with the method for determining the pion spin, the ratio between the rates of the reaction $K^- + p \to \Sigma^\pm + \pi^\mp$ and its reverse at the same energy. This method has been considered also in [19] where the complications arising from the parity doublet assumption have been discussed.

Finally it is also evident that indications on the spins of the new particles will be given, at least if one spin is supposed to be known, from the momentum dependence of some production cross sections.

virtual π meson from another proton in the nucleus; notice that we are not speaking of the case in which the Λ^0 emits a real pion which is afterwards reabsorbed by some other nucleon in the nucleus; the distinction is important because the two cases differ essentially in the fact that the momentum of the exchanged pion is in the first case much higher than in the second in which the momentum is just that corresponding to a real decay.

With this interpretation of the non mesonic decay the (Feynman) graphs representing respectively the mesonic and non-mesonic decay of a Λ^0 inside a nuclear fragment are reported below (Fig. 1); graph α corresponds to the real disintegration of the Λ^0 which emits at A the usual π^- and p, where the π has the usual momentum $q \simeq 0.73\, mc$ (m = pion mass); graph β instead corresponds to the previously described process of non-mesonic decay in which at the end no pion is present, but only a neutron and a proton having a kinetic energy corresponding to the total excitation energy of the Λ^0 because no energy is spent in creating the rest mass of the pion; in such case, assuming a Λ^0 initially at rest in the nucleus, the final two nucleons move in opposite directions with a momentum k of $\sim 3mc$ each; which corresponds to a momentum of the virtually exchanged pion of $\sim 3mc$.

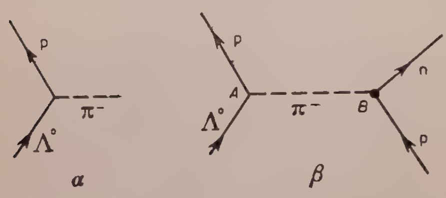

Fig. 1-15'7.

It is understood that in both processes, the vertex at A is due to the very weak interactions responsible for the decay of the new particles; on the contrary in graph β the interaction operating at the vertex B is the strong Yukawa interaction between pions and nucleons, which is assumed to be, for example, the ordinary pseudovector interaction extensively used by CHEW.

In comparing the rates of the mesonic and non-mesonic decay one has to calculate the matrix elements corresponding to the graphs of Fig. 1α and Fig. 1β respectively; both such matrix elements contain as a factor the matrix element of the weak interaction at the vertex A; on account of the fact that the pion which is emitted at A has, as explained above, a very different momentum in the two cases, the matrix element of the weak interaction at A differs in the two cases, the more so the larger is the assumed spin S of the Λ^0; the reason for this is that the amplitude of the low momentum pion wave produced in the mesonic decay is smaller than the amplitude of the high momentum virtual pion wave in the non-mesonic decay on account of the centrifugal barrier; this is the more so, the larger is the spin of the Λ^0, that is, for the conservation of the angular momentum, the orbital angular momentum l carried by the pion.

More precisely if l is the orbital angular momentum of the pion proton system produced at A the amplitude of the pion wave emitted is proportional

to the momentum of the pion to the power l; notice that:

$$l = S \pm \tfrac{1}{2}\,, \tag{1-15.7}$$

where the sign depends on the parity of the Λ^0.

The ratio between the amplitudes for the two cases will then be proportional to $(k/q)^l$ where as already mentioned, q is the momentum of the emitted pion in the mesonic decay, and k is the momentum of the virtual pion in the non-mesonic decay; the ratio $Q^{(-)}$ between the two rates of decay is then proportional to $(k/q)^{2l}$.

$$Q^{(-)} = \frac{R^{(-)}\,(\text{non-mesonic})}{R^{(-)}\,(\text{mesonic})} \cong C^{(-)}(k/q)^{2l} \cong C^{(-)}(17)^l\,, \tag{2-15.7}$$

where $C^{(-)}$ is a quantity independent of the spin of the Λ^0.

The essential point in formula (2) is the large factor $(k/q)^{2l} \cong (17)^l$; passing from one l value to the next the ratio between the rate of non-mesonic and the rate of mesonic decay of a given kind of excited fragment should increase by a factor $(17)^l$.

This shows that the ratio in question is a very sensitive function of the orbital momentum of the pion in the process $\Lambda^0 \to p + \pi^-$ and therefore of the spin S of the Λ^0.

To determine which value of l gives the best agreement with the experiments, it is however necessary to determine the constant $C^{(-)}$ in (2).

Considering the Feynman graphs α) and β) we see that the rate of non-mesonic decay will be proportional to: 1) the density of the nucleons capable of absorbing the emitted pion, 2) the square of the Yukawa g constant for the absorption of a pion by a nucleon, 3) the phase space volume of the final products; on the other hand the rate of mesonic decay is simply proportional to the phase space volume of the final products.

Assuming for the Yukawa constant at the vertex B the value $g^2/4\pi\hbar c = 0.08$ with a pseudovector interaction, we get for the quantity $C^{(-)}$ in (2) the expression:

$$C^{(-)} \cong 5\varrho_p P\,, \tag{3-15.7}$$

where ϱ_p is the density of the protons in the nucleus, and P the probability that the Λ^0 is in the nucleus.

We may also mention that a completely similar expression is obtained for the ratio between the non-mesonic and the mesonic decay corresponding to the decay mode $\Lambda^0 \to n + \pi^0$ of the Λ^0; RUDERMAN and KARPLUS give the

expression analogous to (3):

$$C^{(0)} = 2.5\varrho_p P\,,$$

where the factor 2 of difference is simply due to the fact that in a symmetrical theory the effective coupling constant for neutral pions is $g/\sqrt{2}$.

15·8. – Comparison with the experiments; the results and some remarks on their validity.

We now apply (2-**15**·7) with the calculated values of $C^{(\;)}$ (3-**15**·7) to the light nuclei. The binding energy B_Λ of the Λ^0 to the light hyperfragments ($Z \leqslant 2$) is small and in these conditions the probability for the Λ^0 of being in the nucleus, is essentially determined by B_Λ: We then get:

$$\varrho_p P \simeq \frac{3Z}{4\pi R^3}\, R\sqrt{2M_\Lambda B_\Lambda}\,, \tag{1-15.8}$$

where R is the radius of the nucleus and M_Λ the reduced mass of the Λ^0. We leave aside the case of the excited triton on account of the fact that in such case a more detailed treatment (than that leading to (1)) of the probability of finding the Λ^0 inside the nucleus (deuteron) should be necessary. Moreover out of eleven cases of decay of excited tritons identified with certainty, not one of non-mesonic decay has been observed so that it is presently difficult to know something of the experimental value of the ratio.

We instead want to compare the theory with the experiments for a system of charge $Z = 2$; using (1) and assuming $R \simeq A^{\frac{1}{3}} \simeq 3^{\frac{1}{3}}$ we obtain the values of $Q^{(-)}$ reported in Table I-**15**·8 as a function of l; the binding energy B_Λ still appears as a parameter.

TABLE I-**15**·8. – *Ratio of non-mesonic to mesonic decay for helium hyperfragments as a function of the pion angular momentum l and the binding energy B_Λ of the Λ^0 in* MeV.

$l =$	0	1	2	3
$Q^{(-)} =$	$0.4B_\Lambda^{\frac{1}{2}}$	$6.8B_\Lambda^{\frac{1}{2}}$	$120B_\Lambda^{\frac{1}{2}}$	$200B_\Lambda^{\frac{1}{2}}$

Very little is presently known about the binding energies of the excited fragments in question; its value may vary from a fraction of a MeV to, say, 2 MeV. Assuming a value $B_\Lambda \simeq 1$ MeV the ratio between non-mesonic and mesonic decay in the hyperfragments considered, should be 0.4 for $l = 0$, 6.8 for $l = 1$, 120 for $l = 2$, increasing at each step by a factor 17. The present experimental ratio is 1.7 (Sect. **21**·4); it appears therefore that $l = 0$

or 1 are the assignments which best fit the data. On account of (1-**15**'7) the spin of the Λ^0 should therefore be $\frac{1}{2}$ or $\frac{3}{2}$; a value $\frac{5}{2}$ would imply, a binding energy B_Λ of the order of 1 keV, to be consistent with the experimental ratio.

An independent test of the above result may be obtained, according, to KARPLUS and RUDERMAN, considering the ratio non-mesonic decay to mesonic decay in the heavier ($Z > 2$) hyperfragments; in such case we assume that the Λ^0 has probability ~ 1 of being inside the nuclear volume, in agreement with the experimental fact that the binding energy of the Λ^0 in heavier hyperfragments seems to be larger (about 6 MeV in Be); the protonic density ϱ_p is constant for all the nuclei and approximately equal to $3/8\pi$; several corrections due to the Pauli principle, (see also [21]), the reabsorption of mesons (compare also [22]), have to be made before comparing the experimental values with the ones obtained from the formulae (2-**15**'7) and (3-**15**'7) above. The corrected $Q^{(-)}$ values, $Q_c^{(-)}$, are reported in Table II-**15**'8, as a function of l.

TABLE II-15'8. - *Corrected ratio $Q_c^{(-)}$ of non-mesonic to mesonic decay expected for hyperfragments with $Z > 2$, as a function of the pion angular momentum l (from [1]).*

$l = 0$	1	2	3
$Q_c^{(-)} = 4.8$	80	1400	24000

The experimental ratio is ~ 20; FRY (see Sect. 21'4) finds 138 hyperfragments with $Z > 2$ decaying non-mesonically and just 8 decaying mesonically Again from Table II-**19**'8 is apparent that the $l = 0$, 1 values are the best ones. RUDERMAN and KARPLUS conclude that on the basis of the above evidence the Λ^0 should have either spin $\frac{3}{2}$ or $\frac{1}{2}$. (*)

We may close this section with the remark that two different reasons may possibly influence and modify the above conclusion although it seems improbable; one is that, as we shall see from the discussion in Chapter **21**, the experimental ratio non-mesonic versus mesonic decay is not surely established, and the figures given above are subject to modifications also in view of some bias which may tend to favour the observation of some modes of decay with respect to others; we do not include in such biasses the fact that the Λ^0 has the alternate decay mode $n+\pi^0$ and that the mesonic decays with emission of a π^0 either may escape detection or not be correctly interpreted; this has already been taken in to account in the figures given in Table II.

The second remark is that, as pointed out by the Authors themselves, the factor $(17)^l$ in formula (2-**15**'7) which allows easily to distinguish between

(*) The distinction between these two cases by this method appears difficult and is further complicated if parity is not conserved in the decay of the Λ^0.

one l value and the next one, may have to be replaced by a much smaller figure if the « radius » of the Λ^0 becomes larger than the Compton length of the pion; the above factor of 17 is in fact obtained only, as is apparent from the derivation, if it is assumed that the « radius » of the Λ^0 is small with respect to the wave length of the intervening pions.

Unfortunately nothing can be said, at present, on the value of such « radius ».

REFERENCES (CH. 15)

[1] R. KARPLUS and M. RUDERMAN: *Phys. Rev.*, **102**, 247 (1956).
[2] M. SCHWARTZ, J. STEINBERGER, R. PLANO and N. SAMIOS: to be published.
[3] J. STEINBERGER: *Proceed. of the Rochester Conf.* (1956).
[4] R. BUDDE, M. CHRETIEN, J. LEITNER, N. P. SAMIOS, M. SCHWARTZ and J. STEINBERGER: *Phys. Rev.*, **103**, 1827 (1956).
[5] L. W. ALVAREZ, H. BRADNER, P. FALK VAIRANT, J. D. GOW, A. H. ROSENFELD, F. T. SOLMITZ and R. TRIPP: U.C.R.L. 3583 and *Nuovo Cimento*, **5**, 1026 (1957), and added note by the same authors.
[6] W. D. WALKER and W. D. SHEPHARD: *Phys. Rev.*, **101**, 1810 (1956).
[7] S. B. TREIMAN, G. T. REYNOLDS and A. L. HODSON: *Phys. Rev.*, **97**, 244 (1955).
[8] G. MORPURGO: *Nuovo Cimento*, **3**, 1069 (1956).
[9] S. B. TREIMAN and H. W. WYLD: *Phys. Rev.*, **100**, 879 (1955).
[10] T. D. LEE and C. N. YANG: *Phys. Rev.*, **104**, 822 (1956).
[11] G. MORPURGO: *Nuovo Cimento*, **4**, 1222 (1956).
[12] G. MORPURGO: *Nuovo Cimento*, **5**, 1787 (1957).
[13] R. K. ADAIR: *Phys. Rev.*, **100**, 1540 (1955).
[14] S. B. TREIMAN: *Phys. Rev.*, **101**, 1217 (1956).
[15] R. GATTO: *Nuovo Cimento*, **2**, 841 (1956); **4**, 197 (1956).
[16] S. HORI: *Progr. Theor. Phys. (Japan)*, **16**, 189 (1956).
[17] R. GATTO: *Nuovo Cimento*, **3**, 1394 (1956).
[18] T. D. LEE: *Phys. Rev.*, **99**, 337 (1955).
[19] K. NISHIJIMA: *Progr. Theor. Phys. (Japan)*, **14**. 523 (1955).
[20] H. PRIMAKOFF and R. CHESTON: *Phys. Rev.*, **92**, 1538 (1953).
[21] H. PRIMAKOFF: *Nuovo Cimento*, **3**, 1394 (1956).
[22] T. K. FOWLER: *Phys. Rev.*, **102**, 844 (1956).

CHAPTER 16.

General Considerations on the Decay Interactions.

16·1. – Introduction.

Apart from some scattered remarks (in Chapter 10) nothing has been said up to this point, about the properties of the interactions responsible for the decays of the new particles, besides the general statement that they are « weak » and do not conserve T and T_3.

To such a discussion this chapter will be devoted; of course the question of the bosons, which has already been considered in detail has to be kept in mind in some of the considerations which will follow.

The goals of a complete theory should be two: 1) to explain the values of the lifetimes and branching ratios of the various particles, with some few assumptions concerning the interactions responsible for the decay; 2) to explain why the observed modes of decay actually take place, while other « a priori » possible modes do not or are much less effective.

We begin by summarizing the situation, with respect to the above two questions, for the old particles; this is necessary to understand the situation for the new ones.

16·2. – The decay processes of the old particles.

The reactions which we shall consider here are the β decay of the neutron

(1-16·2) $$n \to p + e^- + \nu$$

the μ decay,

(2-16·2) $$\mu \to e + \nu + \begin{pmatrix} \nu \\ \text{or} \\ \tilde{\nu} \end{pmatrix},$$

the μ capture:

(3-16·2) $$\mu^- + p \to n + \nu$$

and the pion decay:

(4-16·2) $$\pi \to \mu + \nu\,.$$

We shall have also to consider the reactions:

(5-16·2) $$\pi \to e + \nu \qquad \text{and} \qquad \pi \to e + \nu + \gamma$$

which have never been observed.

The first thing which one can do in discussing the above reactions is to compare the total phase space volumes available for the final products. We refer first to the three reactions (1), (2), (3).

The remarkable fact is, then, that for these reactions the three phase space volumes are, with good approximation, in the same ratios as the rates of decay (+) (which are simply the inverses of the lifetimes).

(+) The phase space for the μ-decay is:

(*) $$\varrho_{\mu\ \text{decay}} = \frac{8V^2}{(2\pi\hbar)^6}\int\limits_0^{E/2} p^2(3E^2 + 2p^2 - 6Ep)\,\mathrm{d}p \simeq \frac{8V^2}{(2\pi\hbar)^6}\,\frac{\pi^2E^5}{32}\,,$$

where E is the total energy available to the end products (in this case $E = m_\mu$), expressed in electron masses.

The phase space for the β-decay of the neutron is:

$$\varrho_{\beta\ \text{decay}} = \frac{8V^2}{(2\pi\hbar)^6}\,16\pi^2\int\limits_0^{(E^2-1)^{\frac{1}{2}}} p^2(\sqrt{p^2+1} - E)^2\,\mathrm{d}p = \frac{8V^2}{(2\pi\hbar)^6}\,16\pi^2 f(0, E)\,,$$

with the same meaning of E, all being expressed taking the electron mass as unit; here f is the well known tabulated function.

The ratio between the two phase spaces is:

$$\frac{\varrho_{\beta\ \text{decay}}}{\varrho_{\mu\ \text{decay}}} \simeq \frac{500 f(0, E)}{E^5} \simeq 2\cdot 10^{-9}\,,$$

where the last figure is obtained inserting for $f(0, E)$ the value 1.5 and for E the value $2.1\cdot 10^2$.

The experimental ratio between the rates of decay of the neutron and of the muon is: $2.15\cdot 10^{-6}/750 = 3\cdot 10^{-9}$ in good agreement with the above figure.

Considering now the process (3) the comparison is a little more involved. The phase space for the final products in the reaction (3) is:

(**) $$\varrho_{\mu\ \text{absorption}} \simeq \frac{4V}{(2\pi\hbar)^3}\,4\pi m_\mu^2\,.$$

This shows that the matrix elements of the interaction hamiltonian inducing the transitions (1), (2), (3) must be the same in the three cases [1-5].

This is rather satisfactory because the possibility arises of explaining the three processes (1), (2), (3), through the same kind of interaction.

The problem is then to construct such an interaction so that the matrix elements for the three processes (1), (2), (3) result to be the same. Of course due account has to be taken of the processes (4) and (5) since the interaction chosen must not be in contradiction with them. It is instructive to consider two of the possibilities which appear to exist.

16·3. – Weak boson-fermion interaction.

A first possibility which one may consider [6] but which will have to be rejected, is illustrated in the following coupling diagram (Fig. 1-16·3). The pion is strongly coupled to the nucleon field (as indicated by the full line) and weakly coupled (as indicated by the dashed lines) both to the eν and to the μν fields.

The three couplings in question may be provided by a trilinear interaction of the kind:

$$H'_{ab} = g_{ab}\int \varphi_\pi(\overline{\psi}_a \Gamma \psi_b)\,\mathrm{d}^3\boldsymbol{x} + \text{h.c.}\,, \tag{1-16·3}$$

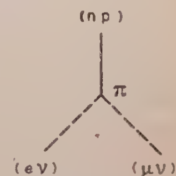

Fig. 1-16·3.

where φ_π is the pion field operator and ψ_a, ψ_b are the spinors respectively of the n, p, or of the μ, ν or of the e, ν fields; $g_{a,b}$ are the corresponding coupling constants. Γ is a Dirac matrix chosen in such a way as to make the expression an invariant.

This formula however refers to a situation in which everything is free to move in the volume V, which has been taken as our normalization volume. In other words the rate of absorption (3) would be that given by (**) if the muon density near the proton were V^{-1}. This is not so however, because the μ^- is bound to the proton and its density near the proton is $1/\frac{4}{3}\pi R^3$ were R is the radius of the lowest muonic orbit. To obtain the correct rate of absorption in this situation one has to multiply $\varrho_{\mu\,\text{absorption}}$ given by (**) with the ratio of the two densities: $(\frac{4}{3}\pi R^3)^{-1}/V^{-1}$. We then obtain:

$$\varrho'_{\mu\,\text{absorption}} \cong \frac{8V^2}{(2\pi\hbar)^6}\,\frac{4\pi m_\mu^2}{2\cdot\frac{4}{3}\pi R^3} = \frac{V^2\pi^2}{(2\pi\hbar)^6}\,\frac{24 m_\mu^5\pi}{(137)^3}\,,$$

where we have inserted for R its expression $\hbar^2/m_\mu e^2$. Comparing this expression with (*) we obtain:

$$\varrho'_{\mu\,\text{absorption}}/\varrho_{\mu\,\text{decay}} = 5\cdot 10^{-4}\,.$$

The extrapolated (according to the $(Z/10)^4$ law) experimental value is around 10^{-4}; the agreement is rather satisfactory; the main reason of the factor 5 discrepancy is probably the fact that the extrapolation is not correct (Pauli principle [1]).

The coupling constant g_{np} is now determined by the known facts about the pion-nucleon interactions (nuclear forces and so on); such facts establish also the pseudoscalar nature of the pion (with respect to the nucleons) so that the interaction H'_{np} may be considered fixed by these facts.

We may next adjust the coupling constants $g_{\nu\mu}$ and $g_{e\nu}$ in such a way that the matrix element for the process (5-**16**·2) ($\pi \to e$ decay) is much smaller than that for the process (4-**16**·2) $\pi \to \mu + \nu$ decay, in agreement with the experiment.

But then it turns out that the two step matrix element for process (1-**16**·2) (β decay) is by the same amount smaller than the two step matrix element for the process (3-**16**·2) (absorption of μ^-) contrary to the experiment [6] (*).

This is already sufficient to exclude the coupling scheme; in addition a second, and even more obvious difficulty against such a scheme is that the process (2-**16**·2) (decay of muon) would be much too slow, because taking place through two weak steps ($\mu \to \pi + \tilde{\nu}$, $\pi \to e + \nu$).

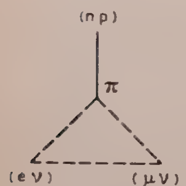

Fig. 2-**16**·3.

Of course this last fact does not happen if one introduces in the coupling scheme a direct connection between the μ, ν fields and the e, ν fields, as indicated in Fig. 2.

This however ruins completely the spirit of the scheme; moreover the first difficulty remains.

It may be added that not only the scheme in question is unsatisfactory as a whole; but already the branch (np) $\rightleftarrows \pi \rightleftarrows$ (eν) even apart from all the difficulties mentioned above, is insufficient to explain the β decay spectra; this is due to the fact [6] that the pion is pseudoscalar and the equivalent quadrilinear coupling produced by a pseudoscalar pion is the axial vector one giving rise to the Gamow-Teller selection rules, which as known, are not the only ones exhibited by the β phenomena; see also [7].

We therefore discuss briefly the second possibility.

16·4. – Interaction between four fermions.

This coupling scheme is illustrated in the diagram below (Fig. 1-**16**·4). Here a weak direct coupling between the three pairs of intervening fermions is assumed from the start. In addition the pion interacts with the nucleon field, of course, through a strong coupling.

The direct coupling between pneν is of the kind well known from the

(*) Choosing $g_{\mu\nu}$ and $g_{e\nu}$ equal there are other similar difficulties [7] even assuming a pseudovector interaction (1).

β theory:

$$H'_{\text{pnev}} = \sum_i g_i^{(e,N)} (\psi_p^+ \Gamma_i \psi_n)(\psi_e^+ \Gamma'_i \psi_\nu) \tag{1-16·4}$$

a linear combination of the five invariants with constant $g_i^{(e,N)}$. The other direct couplings are assumed to be of the same general structure; the coupling between np and μν will be characterized by a set of constant $g_i^{(\mu N)}$ and the interaction between μ, ν and e, ν will be characterized by a set of constants $g_i^{(\mu,e)}$. The important point is now that, with the interactions chosen in this way, one gets matrix elements having the same order of magnitude for the three transitions (1-16·2), (2-16·2) and (3-16·2) simply by assuming that the coupling constants which characterize the β interaction, or the muon decay interaction or the muon capture interaction have the same order of magnitude.

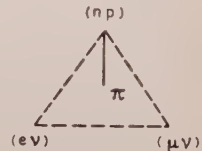

Fig. 1-16·4.

Therefore we may conclude that the order of magnitude of the rates of the three processes (1-16·2), (2-16·2), (3-16·2) are explained simply by: *a*) the phase space available to the final products, *b*) the assumption of the same interaction between the three pairs of fermions involved; the expression « the same interaction » being understood in the not yet precise way explained above.

16·5. – Something more on the constants g_i.

We begin by rewriting the β interaction in the more explicit form:

$$H'_{\text{pnev}} = g_S S + g_V V + g_T T + g_A A + g_P P + \text{h. c.}, \tag{1-16·5}$$

where the expressions S, V, T, A, P are the invariants defined below, the signs being chosen so as to be in agreement with MICHEL [8],

$$\begin{cases} S = (\psi_p^+ \beta \psi_n)(\psi_e^+ \beta \psi_\nu) \\ V = -\{(\psi_p^+ \psi_n)(\psi_e^+ \psi_\nu) - (\psi_p^+ \boldsymbol{\alpha} \psi_n)(\psi_e^+ \boldsymbol{\alpha} \psi_\nu)\} \\ T = (\psi_p^+ \beta \boldsymbol{\sigma} \psi_n)(\psi_e^+ \beta \boldsymbol{\sigma} \psi_\nu) + (\psi_p^+ \beta \boldsymbol{\alpha} \psi_n)(\psi_e^+ \beta \boldsymbol{\alpha} \psi_\nu) \\ A = (\psi_p^+ \boldsymbol{\sigma} \psi_n)(\psi_e^+ \boldsymbol{\sigma} \psi_\nu) - (\psi_p^+ \gamma_5 \psi_n)(\psi_e^+ \gamma_5 \psi_\nu) \\ P = -(\psi_p^+ \beta \gamma_5 \psi_n)(\psi_e^+ \beta \gamma_5 \psi_\nu) . \end{cases} \tag{2-16·5}$$

It is well known that, the neutrino having mass zero, the expression (1) is [8] the most general expression linear in the four fields, not containing derivatives,

conserving parity (*), and, if the g_i are real, invariant with respect to time reversal. The order in which the operators of the intervening fields have been written in (2) is important because, if such order is changed (for example if ψ_n is changed into ψ_ν) any of the interactions (2) is changed into a combination of the others [8-10].

The meaning of the g_i then changes. In the following the order will be supposed to be fixed in the way indicated in (2).

We may now ask what may be said about the g_i. A first fact which is derived [11]: *a*) from the observation that in some β transitions the Fermi selection rules are operating and in other the Gamow-Teller, *b*) from the absence of Fierz terms in the Curie plots of the allowed decays, is that either S or V are present but not both with comparable magnitude; similarly either T or A are present but not both with comparable magnitude. Nothing sure can be said, as we shall see in a moment, about the P interaction.

So leaving aside for the moment the question of the P interaction the β interaction might be of the form (ST) or (SA) or (VT) or (VA). Very difficult angular correlation measurements indicate the first combination (ST) as the preferred one; the evidence for this last conclusion has still be to improved.

We may thus conclude that the largest coupling constants should be g_S and g_T. However one cannot exclude very small admixtures of V and A interactions; the present limit for g_A is [Wu 11]:

$$|g_A|^2 \leqslant 0.01\,|g_T|^2\,. \tag{3-16.5}$$

Coming back to the P interaction we may say that, even if g_P is larger than g_S and g_T by a factor 10, it still produces very small effects in the most part of the β phenomena. A larger g_P should however be in contradiction with the experiments.

As far as the relative magnitudes of g_S and g_T are concerned the present data show, according to KONOPINSKY [11] that:

$$0.75 \leqslant |g_S|^2/|g_T|^2 \leqslant 1.15\,. \tag{4-16.5}$$

In the following we shall assume:

$$|g_S| = |g_T| = g/2\,. \tag{5-16.5}$$

(*) Of course the non conservation of parity implies a linear combination between the interaction (1-16.5) and a similar one with all the couplings replaced by pseudo-couplings. Although many consequences are unchanged by this (compare the Appendix of the paper [3] quoted in Ch. **12**) it is too early to say which modifications non conservation of parity may produce in the ensuing discussion.

This being the situation it can be shown [8] that the half life of the neutron, expressed only through the dominant constants g_S, g_T may be written:

$$\tau_n' = \pi^3 \ln 2 \frac{\hbar^7}{m^5c^4} \frac{1}{g_S^2 + 3g_T^2} \frac{1}{f(E)} = \frac{\pi^3}{g^2} \ln 2 \frac{\hbar^7}{m^5c^4} \frac{1}{f(E)}, \tag{6-16.5}$$

where

$$f(E) = \int_0^{\sqrt{E^2-1}} p^2\, dp(\sqrt{p^2+1} - E)^2 , \tag{7-16.5}$$

E being the available energy of the final products.

This determines g as:

$$g = 3.13 \cdot 10^{-49} \text{ erg cm}^3 . \tag{8-16.5}$$

We may now turn to the μ meson decay and see [8, 12] whether the same constants as determined from the β decay, explain also the μ meson decay.

This question has not however a definite meaning until we do not specify in which order we want to insert the $\mu e \nu \nu$ operators in the quadrilinear interactions above; changes of order correspond to changes in the constants [5]. Also one has to specify if the two neutrinos emitted are both neutrinos or are one neutrino and one antineutrino.

We may now fix the attention on one order, namely that specified by the correspondence:

$$\begin{pmatrix} \mathrm{p} & \mathrm{n} & \mathrm{e} & \nu \\ \mu & \nu & \mathrm{e} & \nu \end{pmatrix} \tag{9-16.5}$$

and show that with this order the same constants used for the β decay may be used for the muon decay obtaining results consistent with the experiments; this both in the case of two like neutrinos emitted and of two unlike neutrinos.

The quantities to be compared with the experiment in the case of the μ decay are two: one is the lifetime τ_μ and the other is the parameter ϱ introduced by MICHEL which determines the shape of the spectrum of the decay electrons. The experimental value of ϱ is around $\frac{1}{2}$ (*) and that of τ_μ is $2.22 \cdot 10^{-6}$ s.

Choosing the correspondence (9) and using the same constants used in

(*) The best value of ϱ (H. ANDERSON: private communication) appears now to be $\simeq 0.67 \pm 0.05$. The additional freedom which one has, due to non conservation of parity, is such that there are probably many more possible choices for the g_i and g_i' in the ensuing discussion (*note added in proof*).

the β decay (that is making use only of g_S and g_T) one gets [8] for the case of different neutrinos:

$$(10\text{-}16.5)\qquad \begin{cases} \varrho = \dfrac{3}{4}\,\dfrac{(2g_T^2 + g_S^2 + 2g_S g_T)}{g_S^2 + 6g_T^2}\,, \\[2ex] \tau_\mu = \dfrac{3\cdot 2^9}{m_\mu^5}\,\dfrac{\hbar^7}{m^5c^4}\,\dfrac{1}{g_S^2 + 6g_T^2}\,. \end{cases}$$

Assuming again as in (5)

$$(11\text{-}16.5)\qquad |g_S| = |g_T| = \tfrac{1}{2}\ 3.13\cdot 10^{-49}\ \text{erg}\cdot\text{cm}^3$$

and in addition assuming now that the sign of g_S is the same as the sign of g_T, one gets [12]:

$$\varrho = 0.54\,, \qquad \tau_\mu = 2\cdot 10^{-6}\,,$$

in reasonable agreement with the experiment. We may thus say that, assuming the correspondence (9), in the case of different neutrinos an interaction

$$(12\text{-}16.5)\qquad g_S S + g_T T\,,$$

with $g_S \simeq g_T \simeq \frac{1}{2}\ 3.13\cdot 10^{-49}$ erg·cm³ is in reasonable agreement with the β and μ decays.

For the case of identical neutrinos, an interaction which explains the μ decay is instead of the form [12]:

$$(13\text{-}16.5)\qquad g_S S + g_T T + g_P P\,,$$

with

$$(14\text{-}16.5)\qquad g_T = -g_S = \frac{1}{3.5}\,g_P\,.$$

Here a term P is necessary but its magnitude is not contradictory with the β decay evidence. So again (14) may, at present, explain both the β and μ decay phenomena.

We may finally add that no similar discussion on the constants may be made for the μ^- capture in view of the complexity of the nuclear dynamics.

16·6. – The Universal Fermi interaction.

Being possible to explain β and μ decay with exactly the same constants, one may ask [5] whether the interaction written down in the previous sections should not be a Universal one, in the sense that, substituting in place of pnev

any four fermions would give the interaction among the four fermions in question. There are however two serious limitations to this Universality:

1) Not all the processes « a priori » possible among four fermions are seen to take place, for example one has to avoid the processes which do not conserve the charge or the nucleonic number; and one has to avoid also other processes such as:

$$(1\text{-}\mathbf{16}{\cdot}6) \qquad \begin{cases} \mu^{\pm} \to e^{\pm} + e^{\mp} + e^{\pm} \\ \\ \mu^{-} + p \to p + e^{-} \end{cases}$$

which have not been observed.

Thus the terms corresponding to such processes should not be written down in the Universal interaction, which fact already limits its character of Universality.

2) Assume we write the interaction in the form (1-**16**·5) and assume that, for the npeν interaction, the order in which the fields appear be that specified in (2-**16**·5); we then would like to say that the same interaction represents the interaction among four other fermion fields a, b, c, d if simply these fields are inserted in (2-**16**·5) at the place of p, n, e, ν; however, as already mentioned, this is not a definite statement until one does not specify the order in which the a, b, c, d fields are inserted, or, in other words, the correspondence $a, b, c, d \rightleftarrows$ p, n, e, ν. Only one interaction is independent from such specification, being totally antisymmetric.

It is the interaction with:

$$(2\text{-}16.6) \qquad g_S = -g_A = g_P \neq 0\,, \qquad g_T = g_V = 0$$

proposed by Chritchfield and Wigner [13]. This choice seems however incompatible with the evidence from the β spectra.

As far as the first objection is concerned one might be inclined to maintain the assumption of a Universal Interaction simply adding a prescription to insert some terms and exclude others. The first proposal along this line is due to Yang and Tiomno [14]. A more recent attempt is that by Konopinsky and Mahmoud [15].

The second objection is more difficult to answer; several attempts have been made to find reasonable ways to establish in an unique way the correspondence but without too much success. This difficulty is unsolved also in the scheme of Mahmoud and Konopinsky.

Therefore we may conclude that, while it is certain that there is a class of phenomena governed by quadrilinear matrix elements with the coupling constants of comparable magnitude (5-**16**·5), it has not yet been possible to express this idea in a rigorous and unambiguous way.

16·7. – The decay of the pion.

The last question [16-18] which we want to mention is whether the coupling scheme (Fig. 1-**16**·4) is able to explain the rates for the processes $\pi \rightarrow \mu+\nu$ and $\pi \rightarrow e+\nu$ and to predict that the rate of the first process must be much larger than that of the second (which has not yet been observed).

Of course both these processes are possible according to the coupling scheme in question: the π first goes into an intermediate state in which a nucleon and an antinucleon are present and this state next goes into a $\mu+\nu$ state or into an $e+\nu$ state. Perturbation theory calculations have been made for both processes assuming a pseudoscalar pion and various kinds of Fermi coupling between pneν and between pn$\mu\nu$.

The order of the β interaction being that written down in (2-**16**·5), all the Authors implicitly assume in their calculations a correspondence:

$$\begin{pmatrix} p\;n\;e\;\nu \\ p\;n\;\mu\;\nu \end{pmatrix}. \tag{1-16·7}$$

However, it must be emphasized that, since we know nothing from the μ^- absorption on the correspondence, any other correspondence, for instance

$$\begin{pmatrix} p\;n\;e\;\nu \\ p\;\nu\;\mu\;n \end{pmatrix} \tag{2-16·7}$$

would be equally admissible.

Until explicitly mentioned we shall however refer, from now on, to the correspondence (1-**16**·7).

When we proceed to calculate the rates of decay $\pi \rightarrow \mu+\nu$, $\pi \rightarrow e+\nu$ we immediately see that such rates either vanish (due to the operation of some particular selection rule), or diverge.

The divergence is due to the fact that the relative momentum of the virtual nucleon antinucleon produced is arbitrary and one has to sum over all the intermediate momenta. More precisely it is seen that, for a pseudoscalar pion, coupled to the nucleons, either with ps/ps or with ps/pv coupling, the rates in question vanish when the coupling among the four fermions is S and/or V and/or T. In the last two cases this is due to the operation of the Furry theorem, in the first case it is an absolute selection rule. On the contrary, for an A or P coupling among the fermions the rates diverge, unless some cut-off is introduced. The situation is summarized in the following Table I-**16**·7.

This being the situation, an interaction which has to produce the decay $\pi \rightarrow \mu+\nu$ must contain a part of A or P interaction. Then, though the rates

TABLE I-16·7.

Kind of coupling (*) ($\bar{N}N\mu\nu$) . . .	S	V	A	T	P
Rate of decay $\pi \to \mu + \nu$	0	0 (F)	diverg.	0 (F)	diverg.

(*) The correspondence 1-16·7 is assumed.
(F) Means that the process is forbidden by the Furry theorem.

of ($\pi \to \mu + \nu$) and ($\pi \to e + \nu$) will be both divergent, one may still consider the ratio R between the rate of the transition $\pi \to e + \nu$ and that of the transition $\pi \to \mu + \nu$. This ratio is finite even if both the numerator and denominator diverge. For an A coupling this ratio is:

$$R_A \simeq 3.3 \left(\frac{m_e}{m_\mu}\right)^2 \simeq 10^{-4}\,, \tag{3-16·7}$$

while for a P coupling it is:

$$R_P \simeq 5.1\,. \tag{4-16·7}$$

It has to be recalled that according to LOKANATHAN and STEINBERGER [19] the experimental value of R is $(-0.3 \pm 0.9)\cdot 10^{-4}$; ANDERSON and LATTES (private communication) obtain an even smaller value. Comparing these figures with that given in (3) one may conclude that it is difficult that the theory may explain the experimental ratio, but, if so, the quadrilinear interaction must contain an axial vector part and no comparable P part. If we remember the conclusions arrived at from the β decay (compare form. (3-**16**·5)) we see that the situation is not particularly encouraging. However, it cannot be excluded that the very small fraction of A interaction which is still compatible with (though not requested from) the β data is just that needed to explain the situation.

But even if it explains the value of R, is it sufficient to explain the absolute value of the $\pi \to \mu + \nu$ rate of decay? On account of the divergences it is difficult to answer this question, but if a reasonable cut-off is introduced, the answer is negative.

We may at this point remark that a much simpler situation is obtained if we renounce [20] to the correspondence (1) and choose the correspondence (2). Then we may assume that both for the β interaction and for the (pneν) interaction just a coupling $S+T$ exists. Then the decay $\pi \to e + \nu$ is strictly forbidden. On the contrary for the (pn$\nu\mu$) interaction, an interaction $S+T$ with the correspondence (2) is equal to an interaction containing a pseudoscalar and an axial vector term with the correspondence (1). Therefore the decay $\pi \to \mu + \nu$ may take place.

A further complication to this problem is due to the fact that even if it is possible to explain the absence of the $\pi \to e+\nu$ decay with respect to the $\pi \to \mu+\nu$, the absence of the decay:

$$\pi \to e + \nu + \gamma \tag{5-16.7}$$

may be difficult to understand. While, with a T interaction the decay $\pi \to e+\nu$ is certainly forbidden, the decay (5) is not; the Furry theorem does not operate any more, in this case, due to the presence of a γ-ray. According to a paper by Treiman and Wyld [21] the rate of the decay (5) would then turn to be ~ 0.025 of the rate of the $\pi \to \mu+\nu$ decay, contrary to the observation (compare also [22]).

We notice again [20] that the choice (2) of the correspondence proposed here solves also this difficulty.

16.8. – The decays of the new particles.

In examining the decays of the new particles we begin, as in Sect. **16**.2, by comparing the decay rates with the available phase spaces. We therefore report in the following Table I-**16**.8 the value of the phase space for the decay of the particle indicated in the first column into the channel indicated in the second column taking the phase space for the K_1^0 decay into $\pi^+\pi^-$ as unit.

Table I-**16**.8.

Particle	Decay mode	Phase space ($K_1^0 \to \pi^+ + \pi^-$ taken as unit)	Exper. *rate* of decay	Exper. rate of decay; that for the $K_1^0 \to \pi^+ + \pi^-$ decay taken as unit
K_1^0	$\pi^+\pi$	1	$\sim 10^{10}\ s^{-1}$	1
K^+	$\pi^+\pi^0$	1	$\sim 2.5\cdot 10^7\ s^{-1}$ (**)	$2.5\cdot 10^{-3}$
K^+	3π	$2\cdot 10^{-4}$ to $7\cdot 10^{-3}$ (*)	$7\cdot 10^6\ s^{-1}$ (**)	$7\cdot 10^{-4}$
Λ^0	$p+\pi^-$	1.5	$0.25\cdot 10^{10}\ s^{-1}$	$2.5\cdot 10^{-1}$
Σ^-	$p+\pi^0$	3	$\sim 10^{10}\ s^{-1}$	1
Ξ	$\Lambda^0+\pi$	$0.9(2S_\Lambda+1)$	$\sim 10^{10}\ s^{-1}$	1
K^+	$\mu^+ + \nu$	5	$\sim 6\cdot 10^7\ s^{-1}$	$6\cdot 10^{-3}$
π^+	$\mu^+ + \nu$	10^{-1}	$\sim 5\cdot 10^7\ s^{-1}$	$5\cdot 10^{-3}$

(*) For the $K^+ \to 3\pi$ decay (compare [3]) it is necessary to specify the interaction volume in order to compare the available phase space with those for the other two body decays. The first figure refers to a volume $V = \frac{4}{3}\pi(\hbar/M_K c)^3$, the second to a volume $V = \frac{4}{3}\pi(\hbar/m_\pi c)^3$.

(**) These values for the rates of decays are on the assumption that just one K^+ exists which may decay into 3π and 2π by non conservation of parity.

We report both some decays into nucleons and pions only, and some decays into leptons; the phase space for decay of a particle of mass M into two particles of masses $m_1 = xM$ and $m_2 = yM$ is proportional to:

$$\varrho = M^2 \sqrt{1 + (x^2 - y^2)^2 - 2(x^2 + y^2)[1 - (x^2 - y^2)^2]}(2S_1 + 1)(2S_2 + 1)\,,$$

where S_1 and S_2 are the spins of the final particles.

For comparison the experimental value for the partial rate of decay into the channel in question is also reported in column four. For deriving such value from the measured lifetime it is necessary to know the branching ratio leading to the channel in question; if there is just one decay channel, or one is predominant, the rate of decay is simply the inverse of the lifetime; this is the case, usually, for the old particles; if there is more than one decay channel the rate of decay into that channel is the inverse of the observed lifetime, times the branching ratio for decay into the channel in question. The poor knowledge of the branching ratios in the most part of the cases, prevents a good knowledge of the rates of decay; in completing the table the values of the branching ratios reported in table I-1·3. have been used.

Finally we have reported in column 5 the rates of decay from column 4 expressed in terms of the $K_1^0 \to \pi^+ + \pi^-$ rate of decay.

It is apparent from the table that the ratios between the rates of decay for the Λ^0, Σ^+, Ξ^- and the rate of decay of the K_1^0 are in agreement with what is predicted from the phase space within a factor of 10; the same may be said for the rate of the $K^+ \to 3\pi$ decay (for this compare also Sect. **16**·11). Therefore the strength of the interaction responsible for the above decays should be, more or less, the same in all the cases above. However the K^+ decay into $\pi^+\pi^0$ has a rate 400 times smaller than that predictable from the phase space; also the rate of the $K \to \mu + \nu$ decay is $\sim 10^3$ times smaller than the value given by the phase space.

Even including these two cases (the case of the $K_{\pi 2}^+$ will be discussed in Sect. **16**·10; remark that the difference by a factor 400 between the $K^+ \to \pi^+ + \pi^0$ and $K_1^0 \to \pi^+ + \pi^-$ rates appears at first rather strange), the general impression one has from the table is that a unique coupling constant is responsible for all the decays; and that the deviations are attributable to such things as weak selection rules, or final state interactions etc. It is also apparent from the last row of the table that the coupling constant into play here is not essentially different from the one which appears in the $\pi \to \mu + \nu$ decay.

This being the situation, all the attempts to bring order in the field of the decay interactions, have been, as it was the case for the old particles, of the following nature: write down some « fundamental » elementary weak interaction between different fields, with a given coupling constant and interprete the decay processes as due to this interaction (with the partecipation, in some intermediate steps, if necessary, of the known strong interactions).

An elementary interaction in this sense might be one connecting directly two fermions and one bosons (as is was the case for the coupling scheme of Sect. **16**˙3); one speaks then of weak boson fermion interaction; for instance:

$$H' = g\overline{\psi}_a\gamma_\mu\psi_b\frac{\partial\varphi_c}{\partial x_\mu} + \text{h.c.}\,,$$

where ψ_a and ψ_b are two fermion fields and φ_c a boson field; one would like to have the same coupling constant g whatever a, b, c are.

The results which one obtains in this way have been discussed especially in [1, 2], We report, as an example a part of a table from [2] in which the rates of decay for various processes are given assuming g in H' to have the value needed to explain the $\pi\to\mu+\nu$ lifetime (namely $g^2/4\pi = 3.6\cdot 10^{-15}\,m_\pi^2$).

For other tables calculated using non gradient couplings or for higher spins compare [2]. The general conclusion is that the order of magnitude of several processes may be obtained although a quantitative agreement is still lacking.

Another mechanism which may, again qualitatively, explain the various processes is that of the Universal Fermi Interaction; such a mechanism appears to be capable of giving a rather unified picture of all the decay processes, including those of the old particles (for a possible solution of the $\pi\to e+\nu$ difficulty compare Sect. **16**˙7).

TABLE II-**16**˙8. – *Values of the rates of decay obtained for several processes with the interaction* $g\overline{\psi}_a\gamma_\mu\psi_b(\partial\varphi_c/\partial x_\mu)$ *and with* $(g^2/4\pi)m_\pi^2 = 3.6\cdot 10^{-15}$ *(the value needed to explain the* $\pi\to\mu+\nu$ *rate) (from* [2]).

Decay process	Calculated rate (s^{-1})	Exper. rate (s^{-1})
$\Lambda^0\to p+\pi^-$	$0.14\cdot 10^{10}$	$0.25\cdot 10^{10}$
$\Sigma^+\to p+\pi^0$	$0.7\ \cdot 10^{10}$	10^{10}
$\Xi^-\to\Lambda^0+\pi^-$	$0.27\cdot 10^{10}$	$\sim 10^{10}$
$K^+\to\mu+\nu$	$7\cdot 10^3$	$6\cdot 10^7$

Although quantitative calculations are, in the most part of the cases, beset by divergences, the order of magnitude of the coupling constant which intervenes in these decays is the same as for the processes involving old particles [4]. The kind of quadrilinear interaction which one has to assume (in addition to the ones discussed in the previous sections involving nucleons and leptons) is [4]:

$$g(\psi_N^+\Gamma\psi_Y)(\psi_a^+\Gamma'\psi_b)\,, \tag{1-16˙8}$$

where Y is an hyperon and N a nucleon while a and b may be either two nucleonic fields or two leptonic ones.

With this scheme the decay of a Λ^0, is, for instance, interpreted as:

(2-**16**·8)
$$\begin{cases} \Lambda^0 \to n + \overline{n} + n \to n + \pi^0 \\ \qquad\qquad \text{or} \\ \quad \to p + \overline{p} + n \to p + \pi^- . \end{cases}$$

The decay of the Σ^+ takes place according to:

(3-**16**·8)
$$\Sigma^+ \to p + \overline{n} + n \begin{cases} p + \pi^0 \\ n + \pi^+ \end{cases}$$

and so on.

As far as the Ξ^- is concerned, it might decay either according to:

(4-**16**·8)$_a$
$$\Xi^- \to n + n + \overline{p} \to n + \pi^-$$

or according to:

(4-**16**·8)$_b$
$$\Xi^- \to \Lambda^0 + n + \overline{p} \to \Lambda^0 + \pi^- .$$

Apparently the Ξ^- prefers the second way; we shall return to this point in Sect. **16**·10.

Notice that according to the general interaction (1-**16**·8) chosen we might expect also decays such as:

(5-**16**·8)
$$\begin{cases} a & \Lambda^0 \to p + \mu^- + \nu \\ b & \Lambda^0 \to p + e^- + \nu \\ & \cdots\cdots\cdots \\ & \cdots\cdots\cdots \\ c & \Sigma^+ \to n + e^+ + \nu \\ & \cdots\cdots\cdots \\ d & \Xi^- \to n + e^- + \nu . \end{cases}$$

However, as we shall see in the next section, all these decays, except possibly the last one should be rather improbable, as compared to the usual decay modes (2), (3), (4). Up to now they have not been observed (except perhaps one case [5]).

Let us now consider, in the same qualitative way, the decays of the heavy bosons; the postulated interaction may account also for them.

The scheme which applies is then [6]

$$(6\text{-}16{\cdot}8)\qquad \mathrm{K} \xrightarrow[\mathrm{s}]{\mathrm{I}} \overline{\mathrm{Y}} + \mathrm{N} \begin{array}{l} \overset{\mathrm{II}}{\nearrow} \mathrm{N} + \overline{\mathrm{N}} \xrightarrow[\mathrm{s}]{\mathrm{III}} \text{pions} \\ \underset{\mathrm{w}}{\searrow} \text{leptons} \end{array}$$

The first step, indicated as I, is due to a strong interaction, the same interaction which, when inverted, gives rise to the process of associated production; it conserves the strangeness. The second step (II) is instead due to the *weak* Universal Fermi interaction postulated above (1). It may give rise either to a state in which a nucleon antinucleon pair is present, which then decays into two or more pions through a *strong* Yukawa interaction III; or to a state in which two leptons are present.

Of course, in addition to the leptons, it is always possible to have pions, without decreasing appreciably the matrix element; they may be produced, through virtual Yukawa steps, in the first intermediate state:

$$\mathrm{K} \xrightarrow[\mathrm{s}]{} \overline{\mathrm{Y}} + \mathrm{N} \xrightarrow[\mathrm{s}]{} \overline{\mathrm{Y}} + \mathrm{N} + \pi \xrightarrow[\mathrm{w}]{} \text{leptons}\,.$$

It is worth of mention [6] that with this interaction one may explain the fact that the decay processes of the bosons which involve finally leptons are not discouraged with respect to those which involve final pions only. We may remark that, if we had for instance assumed a primary weak coupling of the kind $(\mathrm{KN\overline{N}})$ this would have given rise to the K decay into pions according to

$$\mathrm{K} \xrightarrow[\mathrm{w}]{} \mathrm{N} + \overline{\mathrm{N}} \xrightarrow[\mathrm{s}]{} \text{pions}\,,$$

but the decay into leptons would have been very much discouraged, needing, to take place, a second weak interaction:

$$\mathrm{K} \xrightarrow[\mathrm{w}]{} \mathrm{N} + \overline{\mathrm{N}} \xrightarrow[\mathrm{w}]{} \text{leptons}\,.$$

Referring, specifically to the K^+ decays we thus see that the scheme may in principle account for all the established decay modes; of course it is not clear why other, « a priori » possible processes, do not seem to take place: for instance:

$$\begin{aligned} &\mathrm{K}^{\pm} \to \mu^{+} + \mu^{-} + \pi^{\pm} \\ &\mathrm{K}^{\pm} \to \nu \;\; + \nu \;\; + \pi^{\pm} \\ &\cdots\cdots\cdots\cdots \\ &\cdots\cdots\cdots\cdots \end{aligned}$$

In this respect however the situation is not different from that for old particles (recall for instance the processes (1-**16**·6)).

Similarly all the observed decay modes of K_1^0 and K_2^0 may be inserted in the scheme, though the (strongly presumed) absence of many modes like $\nu+\tilde{\nu}$ or $\nu+\tilde{\nu}+\pi^0$ or $\mu^++\mu^-$ or e^++e^- is not entirely clear.

16·9. – The β-decays of the hyperons.

With the assumed scheme one may easily [6] calculate the expected rates of decays of the hyperons through the reactions (5-**16**·8).

We shall assume here that the interaction and the coupling constant for such decays is exactly the same responsible for the β decay of the neutron.

Then the lifetime for the β decay of the Λ^0 turns out to be:

(1-**16**·9) $$\tau^{(\beta)}_{\Lambda^0} = \tau_n \frac{f_n}{f_{\Lambda^0}},$$

where τ_n is the lifetime of the neutron. Here f_n is the phase space for the β decay of the neutron and f_{Λ^0} is the same quantity for the process $(4\text{-}\mathbf{16}{\cdot}8)_b$. Since $f_n \simeq 1.6$ and $f_{\Lambda^0} \simeq (1/30)Q^5_{\Lambda^0}$, Q_{Λ^0} being the Q of the decay process $(5\text{-}\mathbf{16}{\cdot}8)_b$ expressed in electron masses ($\simeq 340$), we get:

(2-**16**·9) $$\tau^{(\beta)}_{\Lambda^0} \sim 110 \cdot 10^{-10}\ \text{s}.$$

We thus see that the lifetime for the β decay mode of the Λ^0 is about 50 times larger than the observed one for decay into nucleon and pion. In other words one Λ^0 over 50 should decay according to this process.

The situation, in this respect, is more favourable for the $\Sigma^\pm$ where in 1 case over 20 the β decay should take place (always if the coupling constant is the same as for the neutron and the transition is an allowed one).

For the Ξ^-, if a direct interaction of the kind ($\Xi^- n e^- \nu$) is present with the same coupling constant (see however the next section), the lifetime for the β decay should be [7]:

(3-**16**·9) $$\tau^{(\beta)}_{\Xi^-} \simeq (2 \div 3)\cdot 10^{-10}\ \text{s}$$

which is of the order of that observed for the decay process $\Xi^- \to \Lambda^0 + \pi^-$.

One might at this point try to calculate, one after another, all the decay processes described in the previous section through the Universal Interaction, making detailed assumptions on the form (S, T, V, A, P or a combination of them) of quadrilinear terms which include the hyperons; the divergences make such an analysis not much fruitful and we shall interrupt at this point these considerations.

16·10. – The rules $\Delta S = \pm 1$, $\Delta T = \pm \frac{1}{2}$.

Two points which have not been touched in the past section deserve consideration here: they are: 1) Why does the Ξ^- decay into a $\Lambda^0+\pi^-$ and not into a $n+\pi^-$, a decay which would be also favoured by the phase space? 2) Why does the $K^0_{\pi 2}$ (or better the $K^0_{1\pi 2}$) decay at a rate 100 times larger than that of the $K^+_{\pi 2}$?

As far as the first question is concerned, it is necessary to notice that on account of the small number of Ξ^- observed, the fact that the decay $\Xi^- \to n+\pi^-$ does not take place at all, cannot be considered very well founded; however it is perfectly true that one would have expected the decay to take place in the above way and this, so far, has not been the case.

Gell-Mann and Pais [8] have noticed that the absence of the $\Xi^- \to n+\pi^-$ decay may be the expression of a general property of the weak interactions, namely that in the decays in which only strongly interacting particles intervene the strangeness S may change only by $+$ or -1:

$$\Delta S = \pm 1 .$$

In fact all the other decays of hyperons and heavy bosons into states in which only strongly interacting particles are present, are consistent with the rule $\Delta S = \pm 1$, and, assuming this rule to hold generally the decay $\Xi^- \to n+\pi^-$ is avoided in favour of the decay $\Xi^- \to \Lambda^0+\pi^-$ (*).

According to this point of view decays with $\Delta S = \pm 1$ should be of the first order in the weak decay coupling constant, and decays with $\Delta S = \pm 2$ should be of the second order, and so completely negligible. In a sense this is not an explanation for the absence of the decay in question, but its inclusion in a more general rule which has to be understood; also, a possible criticism with respect to the rule is that it is not very general: it looses meaning in those decays in which leptons are emitted. There, either one attributes a strangeness to the leptons, but this is not easy, if the strangeness has to be related to the third component of the isotopic spin; or one assumes that they have no strangeness, in the same way as one does not attribute an isotopic spin to the photon emitted in an electromagnetic transition between two nuclear levels (which *do* have an isotopic spin); in this last case however the β decay of the neutron would be due to a weak interaction with $\Delta S = 0$, this contradicting the rule $\Delta S = \pm 1$.

(*) It must be said that one of the reasons for attributing strangeness -2 and not -3 to the Ξ in Sect. 11·5 has been just this; although now this strangeness assignment appears to be confirmed by the event mentioned in Sect. 18·2.

Confining oneself however to decays in which only strongly interacting particles are produced, it is immediately seen that the rule $\Delta S = \pm 1$ is equivalent to the rule $\Delta T_3 = \pm \frac{1}{2}$. This suggests, as also remarked by GELL-MANN and PAIS [8] a possible generalization of the rule, which consists in assuming that the weak transitions to final states in which only strongly interacting particles are present, are restricted to satisfy not only $\Delta T_3 = \pm \frac{1}{2}$ but also:

$$\Delta T = \pm \tfrac{1}{2}\,, \tag{1-16.10}$$

T being the total isotopic spin.

Of course to this rule one can apply all the remarks inherent to the weaker rule $\Delta S = \pm 1$ (or $\Delta T_3 = \pm \frac{1}{2}$).

The possible consequences of the rule (1) have been investigated by GATTO, DALITZ, WENTZEL and by several other Authors [9-15]. One of these consequences concerns the second point proposed at the beginning of this section and will be discussed now; others imply limitations to other processes and will be discussed in the next section.

Consider the two pion final states from the decay of the K^+; their isotopic spin must be 1 or 2, since $T = 0$ is not possible, the charge being 1. Since the final wave function must be completely symmetrical, the final state must have $T = 1$ if the spin J of the K is odd and $T = 2$ if it is even.

Consider now the decay of the K_1^0 in two pions. Here the final states with $T = 0$, $T = 1$, $T = 2$ will be accessible, and more precisely, $T = 0$ and $T = 2$ for even J and $T = 1$ for odd J.

Since the initial T of the K is $\frac{1}{2}$ one therefore has:

a) for *even spin*: the decay $K^0_{1\pi 2}$ is allowed by the rule (1) but the decay $K^-_{\pi 2}$ is forbidden, the only possible final state having then $T = 2$;

b) for *odd spin*: both the decay $K^0_{1\pi 2}$ and $K^+_{\pi 2}$ are allowed according to the rule (1) and in fact their rates will be identical.

So, for an even spin of the K, there is in principle the possibility of having, on the basis of (1) a large difference between the rates of decay of $K^+_{\pi 2}$ and $K^0_{1\pi 2}$.

The difference is even too large: the decay of the $K^+_{\pi 2}$ is forbidden. To overcome this situation two possibilities appear: one consists in taking into account the electromagnetic effects, the other in assuming that, even apart from electromagnetic effects the rule $|\Delta T| = \frac{1}{2}$ is not a strict one and that the interactions responsible for the decays have also a component which may change the isotopic spin by $\frac{3}{2}$ or more.

Perhaps [15] the electromagnetic effects are too weak to give rise to a sufficient transition probability to the $T = 2$ state; by electromagnetic effects it is meant here both the emission and the reabsorption of a virtual photon or the effects connected with the π^+, π^0 mass difference.

If this is so, one has to assume that the rule (1) itself must be modified in the sense that the weak interactions contain a dominant part fulfilling (1) but also a part ($6\cdot10^{-2}$ in amplitude) (*) producing transitions with $\Delta T = \frac{3}{2}$.

16·11. – Further consequences of the rule $\Delta T = \pm\frac{1}{2}$.

Other cases to which the rule (1) has been applied include the following:

1) *Branching ratio in the decay of the* K_1^0. – With the pure rule $\Delta T = \pm\frac{1}{2}$ one obtains:

$$\frac{R(K_1^0 \to \pi^0 + \pi^0)}{R(K_1^0 \to 2\pi)} = \frac{1}{3}, \tag{1-16·11}$$

assuming even spin, of course, for the K_1^0. The experiment of the Brookhaven-Columbia group gives for the above ratio 0.14 ± 0.06.

A mixture of $\Delta T = \frac{3}{2}$ interaction (compare the past section, footnote (*)) gives rise to a ratio (1-16·11) in between:

$$0.5\left(1 + 3\sqrt{2}\left|\frac{a_3}{a_1}\right|\right) \qquad \text{and} \qquad 0.5\left(1 - 3\sqrt{2}\left|\frac{a_3}{a_1}\right|\right),$$

depending on the relative phases of a_3 and a_1. For $|a_3/a_1| = 6\cdot10^{-2}$ the limits are 0.62 and 0.38.

2) *Branching ratio for the decay of the* Λ^0. – With the pure $\Delta T = \pm\frac{1}{2}$ rule one has:

$$\frac{R(\Lambda^0 \to n + \pi^0)}{R(\Lambda^0)} = \frac{1}{3}, \tag{2-16·11}$$

which is in agreement with the experimental value [16] of 0.32 ± 0.05 (the assumption is made of the existence of one Λ^0 only).

3) *Ratio between the lifetimes of* Σ^- *and* Σ^+. – Always under the assumption that there is only one Σ^- and one Σ^+ the ratio between their lifetimes with

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(*) This value is obtained as follows [15, 16]: one may write for the probability ratio:

$$\frac{R(K^+ \to \pi^+ + \pi^0)}{R(K_1^0 \to 2\pi)} = \frac{3}{4}\,\frac{|a_3|^2}{|a_1|^2 + |a_3|^2},$$

where a_3, a_1 are the amplitudes of the states with $T = 0$, $T = 2$ respectively. Inserting the observed lifetimes (the value for the lifetime of the K_1^0 is taken to be $0.95\cdot10^{-10}$ s and the branching ratio for decay of the K^+ in $\pi^+ + \pi^0$ is 0.28) we get $|a_3/a_1| = 6\cdot10^{-2}$.

the pure $|\Delta T| = \frac{1}{2}$ rule results:

$$\text{(3-16·11)}\qquad \frac{\tau(\Sigma^-)}{\tau(\Sigma^+)} = \frac{1 + x^2}{3},$$

where x is determined through:

$$\text{(4-16·11)}\qquad f = \frac{R(\Sigma^+ \to p + \pi^0)}{R(\Sigma^+ \to n + \pi^+)} = \frac{2 + x^2 - 2\sqrt{2}x \cos \psi}{1 + 2x^2 - 2\sqrt{2}x \cos \psi}.$$

In (4) ψ is $\delta_{\frac{3}{2}} - \delta_{\frac{1}{2}}$, the difference between the $T = \frac{3}{2}$ and $\frac{1}{2}$ phase shifts in pion nucleon scattering, appropriate to the energy, parity (*), and angular momentum of the final state. According to the experiment described in Sect. **20·6** the experimental value of (3), namely 2.2 ± 0.5, is in contradiction with the one derived by determining x through (4) for any spin parity assignment of the Σ; in such experiments f is ~ 1.

4) *The branching ratio* $(K^+ \to \pi^0 + \pi^0 + \pi^+)/(K^+ \to \pi^+ + \pi^+ + \pi^-)$. – The ratio between the rate of the $K^+_{\pi 3}$ decay into $\pi^+ + \pi^0 + \pi^0$ to that into $\pi^+\ \pi^+\ \pi^-$ is also dependent on the isotopic spin state of the three pions produced.

If the rule $|\Delta T| = \frac{1}{2}$ is fulfilled, then the final three pion state has $T = 1$ and one may obtain for the ratio of $\pi^+ + \pi^0 + \pi^0$ to $\pi^+ + \pi^+ + \pi^-$ decay all the values between 1 and $\frac{1}{4}$ (times 1.295; see later for this factor) [17]. However, if we assume, as indicated in the discussion in Chapter **13** that the $K^+_{\pi 3}$ is pseudoscalar (0 —) then the same discussion shows that the spatial part of the three pion wave function is predominantly symmetrical with respect to interchanges of the momenta of the three pions. If this is so and if the rule $|\Delta T| = \frac{1}{2}$ is fulfilled, the ratio between the two probabilities of decay mentioned above turns out to be determined (+) [9, 15]:

$$\text{(5-16·11)}\qquad \frac{R(\pi^+ + \pi^0 + \pi^0)}{R(\pi^+ + \pi^+ + \pi^-)} = \frac{1}{4}\cdot 1.295 = 0.325\,,$$

(*) If parity is not conserved in the decay of the Σ the final state is a mixture of states with opposite parities and an indeterminate parameter is introduced.

(+) There is only one completely symmetrical function of the isotopic spin with $T = 1$, $T_3 = 1$; it is [15]:

$$\chi = \frac{\sqrt{5}}{3}\chi_0^1 + \frac{2}{3}\chi_2^1\,,$$

where

$$\chi_0^1 = \frac{1}{\sqrt{3}} u_3(u_1 w_2 + u_2 w_1 - v_1 v_2)\,,$$

$$\chi_2^1 = \frac{1}{\sqrt{60}} [u_3(u_1 w_2 + u_2 w_1 + 2v_1 v_2) - 3v_3(u_1 v_2 + u_2 v_1) + 6w_3 u_1 v_2]\,,$$

where u, v, w are the isotopic spin functions of π^+, π^0, π^- respectively. The final three

where the factor 1.295 is the ratio between the available phase space volumes for the two cases, due to the π^0-π^+ mass difference.

Presently (Sect. **1**·3) the ratio (5) is in reasonable agreement with the above value.

The ratio (5) may depend sensitively on the $|\Delta T| = \frac{5}{2}$ part of the decay interaction, if it is present in some amount; on the contrary it is independent of the amount of the $|\Delta T| = \frac{3}{2}$ interaction, if the spatial part of the three pion state formed has to be symmetrical.

5) *Branching ratio* $K^+_{\pi 3}/K^+_{\pi 2}$. – The problem of the two bosons has already been extensively discussed and various ideas illustrated. Here we simply want to notice that if the $K^+_{\pi 3}$ and the $K^+_{\pi 2}$ are different decay modes of the same particle the rule $|\Delta T| = \frac{1}{2}$ might be relevant to explain their branching ratio. The problem is in fact that of explaining why the branching ratio $\sim \frac{1}{5}$ of $K^+_{\pi 3}$ to $K^+_{\pi 2}$ is so large as compared to the phase space available; in fact for a radius of the K equal to $\hbar/M_K c$ one gets a phase space ratio $\sim 2 \cdot 10^{-4}$ and for a radius equal to $\hbar/m_\pi c$ a ratio of the order $7 \cdot 10^{-3}$ (Table I-**16**·8). But, if a rule like the one mentioned above is effective, the decay probability of the $K^+_{\pi 2}$ is reduced by a factor ~ 400 with respect to that of the $K^0_{\pi 2}$ which may be thought as taking place without limitations; on the other hand the rule $|\Delta T| = \frac{1}{2}$ does not affect the $K^+_{\pi 3}$ decay rate appreciably. So the ratio between the $K^+_{\pi 2}$ and $K^+_{\pi 3}$ decay rates may be understood. Of course if the $K^+_{\pi 2}$ and $K^+_{\pi 3}$ decays proceed from the same initial K through the mechanism of non conservation of parity, one might also think that the interaction leading to states of parity minus ($K^+_{\pi 3}$) is much stronger in this decay that the part of interaction leading to states of parity plus ($K^+_{\pi 2}$); but this would not be much in line with what seems to be the case in the β interactions.

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pion state is therefore $F(p_1, p_2, p_3)\chi$ with F symmetrical and the probability of decay in the mode $\pi^+ + \pi^+ + \pi^-$ is obtained adding the squares of the projections of this function over the states $u_1u_2w_3$, $u_1w_2u_3$, $w_1u_2u_3$. Similarly the probability of decay into the state $\pi^+ + \pi^0 + \pi^0$ is obtained making the square of the projection on the state $u_3v_1v_2$. In this way the ratio $\frac{1}{4}$ is obtained.

We also report here the completely symmetrical T function with $T = 1$, $T_3 = 0$; it is useful for the considerations of the end of Sect. **14**·8; it is $(\sqrt{5/3})\chi_0^{1'} + \frac{2}{3}\chi_2^{1'}$ where:

$$\chi_0^{1'} = \frac{1}{\sqrt{3}}(u_1w_2 + u_2w_1 - v_1v_2)v_3\,,$$

$$\chi_2^{1'} = \sqrt{\frac{3}{20}}\left((w_1v_2 + w_2v_1)u_3 - (u_1v_2 + u_2v_1)w_3\right) - \frac{1}{\sqrt{15}}(u_1w_2 + u_2w_1 + 2v_1v_2)v_3\,.$$

16·12. – Electromagnetic weak processes.

We shall list here the results of some calculations concerning the electromagnetic decays of the new particles. For more detailed discussions we refer to the original papers.

The radiative decays of the heavy bosons have been considered by DALITZ [18]. Such processes may take place in two ways: *a*) the photon may be emitted by one of the *final* charged products, by a bremsstrahlung process *b*) the photon may be emitted « directly » that is during some other intermediate state of the decay process of the particle in question. Processes of kind *a*) may be calculated rather accurately, their probability being usually 1/137 times smaller than the corresponding non radiative decay and the photon spectrum being a dk/k one.

Processes of kind *b*) are more difficult to estimate. Of course if both processes may take place with comparable matrix elements interferences may also appear.

The processes considered by DALITZ are:

$$K^+ \to \pi^+ + \pi^+ + \pi^- + \gamma \qquad (K_{\pi 3/\gamma}) \tag{1-16·12}$$

$$K^+ \to \pi^+ + \pi^0 + \gamma \qquad (K_{\pi 2/\gamma}) \tag{2-16·12}$$

$$K^+ \to \pi^+ + \gamma \qquad (K_{\pi/\gamma}) \tag{3-16·12}$$

$$K^+ \to \pi^+ + e^- + e^+ \qquad (K_{\pi/ee}) \tag{4-16·12}$$

If the spin of the decaying particle is zero the $K_{\pi 3/\gamma}$ process is dominantly of kind *a*) because the centrifugal barriers reduce much the probability of *b*). The bremsstrahlung spectrum obtained is plotted in the Fig. 1-16·12 (where also the case of spin 2 — is reported).

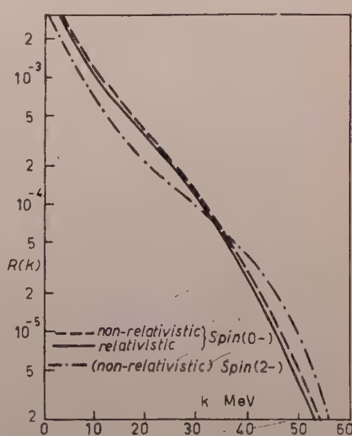


Fig. 1-16·12. — (From [18]). The probability $R(k)$ for emission of a photon exceeding k MeV.

As far as the process $K_{\pi 2/\gamma}$ is concerned (compare also [19]) the bremsstrahlung process will again be α times more unfrequent than the $K_{\pi 2}$ process. The direct process, for a K^+ having spin zero, will produce a state 1 — of the two pions, thus emitting an electric dipole photon if the initial K had parity + and a magnetic dipole photon if it had parity —. One may expect the competition between $K_{\pi 2/\gamma}$ (direct transitions) and $K_{\pi 2}$ or

$K_{\pi 3}$ to be $10^{-4} \div 10^{-5}$ for a K radius $R \sim \hbar/M_K c$:

$$\frac{\text{Rate }(2\pi+\gamma)\text{ (dir. trans.)}}{\text{Rate }(3\pi)} \simeq 0.01\,(\mu R)^4 \quad \text{(spin zero)},$$

where μ is the inverse pion Compton wave length.

Finally, always considering the case of spin zero for the K the process (3-**16**·2) is forbidden and the process (4-**16**·2) can be seen to be very improbable.

For higher spins of the decaying K the radiative processes acquire much more importance. For instance for a spin parity assignment 2 — the ratio $R(\pi+\gamma)/R(3\pi)$ may be, according to DALITZ, as high as 24 although there are, of course, uncertain factors in the calculation; as already noticed this is a possible argument against assigning a high value of the spin to the $K_{\pi 3}$.

The radiative decays of the hyperons, such as $\Sigma^+ \to p+\gamma$, $\Sigma^0 \to n+\gamma$ have been discussed in [12]; they are quite unimportant from a practical point of view and we refer for them to the mentioned paper.

REFERENCES (CH. 16, Sects. 1 to 7)

[1] J. TIOMNO and J. A. WHEELER: *Rev. Mod. Phys.*, **21**, 153 (1949).
[2] O. KLEIN: *Nature*, **161**, 897 (1948).
[3] G. PUPPI: *Nuovo Cimento*, **5**, 587 (1948); **6**, 194 (1949).
[4] T. D. LEE, M. ROSENBLUTH and C. N. YANG: *Phys. Rev.*, **75**, 905 (1949).
[5] E. FERMI: *Elementary Particles* (Yale, 1951).
[6] L. MICHEL: *Progress in Cosmic Ray Physics* (1952), p. 124; also H. YUKAWA: *Rev. Mod. Phys.*, **21**, 474 (1949).
[7] E. R. CAIANIELLO: *Phys. Rev.*, **81**, 625 (1951).
[8] L. MICHEL: *Thèse.*
[9] M. FIERZ: *Zeits. f. Phys.*, **104**, 553 (1937).
[10] E. R. CAIANIELLO: *Nuovo Cimento*, **8**, 479 (1951).
[11] E. J. KONOPINSKY: *Rev. Mod. Phys.*, **27**, 254 (1954); C. S. WU: *Proceedings Glasgow Conf.* (1954).
[12] L. MICHEL and A. WIGHTMAN: *Phys. Rev.*, **93**, 354 (1954).
[13] C. CHRITCHFIELD and E. WIGNER: *Phys. Rev.*, **60**, 412 (1941).
[14] C. N. YANG and J. TIOMNO: *Phys. Rev.*, **79**, 495 (1950).
[15] E. J. KONOPINSKY and H. MAHMOUD: *Phys. Rev.* 92 (1953).
[16] B. D'ESPAGNAT: *Compt. Rend.*, **228**, 744 (1949).
[17] J. STEINBERGER: *Phys. Rev.*, **76**, 1180 (1949).
[18] M. RUDERMANN and R. FINKELSTEIN: *Phys. Rev.*, **76**, 1458 (1949).
[19] S. LOKANATHAN and J. STEINBERGER: *Suppl. Nuovo Cimento*, **2**, 151 (1955).
[20] G. MORPURGO: *Nuovo Cimento* **5**, 1159 (1957).
[21] S. B. TREIMAN and H. W. WYLD jr.: *Phys. Rev.*, **101**, 1552 (1956).
[22] E. T. EGUCHI: *Phys. Rev.*, **102**, 879 (1956).

REFERENCES (Ch. 16, Sects. 8 to 12)

[1] K. Iwata, S. Ogawa, H. Ogonoki, B. Sakita and S. Oneda: *Progr. Theor. Phys. (Japan)*, **13**, 19 (1955).

[2] S. Oneda and A. Wakasa: *Nucl. Phys.* **1**, 445 (1956); S. Oneda: to be published.

[3] A. H. Rosenfeld and M. L. Stevenson: U.C.R.L. 3314 (unpublished).

[4] N. Dallaporta: *Nuovo Cimento*, **1**, 962 (1955).

[5] J. Hornbostel and E. O. Salant: *Phys. Rev.*, **102**, 552 (1956).

[6] G. Costa and N. Dallaporta: *Nuovo Cimento*, **2**, 519 (1955).

[7] G. Morpurgo: *Proceed. of the Turin Conf.* (Sept. 1956).

[8] M. Gell-Mann and A. Pais: *Proceed. of the Glasgow Conf.* 1954 (London, 1955); M. Gell-Mann: *Suppl. Nuovo Cimento*, **4**, 848 (1956).

[9] G. Wentzel: *Phys. Rev.*, **101**, 1214 (1956).

[10] R. Gatto: *Nuovo Cimento*, **3**, 318 (1956).

[11] M. Kawaguchi and K. Nishijima: *Progr. Theor. Phys. (Japan)*, **15**, 180 (1956).

[12] C. Iso and M. Kawaguchi: *Progr. Theor. Phys. (Japan)*, **16**, 177 (1956).

[13] B. D'Espagnat and J. Prentky: *Nuovo Cimento*, **3**, 1045 (1956).

[14] G. Takeda: *Phys. Rev.*, **101**, 1547 (1956).

[15] R. Dalitz: *Proc. Phys. Soc.*, A **69**, 527 (1956).

[16] M. Schwartz, J. Steinberger, R. Plano and N. Samios: to be published.

[17] R. Dalitz: *Proc. Phys. Soc.*, A **66**, 710 (1953).

[18] R. Dalitz: *Phys. Rev.*, **99**, 915 (1955).

[19] S. B. Treiman: *Phys. Rev.*, **95**, 1360 (1954).

PART III

INTERACTION PROPERTIES OF THE NEW PARTICLES WITH THE OLD ONES

Introduction.

The purpose of this chapter is to compare the classification scheme of Gell-Mann and Nishijima with experiments; more precisely, recalling that the predictions of the scheme as far as the strong processes are concerned, are summarized in the Fig. 1-**11**·7 and 2-**11**·7 we may ask:

1) Is there any evidence that the reactions described in the Fig. 1-**11**·7 and 2-**11**·7 actually take place?

2) Is there, on the other hand, any evidence for reactions which contradict the classification scheme?

At present the answer is positive for the first and negative for the second question.

We shall now examine the single processes and try to justify the above statements; it is important in this analysis to recall that one thing is to confirm generically the validity of the associated production and a different thing is to confirm the particular scheme of associated production of Gell-Mann and Nishijima. As we have said, up to now, all seems to be in favour not only of the associated production, but also of the specific scheme of Gell-Mann and Nishijima.

There are four kinds of interaction phenomena which have been studied; production, absorption, scattering and binding of the new particles with or from the old ones. We shall begin with the production processes and consider separately the production in H and in dense materials, both by artificial and cosmic radiations. We next examine the interactions of positive and negative K with the nucleons and finally the interaction of hyperons with nuclei, in particular the hyperfragments.

The first and very strong evidence for the validity of a scheme of associated production came from a certain set of experiments (*) which showed that at energies of the bombarding particles less than the one to be expected according to the scheme of Gell-Mann and Nishijima for the associated production of the new particles no production at all of new particles took place; we may call such experiments the threshold experiments; before proceeding systematically in the way just outlined we mention briefly such experiments.

Thresholds for several processes (*in* GeV).

Process	π nucleon	π nucleus	γ nucleon	γ nucleus	nucleon nucleon	nucleon nucleus
$\Lambda^0+\pi$	0.22	0.16	0.37	0.29	—	—
Λ^0+N	—	—	—	—	0.37	0.18
$\Lambda^0+\Lambda^0$	—	—	—	—	0.77	0.48
$\Lambda^0+K+(N)$	0.75	0.59	0.91	0.72	1.57	1.10
$\Sigma+K+(N)$	0.89	0.69	1.04	0.82	1.78	1.30
$K+\bar{K}+N+(N)$	1.36	1.06	1.51	1.19	2.50	1.84
$\Xi^-+K+\bar{K}+(N)$	2.21	1.73	2.37	1.87	3.73	2.82
$\Lambda^0+\Lambda^0+N+(N)$	4.72	3.76	—	—	7.10	5.62
$\Sigma+\Sigma+N+(N)$	5.22	4.3	—	—	7.75	6.2

For the calculation of thresholds in the laboratory system the following formulas apply:
a) collision against a nucleon at rest (mass m_1) of a particle of mass m_2

$$\text{Threshold} = T^0 = \frac{Q(Q+2(m_1+m_2))}{2m_1} \qquad Q = \sum m_{\text{products}} - m_1 - m_2\,,$$

b) collision against a nucleon moving in the direction opposite to that of the incoming particle with a momentum p_1 (p_1 represents the Fermi momentum of the collided particle inside a nucleus)

$$\text{Threshold} = T' = T^0 - \frac{p_1}{m_1}\sqrt{T^0(T^0+2m_2)} + \frac{p_1^2}{m_1^2}\,\frac{T^0+m_2}{2}\,,$$

(*) They are the following (in parenthesis is reported the nature and energy of the bombarding particle): [1] (310 MeV γ bremsstrahlung on carbon), [2] (290 MeV γ bremsstrahlung), [3] (430 MeV protons on carbon and 345 MeV protons on carbon), [4] (670 MeV protons on carbon), [5] (1.5 GeV protons on hydrogen), [6] (810 MeV protons on hydrogen).

The thresholds for several processes corresponding to different kinds of collisions are reported in the Table of pag. 270; the first three lines refer to processes which are impossible according to the scheme of Gell-Mann and Nishijima. The other refer instead to processes which are possible. The last two refer to reactions in which a hyperon and an anti-hyperon are created, and have been inserted for completeness. For each process two thresholds are given, one referring to a collision of the bombarding particle with a free nucleon, the second to a collision with a nucleon bound in a nucleus. In such case the threshold is somewhat lower on account of the Fermi motion of the struck nucleon. The figure given here corresponds to the assumption that the Fermi momentum of such nucleon is 216 MeV/c and is directed opposite to the impinging particle.

The thresholds given in the table are defined as the minimum kinetic energies of the impinging particle in tha laboratory system (expressed in GeV) needed in the collisions between the particles specified in the first row, to create the particles specified in the first column; there the nucleon in parenthesis is to be inserted in the case of collisions induced by nucleons only.

It is apparent from the table that already at very low energies one should have production of Λ^0 if the production were not associated; for the process $\pi+\text{nucleus} \to \Lambda^0+\pi^0$ the threshold is only 160 MeV and for the process $\text{N}+\text{nucleus} \to \Lambda^0+\text{nucleus}$ it is 180 MeV. Also a γ-ray of 290 MeV might create a Λ^0.

If the production were associated but violating the scheme of Gell-Mann and Nishijima a Λ^0 might be created at 480 MeV in the process $\text{N}+\text{nucleus} \to \Lambda^0+\Lambda^0$ and at 630 MeV in the process $\text{N}+\text{nucleus} \to \Sigma^+ + \Lambda^0$.

On the contrary, if the Gell-Mann and Nishijima scheme is obeyed, the production of a Λ^0 (or of a K) has its threshold at 590 MeV (in a collision pion-nucleus) and at 1.1 GeV (in a collision nucleon-nucleus).

All the experiments listed in (*) made with various detectors, agree in showing that at energies below the threshold indicated by the scheme of Gell-Mann and Nishijima, the production cross-section for new particles is certainly less than 10^{-2} (and in the most part of the cases certainly less than 10^{-4}) times the cross-section for production of a pion in the corresponding conditions. As we have said, no new particle was ever found in such experiments.

Chapter 17.

Production reactions in H.

17·1. – A list of possible reactions.

The reactions listed below are expected to be possible in H, limiting to the minimum number of final products which can satisfy the strangeness conservation law:

a) incident pions:

(1-17·1)	$\pi^- + p \to \Lambda^0 + K^0$	0.75 GeV	$(+\pi$ 1.01)
(2-17·1)	$\to \Sigma^- + K^+$	0.89 »	$(+\pi$ 1.15)
(3-17·1)	$\to \Sigma^0 + K^0$	$\sim$ 0.89 »	»
(4-17·1)	$\to \Xi^- + K^+ + K^0$	2.21 »	
(5-17·1)	$\to K + \overline{K} + n$	1.36 »	
(6-17·1)	$\pi^+ + p \to \Sigma^+ + K^+$	0.89 »	$(+\pi$ 1.15)

The *approximative* threshold energies are reported near each of the reactions. Of course if one considers reactions in which a pion may be among the final products the variety considerably increases; the addition of a pion on the right hand side of each of the reactions (1), (2), (3), (6) raises its threshold to the value given in parenthesis.

We remark:

1) That in the energy interval 0.89 to 1.15 GeV the creation of a Σ^+ is strictly forbidden.

2) That in the same interval of energies the collision of a π^+ with a proton may give rise only to a (Σ^+, K^+) pair; this last remark shows that the study of the collisions of Σ^+ with protons might provide a very definite and unambiguous confirmation of the theoretical scheme.

Beyond 1.15 MeV the addition of a pion on the right hand side of (2), (3) and (6) is possible, so the two above statements are no longer strictly true; however the production of a pion considerably reduces the phase space of the final products and the two statements should remain true with good approximation up to some hundreds of MeV beyond 1.15 GeV.

b) incident protons:

$$
\text{(7-17·1)}\qquad
\left\{
\begin{array}{lll}
p + p \to \Lambda^0 + p & + K^+ & \sim 1.57\ \text{GeV} \\
 \to \Sigma^0 + p & + K^+ & \sim 1.78\ \text{»} \\
 \to \Sigma^+ + p & + K^0 & \sim 1.78\ \text{»} \\
 \to \Sigma^+ + n & + K^+ & \sim 1.78\ \text{»} \\
 \to K + \overline{K} & + N + N & \sim 2.50\ \text{»} \\
 \to \Xi^- + K^+ & + K^+ + p & \sim 3.73\ \text{»}
\end{array}
\right.
$$

with their approximative thresholds.

The np collisions will not be explicitly written down.

Very little has been published [7, 8] on the reactions induced by nucleons. 10 events have been described by FOWLER at the Rochester Conference 1956 [9]. The fact that in many cases a pion is found between the final products makes the analysis more difficult.

Also some preliminary results of production in H from π^- with an energy 4.5 GeV have been reported by the same Authors [9, 10].

They describe 4 events in which there is production of the new particles; in all of them there are four or more final products. One of them may perhaps be interpreted as an associated production of a $\overline{K}^0$ and a K^0.

Most of the production events in H reported up to now refer to collisions of artificially produced pions of energy from 1 to 1.4 GeV in cloud chamber [11-13] and in bubble chamber experiments. The description of such experiments will be the purpose of the next sections.

17·2. – Cloud chamber experiments with artificial pions.

These experiments have furnished the first direct evidence for associated production.

We begin by giving a general description of the experiments; next we shall limit to consider the events produced in H. The observed events may be classified in four categories:

1) Events produced in dense material (the wall of the chamber in most cases) in which the primary is not observed and in which two new particles are observed (either directly if charged, or by their decay if neutral) whose lines of flight intersect;

2) events produced in dense material in which the primary is not observed and only one of the new particles is observed in the chamber;

3) events produced in the gas in which one may observe the primary and two new particles (referred to as $(g2)$);

4) events produced in the gas in which the primary and only one secondary particle are observed, whose line of flight is coplanar with that of the primary $(g1)$.

A suffix h will be added to the above symbols if a magnetic fields is present and at least part of the momenta of the charged particles may thus be measured; it is evident that even with a magnetic field one has to be particularly lucky to measure all the momenta of the charged particles because sometimes the tracks are too short.

Fig. 1, 2, reproduce several events to be described later; the event reported in Fig. 1 belongs to the category $(g1)_h$, the one in Fig. 2 to the category $(g2)_h$.

Fig 1-17·2. – (From [11]) (see text)

The first experiments were performed by Fowler *et al.* [11, 12,] using a pion beam of 1.37 GeV nominal energy and both a pressurized (18 atm) chamber and a diffusion one filled with hydrogen; the first was operated with a magnetic field of 11000 gauss, the second without; another set of observations at 1 GeV pion energy is the one by Walker [13, 14] also with an hydrogen filled cloud chamber equipped with magnetic field; the observations of production events in hydrogen, using the cloud chamber are very lengthy only 12 events being interpreted as production of new particles over a total of ~ 60000 photograms; the observation of H-produced events with the bubble chamber appears to be extremely more fruitful (compare Sect. **17·3**).

From now on we refer only to events produced in hydrogen; the production in dense materials will be treated in Chapter **18**.

Referring to the above mentioned experiments, the following example [11] (the first event interpreted as providing direct evidence for associate production) will show how the analysis is performed; the event in question is of the $(g1)_h$ class, and is reproduced in Fig. 1. We shall discuss it in detail because it will serve as an example for the following.

Fig. 2-17·2. – (From [12], Photograph of a 1.5-Bev π^- producing two neutral V particles in a collision with a proton. Tracks 1a and 2a, believed to be proton and π^-, respectively, are the decay products of a Λ^0. A K^0 is probably seen to decay into π^+ (1b) and π^- (2b).

Track a is most probably a pion of the incident beam because it is parallel to the other pions; it stops at A; its momentum cannot be measured directly because the visible track is too short, so it has to be assumed equal to the average momentum of the pion beam: 1.37 GeV, as mentioned above. From accurate measurements on the angles, tracks b, c and point A resulted coplanar; it was also possible to measure the momenta of b and of c which were found to be: the first (480 ± 80) MeV/c, the second (210 ± 70) MeV/c. Furthermore the ionization of b is three times the minimum, that of c less than 1.5 the minimum; b is positive, c negative; so b is identified as a proton, and c which, on account of its ionization has a mass less than 410 m_e must be a π^-. Assuming then that the two tracks b and c represent the two body decay of Λ^0, a value of 51 MeV is found for its Q in agreement, within the errors, with the known value of ~ 37 MeV.

Better values of the momenta of b and c may then be recalculated from the given angles, assuming the 37 MeV value for the Q of the Λ^0; they result 460 MeV/c for the proton and 180 MeV/c for the π^-; they agree with the previous ones, within the errors. The total energy of the Λ^0 may then be calculated; it is 1.26 GeV and its momentum is 610 MeV/c; to conserve energy and momentum at least another particle must be produced at A; we shall assume that just one particle is produced; then the conservation of momentum and energy allows to determine the mass of the other particle: (669 ± 30) MeV. This value is higher than, but of the order of magnitude of, the mass of a heavy boson; we are therefore entitled to assume, that this is a case of associated production; a possible reason for such a high mass value of the boson will

be given later; it may be mentioned that the time of flight of the Λ^0 may be immediately calculated: it is $4\cdot10^{-11}$ s; from the geometry of the chamber we deduce also that, either the assumed neutral boson decays into two neutral pions, or it has a time of flight larger than $3\cdot10^{-10}$ s.

A similar analysis may be made for all the other events observed in references [11-14]; of course each case has to be discussed separately, and all the possible interpretations examined; if there is no magnetic field the interpretation is much more difficult; one cannot measure the energies and signs of the decay products; one may often distinguish, say, a Λ^0 decay from a $K^0_{\pi2}$ decay by the knowledge of angles and ionization, but it is impossible to measure the mass of the decaying particle from the energies of the decay products (and the angles) as done above; the only thing which in such case may be done is to assign to the particle, identified through its ionization, the value of the mass which it must have, to *assume* that the production process is a two body one and to determine by the conservation of momentum and energy the mass of the other (observed or unobserved) particle.

The results of the analysis performed according to the above criteria are reported for the events observed in the above ref. in the following Table I; in column 1 the kind of event is reported; in column 2 and 3 the measured, observed or assumed produced particles assuming a two body process; in column 4 and 5 the Q (in MeV) of the particles in question derived from the kinematics in the way sketched above; when the Q of one of the particles has been assumed (on the basis of the fact that the nature of the particle has been recognized by angles and ionization) but no independent check of the Q value in question is possible, the value has been bracketed.

TABLE I-17.2.

Case	Type of event	Particles		Q(Y)	Q(K)
$F_1 = A$	$(g1)_h$	Λ^0 measured	K^0 assumed	51 ± 20	375 ± 30
$F_2 = B$	$(g1)_h$	Λ^0 observed	K^0 assumed	(37)	330 ± 40
$F_3 = D$	$(g2)_h$	Λ^0 observed	K^0 observed	27 ± 11	233 ± 41
$F_4 = E$	$(g2)_h$	Σ^- measured	K^+ assumed	$\geqslant 50$	(214)
$F_5 = G$	$(g1)$	Λ^0 observed	K^0 assumed	(37)	$\geqslant 290$
$F_6 = H$	$(g1)$	Λ^0 assumed (?)	K^0 observed (?)	$\geqslant 37$	(214)
$F_7 = I$	$(g1)$	Λ^0 assumed	K^0 observed	240 ± 140	(214)
$F_8 = J$	$(g2)$	$\Sigma^\pm$ observed	$K^\mp$ observed	~ 115	(214)
$F_9 = K$	$(g2)$	$\Sigma^\pm$ observed	$K^\mp$ observed	(115)	(214)
W_1	$(g2)_h$	Λ^0 observed	K^0 observed	37^{+10}_{-5}	200 ± 15 or 309 ± 3
W_2	$(g2)_h$	Λ^0 observed	K^0 observed	43 ± 5	195 ± 20 or 226 ± 6
W_3	$(g2)_h$	Λ^0 observed	K^0 observed	41 ± 5	216 ± 20
W_4	$(g1)_h$	Λ^0 observed	K^0 assumed	22 ± 15	220 ± 40
W_5	$(g1)_h$	Λ^0 observed	K^0 assumed	(37)	182 ± 50

In Table I a particle is said to be « measured » or « observed » when the data are not only sufficient to recognize its nature on the basis of ionization measurements, but they are also such as to allow an independent determination of its mass or Q value; there is a slight distinction between measured and observed particles, namely that in the first case the curvature and length of the tracks are such that the momenta of the decay products of the particle in question may be roughly measured independently, as is the case for the event discussed in detail in the past section, in the second not; measured particles imply all a magnetic field, of course.

Though the statistics are very limited all the events reported are in agreement with the scheme of Gell-Mann; no case of positive hyperon which (compare Sect. **17**·1) would have been in contradiction with such scheme has been observed; it must be mentioned however that, with magnetic field, just two cases of Σ^- have been observed.

Of course in the cases, which are the most part, in which only one particle is effectively observed to decay in the chamber, the confirmation of the associated production scheme has to be looked for in the fact that the mass of the other unobserved particle, necessary to balance the energy and the momentum on the basis of an assumed two body production scheme, turns out to be consistent with, if not exactly equal to, that of a hyperon or respectively a heavy meson.

Having interpreted the above data as indicating associated production, one may ask: 1) why so often, one of the particles is not detected; the only possible answer is that, either it does not decay in the visible region of the chamber or decays into neutral products only (e.g. $K^0_{\pi 2} \to \pi^0 + \pi^0$; $\Lambda^0 \to n + \pi^0$); this question will be discussed in the next section when larger statistics will be considered; 2) why the masses of the $K^0_{\pi 2}$ reported in the Table I-**17**·2 are often larger than the known value. Inspection of the table shows that this is the case for the events A, B, G, I and possibly W_1. To get an agreement, according to FOWLER *et al.*, « one should have to assume unreasonably small momenta for the incident π^- for all of these cases. There are two possible explanations of the discrepancy from the known Λ^0 and $K^0_{\pi 2}$ masses in events A, B, G, I: the first is that possibly neutral pions in addition to heavy bosons and hyperons may have been produced; notice that in none of the four cases it was possible to ascertain that only two particles were produced since only one neutral particle was seen to decay and therefore only one line of flight was established; the threshold for events of the kind $\pi^- + p \to \to \Lambda^0 + K^0 + \pi^0$ and the available phase space is correspondingly decreased, but still there is no reason to exclude such events. The second possibility consists in making use of the neutral Σ^0 predicted in the Gell-Mann and Nishijima scheme, which should have a mass $\sim 2300\ m_e$, and in assuming that the events in question are to be regarded as examples of $\Sigma^0 + K^0$ rather than of

Λ^0+K^0 production, the Σ^0 decaying almost immediately into a Λ^0 and a γ-ray:

$$\pi^- + p \to \Sigma^0 + K^0 , \qquad \Sigma^0 \to \Lambda^0 + \gamma .$$

It is possible to show that under this assumption the recalculated value of the $K^0_{\pi 2}$ mass compares favourably with the known value in all the above cases »; this has been the first evidence for the existence of the Σ^0; for the proof of its existence [15] compare the Ch. **7**.

17·3. – The Brookhaven Columbia bubble chamber experiments.

As we have said, a much larger statistics may be collected making use of the bubble chamber technique; using a 12″ diametre bubble chamber exposed to the pion beam of Brookhaven Cosmotron, SCHWARTZ *et al.* [16] observed 210 production events of new particles in H in about 20 000 photograms. Two are reproduced in Fig. 1-**17**·3, 2-**17**·3.

Three beam momenta were employed: 980 MeV/c, 1300 MeV/c and 1430 MeV/c (all $\pm 1\%$). The detector was a propane (C_3H_8) bubble chamber of such dimensions that about $80\sim 90\%$ of the Λ^0 and of the K^0 particles produced in it also decay inside the chamber. A magnetic field is present (average value 13 400 gauss).

Here we shall give the results of this experiment as far as the following topics are concerned:

a) evidence for associated production, and

b) evidence for two neutral boson components.

Other most important information from these experiments has been already mentioned at the appropriate places:

1) neutral decay modes of Λ^0 (Sect. **4**·2) and of $K^0_{\pi 2}$ (Sect. **3**·5);

2) lifetimes of Λ^0 (Sect. **4**·1.4), of $K^0_{\pi 2}$ (Sect. **3**·4.5) and of the longlived boson component;

3) non-observation of three body decays of Λ^0 (Sect. **4**·1.1);

4) discussion of the rule $|\Delta T| = \frac{1}{2}$ (Sect. **16**·11);

5) parity doublet question (Sect. **14**·5).

a) Evidence for associated production. – Up to now the photograms taken in this experiment have been examined only for events in which no charged new particle was produced. Therefore the evidence for associated production which this experiment gives refers, at the moment, only to the reaction: $\pi^- + p \to \begin{Bmatrix}\Lambda^0\\ \Sigma^0\end{Bmatrix} + K^0$; 168 events were observed in which both a Λ^0 and a $K^0_{\pi 2}$ are produced, either in the carbon or in the hydrogen of the C_3H_8. Moreover 251 events were detected in which only one Λ^0 was observed and 109 events in which only a $K^0_{\pi 2}$ is observed.

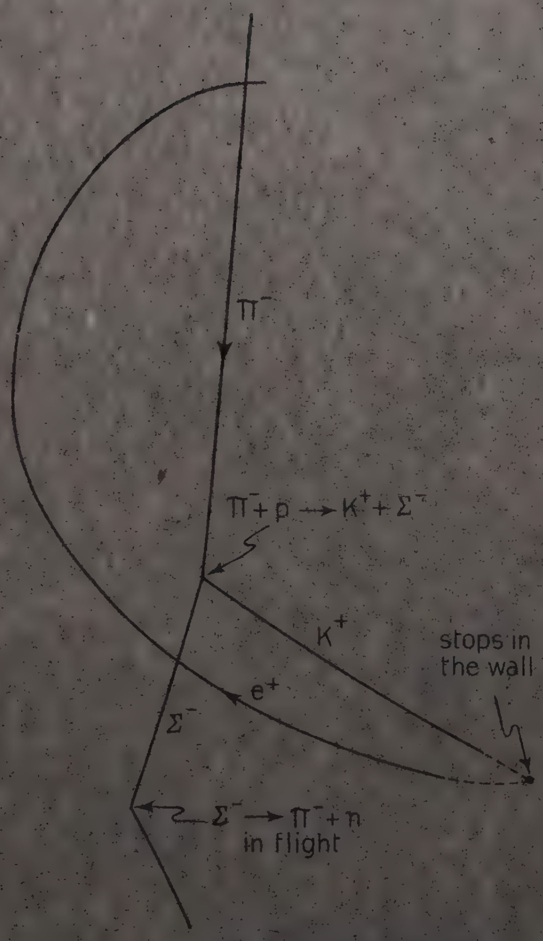
The identification of a Λ^0 or of $K^0_{\pi 2}$ is made in a way completely similar to that of the event discussed in Sect. **17**·2; consider a V^0 event (the plane of which must contain, of course, the line connecting the point of disappearance of a π^- with the origin of the V fork); one may determine the energy of the two branches of the fork, knowing the angle between them and assuming the Q value to be the one corresponding to a Λ^0 or to a $K^0_{\pi 2}$. Also one may determine (independently) this energy from the curvature produced by the magnetic field.

The energy so determined must then [17] « agree with that expected from the observed production angle of the supposed Λ^0 or $K^0_{\pi 2}$ and from the known production kinematics. In the events in which two V's are observed, since the procedure above is applicable for each V, the event is well overdetermined ».

It is clear that the above procedure allows in many cases to separate the production events in H through the reaction $\pi^- + p \to \Lambda^0 + K^0$ from those produced in Carbon through the same reaction and from those produced in Carbon or in Hydrogen through the reaction $\pi^- + p \to \Sigma^0 + K^0$; particularly so when both the Λ^0 and $K^0_{\pi 2}$ are seen to decay in the chamber; according to [16] $\sim 40\%$ of the events are produced in Hydrogen, therefore providing direct evidence for associated production in H.

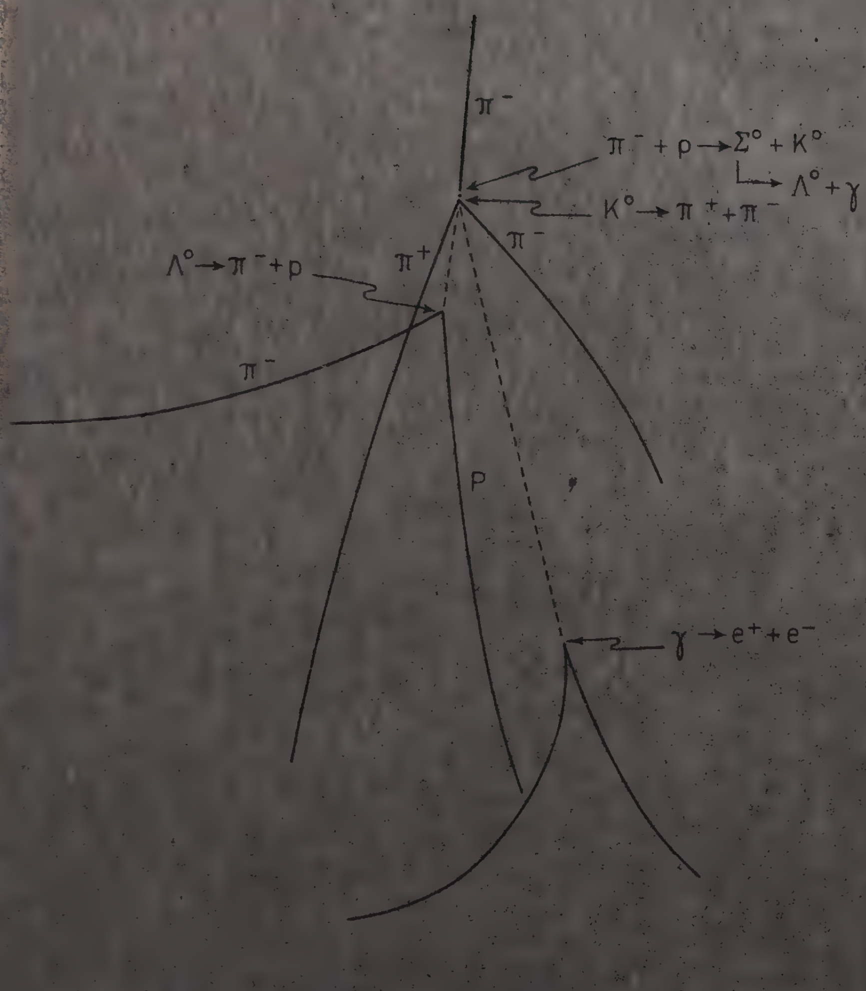
b) Evidence for two neutral boson components. – If associated production is valid and if one could assume that 1) only two neutral boson components exist; 2) the shortlived component K^0_1 and the Λ^0 which are produced all decay in the chamber; 3) none of the longlived bosons decays in the chamber; 4) Λ^0 and K^0_1 decay only into visible prongs; 5) detection efficiencies are 100%; then one should expect to find: α) no event in which only a $K^0_{\pi 2}$ is observed to decay in the chamber without an accompanying Λ^0; β) in half the events in which a Λ^0 is observed a K^0 should be observed and in half not.

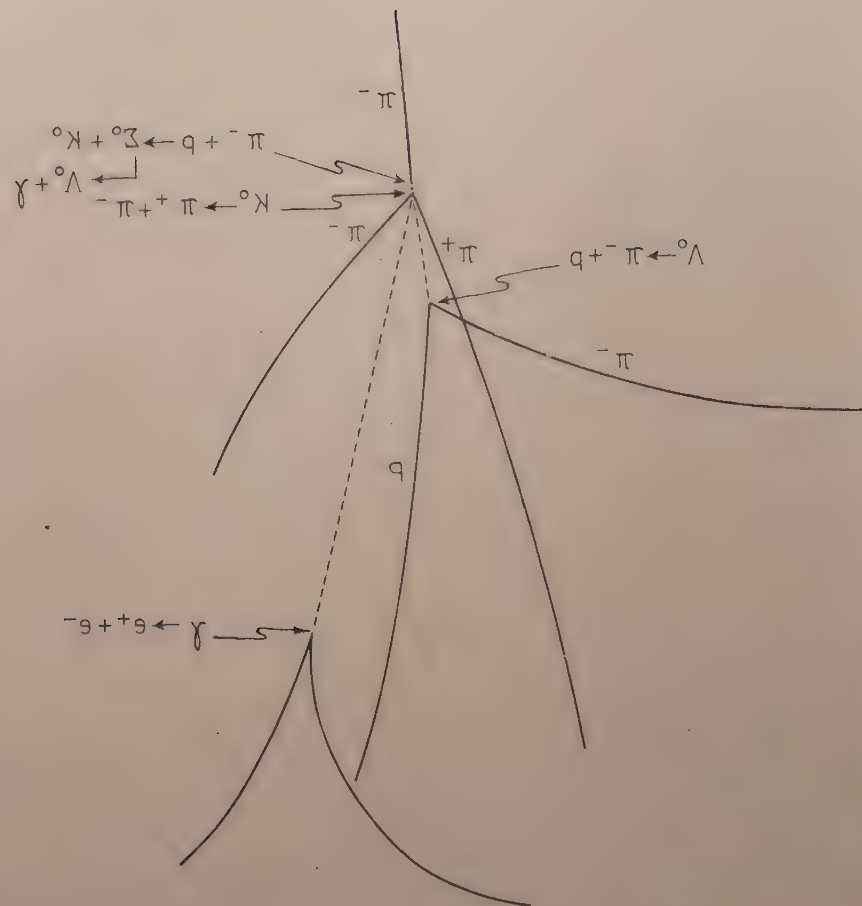
The figures given above show that the situation is different; however, if one determines, through the electron pairs due to γ-ray conversion in the way explained in Sect. **3**·5 and **4**·2, the fraction of the Λ^0 and K^0 which decay in neutral modes; and if one takes into consideration the detection efficiency, and the (rather small) probability which the Λ^0 and shortlived K^0 have of escaping



π⁻
π⁻ + p → K⁺ + Σ⁻
K⁺
stops in
the wall
e⁺
Σ⁻
Σ⁻ → π⁻ + n
in flight

Fig. 1-17·3. – Example of the reaction $\pi^- + p \rightarrow \Sigma^- + K^+$ (bubble chamber experiment of [16]).


π^-
$\pi^- + p \rightarrow \Sigma^0 + K^0$
$\Lambda^0 + \gamma$
$K^0 \rightarrow \pi^+ + \pi^-$
π^-
π^+
$\Lambda^0 \rightarrow \pi^- + p$
π^-
P
$\gamma \rightarrow e^+ + e^-$



π^-
$\pi^- + p \rightarrow \Sigma^\circ + K^\circ$
$\Lambda^\circ + \gamma$
$K^\circ \rightarrow \pi^+ + \pi^-$
π^-
π^+
$\Lambda^\circ \rightarrow \pi^- + p$
π^-
p
$\gamma \rightarrow e^+ + e^-$

Fig. 2-17·3. – Example of the reaction $\pi^- + p \to \Sigma^0 + K^0$ followed by $\Sigma^0 \to \Lambda^0 + \gamma$ (bubble chamber experiment of [16]). The K^0 happens to decay very near the place of production.

the chamber before decaying, then the above figures, appropriately corrected show that:

half (or more precisely 0.49 ± 0.075) *of the produced* K^0 *decay in two pions (charged or neutral) with a lifetime of* (0.95 ± 0.08) s, *half escape from the chamber.*

This is at the same time a confirmation of the associated production mechanism and of the two neutral boson picture of Gell-Mann and Pais; it has already been mentioned (Sec. **14**·5) that it is possible only under rather artificial assumptions to reconcile this result with the parity doublet idea (which would imply four different neutral bosons).

Also the following information may (and probably will) be obtained from an experiment like the one above:

1) frequency of the reaction $\pi^- + p \to$ $\to \Sigma^- + K^+$;

2) angular distributions in the decays of the hyperons;

3) angular distribution, energy dependence and total cross-section for the various production processes of the new particles.

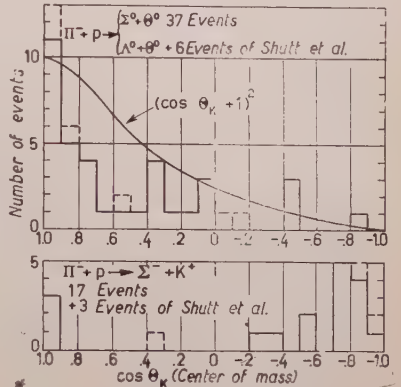


Fig. 3-17·3. – (From [1]). The production angular distribution for the reactions $\pi^- + p \to \Lambda^0 + K^0$, $\pi^- + p \to \Sigma^- + K^+$. θ_K is the centre of mass angle of the heavy meson in each case. The sixth Shutt event should be inserted between .8 and .7.

Preliminary information on the above points was already obtained in a previous experiment by the same group [17] with a smaller bubble chamber without magnetic field, using pions of 1.43 GeV/c.

The results regarding point 2) have been already given in the Sect. **15**·2.

Concerning point 3) the Fig. 3-**17**·3 summarizes the present angular distribution of production events in H. It appears from the diagram that the distribution in the angle θ_k between the line of flight of the incident π^- and that of the outgoing K in the center of mass of the collision is very peaked forward in the case of neutral K (events with production both of Σ^0 and of Λ^0 are included), while it appears rather flat for charged K (*).

(*) According to Goldhaber and to Schwinger (ref. [20] of chapt. 11) this may be evidence for a strong direct interaction between pairs of K's and pions.

Infact, if this interaction exists, the incident π^- will be dissociated, part of the time, in a virtual K^0, K^- pair. When the proton is struck, the K^- is absorbed by the proton with the production of a Λ^0 or a Σ^0 and the K^0 continues its flight conserving its velocity component in the direction of the π^-. For the K^+ a similar mechanism cannot apply and one has a rather isotropic distribution. We do not know, at the moment, of any detailed calculation published along these lines; one is in progress [18].

17·4. – A few data on the total cross sections.

The value of the cross-section for associated production in Hydrogen and its energy dependence are known very poorly; the Brookhaven Columbia data have not yet been analysed to this purpose.

An order of magnitude for the production cross-section by π^- has been given by FOWLER *et al.* [12] on the basis of their 9 observed events of associated production at 1.37 GeV. Taking into account the events which may have escaped detection they estimate 14 events (over a total of 550 pion interactions); these numbers lead to a cross-section of ~ 0.9 mb for associated production and to a *total* cross-section for π^- interaction of (34.6 ± 2.7) mb, the last value being in agreement with previous determinations. WALKER and SHEPHARD [14] give a similar value at 1 GeV derived from 5 production events (always from π^- collisions).

Concerning the cross-section for production in pp collisions BLOCK *et al.* [7] give, from three events of production of new particles at 2.75 GeV a tentative cross-section in between 0.1 and 1.5 mb; notice that the energy available in the center of mass system in pp collisions at this energy is roughly the same as that available in π^- p collisions at 1.37 GeV.

REFERENCES (CH. 17)

[1] G. COCCONI and A. SILVERMAN: *Phys. Rev.*, **84**, 1062 (1951).
[2] G. BERNARDINI and E. SEGRÈ: quoted in the *Phys. Rev.*, **92**, 72 (1953).
[3] R. L. GARWIN: *Phys. Rev.*, **90**, 274 (1953); A. H. ROSENFELD and S. B. TREIMAN: *Phys. Rev.*, **92**, 727 (1953); R. H. HILDEBRAND and C. E. LEITH: quoted in *Phys. Rev.*, **92**, 727 (1953).
[4] M. P. BALANDIN, B. D. BALASHOV, W. A. ZHUKOV, B. M. PONTECORVO and G. I. SELIVANOV: J.E.T.P. USSR, **29**, 265 (1955).
[5] W. B. FOWLER, R. P. SHUTT, A. M. THORNDIKE and W. L. WHITTEMORE: *Phys. Rev.*, **103**, 1479 (1956).
[6] T. W. MORRIS, E. C. FOWLER and J. D. GARRISON: *Phys. Rev.*, **103**, 1472 (1956).
[7] M. M. BLOCK, E. M. HARTH, W. B. FOWLER, R. P. SHUTT, A. M. THORNDIKE and W. L. WHITTEMORE: *Phys. Rev.*, **99**, 361 (1955).
[8] W. B. FOWLER, R. P. SHUTT, A. M. THORNDIKE, W. L. WHITTEMORE, M. M. BLOCK, E. M. HARTH, V. T. COCCONI and E. HARTH: *Phys. Rev.*, **103**, 1484 (1956).
[9] W. B. FOWLER: *Proceed. of the Rochester Conference* (1956).
[10] W. B. FOWLER, G. MAENCHEN, W. M. POWELL, G. SAPHIR and R. W. WRIGHT: *Phys. Rev.*, **103**, 208 (1956).
[11] W. B. FOWLER, R. P. SHUTT, A. M. THORNDIKE and W. L. WHITTEMORE: *Phys. Rev.*, **91**, 1287 (1953).

[12] W. B. Fowler, R. P. Shutt, A. M. Thorndike and W. L. Whittemore: *Phys. Rev.*, **93**, 861 (1954); **98**, 121 (1955).
[13] W. D. Walker: *Phys. Rev.*, **98**, 1407 (1955).
[14] W. D. Walker and W. Shephard: *Phys. Rev.*, **101**, 1810 (1956).
[15] R. Plano, N. Samios, M. Schwartz and J. Steinberger: *Nuovo Cimento*, **5**, 216 (1957).
[16] M. Schwartz, J. Steinberger, R. Plano and N. Samios: to be published.
[17] R. Budde, M. Chretien, J. Leitner, N. Samios, M. Schwartz and J. Steinberger: *Phys. Rev.*, **103**, 1827 (1956).
[18] S. Barshay: *Phys. Rev.*, **104**, 853 (1956).

Chapter 18.

Production in Complex Nuclei.

Introduction.

Observations on the production of K and Y particles in heavy nuclei are, in general, less informative than those relative to bare nucleons. As is easily understood, when particles have to traverse nuclear matter to emerge from the nucleus in which they are produced, some of their characters—such as energy and angular distribution and state of polarization—are often greatly altered.

Despite these obvious difficulties, new knowledge on the production mechanism has been recently derived from the study of the production in complex nuclei. This is chiefly due to the impressive work carried on in the Berkeley and Brookhaven laboratories where intense beams of K and Y particles have been available for sometime.

Out of the copious literature related to this subject we have summarized only the main results. The reader desirous of details not reported here, may find additional information consulting the reference list given at the end of this chapter.

18·1. – Production by artificially accelerated radiation.

18·1.1. *Threshold energy for production of strange particles and determination of the excitation function.* – A detailed study of the production of strange particles as a function of the primary energy has been made by Ridgway *et al.* [1] at Brookhaven. The experimental arrangement that was used has been described in Sect. 3·5. Briefly, protons of an energy varying between 670 MeV and 3 GeV were allowed to hit a 0.97 cm thick copper target and heavy unstable particles were individuated amongst the secondaries produced in the target by detecting the γ-rays resulting from their π^0-modes of decay. The observed γ-ray yield (see Fig. 1) indicates that the position of the threshold for

the production of such particles lies close to 1 GeV. These results, while confirming some early attempts at lower energy (*), agree remarkably well with what expected for the associated production of a $K^{+,0}$ and a Λ^0 particle by a proton hitting a bound nucleon (see the list of threshold energies given in the Table at page 717) and are consistent with the hypothesis that the production of the new particles takes place only when strangeness is conserved.

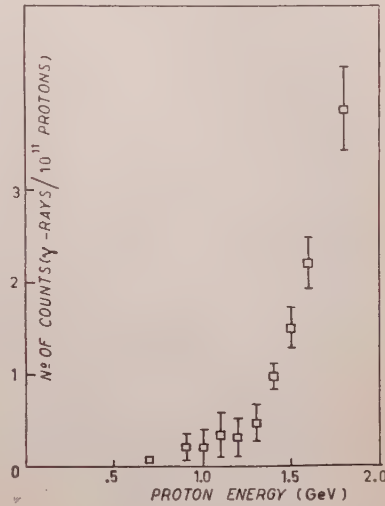


Fig. 1-19·1.

In the parallel experiment by OSHER *et al.* [2] already quoted in Ch. **3**, the production of γ-rays by unstable particles living $\sim 10^{-10}$ s was explored up to primary energy of 6.2 GeV. The behaviour of the excitation function can be deduced from Fig. 2: each curve represents the intensity of γ-rays emitted in the π^0-modes of heavy mesons and hyperons as a function of the distance from the target (see Sect. **3**.4) and the various curves refer to different energies of the primary proton beam. The steep tails are to be attributed to $K^0_{\pi 2}$ and the flatter tails to Λ^0 decays.

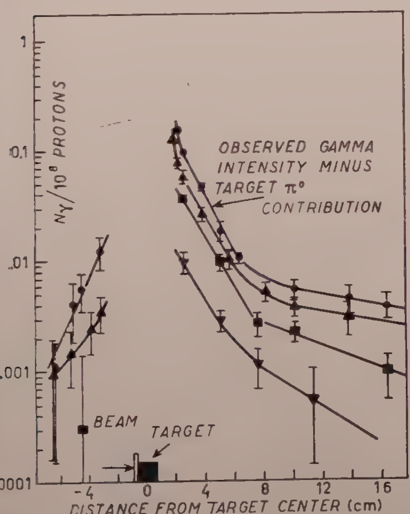


Fig. 2-18·1. – γ-ray intensity as a function of the distance from the target, due to the π^0-modes of decay of K^0's and Λ^0's created by proton beams of different energy (● 6.2 ÷ 5.7 GeV; ▲ 5.3 ÷ 4.8 GeV; ■ 3.6 ÷ 3.2 GeV; ▼ 2.0 ÷ 1.7 GeV) as observed by OSHER *et al.* [2].

18·1.2. *Production of charged* K *particles.* – The production of charged K particles in complex nuclei has been studied in a rather unsystematic way. In fact the cross-section for $K^\pm$ production has not been measured so far (+): on the other hand the yield of K particles relative to that of π mesons of the same

(*) [3-7]. See the discussion at pag. 717 and the references given there.

(+) This statement must be qualified. A value of the differential cross section $d^2\sigma/d\Omega\, dE$ for K^+ emitted with an energy of 114 MeV in the laboratory system, at an angle of 90° from the direction of the primary 5.7 GeV proton beam, was given

sign, emitted in the same solid angle within the same momentum channel has been measured with great accuracy for various energies of the primary radiation by a number of different authors [9-15]. In most of the experiments the produced particles were magnetically separated and the analysis was limited to certain momentum channels.

Exceptions are due to HILL *et al.* [16] who exposed nuclear emulsions to unseparated secondary radiations produced by the Brookhaven Cosmotron at 45° and 90° to the direction of the primary proton beam; and to SCHEIN *et al.* [17] who irradiated the emulsions with energetic π^-'s and analysed the particles emitted in the disintegrations produced there.

The most striking feature of the results listed in Tables I, II, III,

TABLE I-18·1. – *Data on K^+-artificial production.*

Ref.	Primary Radiation		Target	Detector	Time of flight (10^{-8} s)	Angle of observation	Secondary radiation momentum (MeV/c)	Relative yield at the point of detection (*) $N(K^+)/N(\pi^-)\cdot 10^2$
	identity	energy (GeV)						
[19]	p	2.9	Cu	emulsion	1.9	60°	290	0.165
[20]	p	3	Cu	counters	1.6	60°	465	0.25
[21]	p	2.6	—	emulsion	2	90°	250	0.13
[21]	p	4.2	—	»	2	90°	250	0.25
[22]	p	4.8	Cu	»	1.3	90°	350 ± 18	1.7 ± 0.25
[8]	p	5.7	Cu	»	1.3	90°	354	1.3 ± 0.13
[21]	p	6.2	Cu	»	2	90°	250	0.5
[22]	p	6.2	Cu	»	1.3	90°	350 ± 18	1.7 ± 25
[18]	p	6.2	Cu	»	1.3	90°	356	1.86 ± 0.27
[9]	p	6.2	—	counters	1.3	90°	370 ± 30	2.5
[23]	p	6.2	U	emulsion	1.5	120°	340	0.25

(*) In many of the cases listed above it was not explicitly stated whether $N(K^+)$ did or did not include the $K^+_{\pi3}$ mode. In the papers by BIRGE *et al.* [8] and HARRIS *et al.* [18, 19] this point was clear and the values given in the original papers have been multiplied by 1.06 to account for $K^+_{\pi3}$-mesons (see Sect. 1·3). On the other hand as the $K^+_{\pi3}$ mode is comparatively rare with respect to the others, this does not contribute appreciably to the total uncertainty of the final value.

at the Pisa Conference on Elementary Particles, 1955, by the Berkeley group [8]:

$$d^2\sigma/d\Omega\,dE = 2.4\cdot 10^{-32}\ \text{cm}^2\ \text{sr}^{-1}\ \text{MeV}^{-1}\,.$$

This value is uncorrected for decay in flight in the path between the target and the emulsions which were used as a detector. Owing to the imprecise determination of the intensity of the primary proton beam this result is affected by a large uncertainty.

is the large positive excess amongst the K particles produced by protons of energies not higher than 6.2 GeV. This is to be expected if strangeness is conserved in fast reactions. The lowest thresholds for the production of K^+ and K^- respectively are those for the reactions (see Table at page 270).

$$N + N \to \Lambda^0 + K^+ + N \qquad \text{threshold } 1.10 \text{ GeV},$$

$$\to K^- + K^+ + N + N \qquad \text{threshold } 1.84 \text{ GeV}.$$

TABLE II-18·1. – *Data on K^--artificial production.*

Ref.	Primary radiation		Target	Detector	Time of flight (10^{-8} s)	Angle of observation	Secondary radiation momentum (MeV/c)	Relative yield at the point of detection $N(K^-)/N(\pi^-)\cdot 10^4$
	identity	energy (GeV)						
[24]	p	2.8	Be	emuls.	1.4	1°	316 ± 8	0.2
[14]	p	2.8	Be	»	1.4	$4°\pm 3°$	313 ± 8	0.65
[11]	p	5.7	polyethylene	»	—	$0°\pm 5°$	360	1.67
[12]	p	6.2	Ta	»	2	90°	230 ± 25	0.76 ± 0.11
[12]	p	6.2	Ta	»	2	90°	270	0.77 ± 0.7
[22]	p	6.2	Cu	»	1.3	90°	350 ± 18	1.7 ± 0.5

TABLE III-18·1. – *Miscellanous data on K-artificial production.*

Ref.	Primary radiation		Target	Detector	Time of flight (10^{-8} s)	Angle of observation	Secondary radiation momentum (MeV/c)	Relative yields
	identity	energy (GeV)						
[17]	—	3 and 4.6	emuls.	emuls.	—	all	all	$\dfrac{N(K^+)}{N(K^-)} \sim 1$
[16]	p	3	Cu	»	0.14	90°; 45°	350 ± 60	$\dfrac{N(K^+, 90°)}{N(K^+, 45°)} = 1.8^{+0.4}_{-1.2}$

Apart from the higher threshold, the second has also a smaller available phase space volume and these two facts may account for the observed effect at these energies.

A quantitative analysis of the data listed in Table I is—in our opinion—premature. It may be worth mentioning that BLOCK *et al.* [25] have calculated the differential cross-sections for K production from the reactions

$$p + p \to \Lambda^0 + K^+ + p$$

$$\to \Sigma^+ + K^+ + n$$

assuming for the matrix element describing the interaction in the center of mass system the following expressions:

$$|H|^2 = \text{const} \tag{1}$$

or alternatively

$$= \cos^2\theta \tag{2}$$

$$= \gamma^2 - 1 \tag{3}$$

$$= (\gamma^2 - 1)\cos^2\theta\,, \tag{4}$$

θ and γ being the angle of emission of the K^+ in the center of mass system and its total energy in $m_k c^2$ units. The normalization constants were so chosen as to predict a value of $(d\sigma/d\Omega) = 1$ mb/sr in the center of mass system for a primary proton of 2.9 GeV in the laboratory-system. Moreover they assumed that the velocity distribution of the target protons inside the nuclei was given by

a) a Fermi degenerate gas distribution of the velocities,

or alternatively

b) a gaussian distribution.

Of the cross-section calculated taking all the possible combinations between the expression (1) to (4) on one hand and *a*)-*b*) on the other, only (4-*a*) or (4-*b*) yielded a result close to the experimental data.

How significant this type of analysis is in fact, is a question open to discussion. In the first place it is to be expected that an appreciable fraction of the K's emerging from heavy nuclei are due to secondary π's produced in the same nucleus. Besides their angular and energy distribution will be distorted by collisions in nuclear matter before they emerge. An estimate of this effect was made by WIDGOFF *et al.* [23] who measured the flux of K^+ mesons emitted at 120° by 2.9 GeV protons on heavy nuclei (Cu, O) and found it about $\frac{1}{4}$ of that of K^+'s emitted at 60° with respect to the direction of the primary beam. Using BLOCK *et al.*'s calculations, they found that the same ratio should have been 1/20 and they attribute the discrepancy to scattering of K^+ particles inside the nucleus in which they are produced. If one assumes that, of the particles observed to emerge at an angle θ, the fraction $f(\theta)$ were produced at the same angle and emerged unscattered, then $f(120°)/f(60°) \approx \frac{1}{5}$ or $f(120°) \approx 20\%$ (under the most optimistic assumption that $f(60°) = 1$).

18·1.3. *Production of neutral* K *and* Y *particles.* – Using a 36 in. diameter magnet cloud chamber BLUMENFELD *et al.* [26] have studied the production of K^0 and Λ^0 by 1.9 GeV π^- in lead and carbon. For both these elements they

find a cross-section for production of V^0 corresponding to $\sim 1\%$ geometrical (i.e. ~ 0.3 mb per nucleon). In a subsequent experiment [27] on iron the cross-section was found again close to that value ($\sigma_{\pi^-} = 0.3 \pm 0.1$ mb per nucleon) and the ratio $\sigma(1.8 \text{ GeV } \pi^-)/\sigma(2.5 \text{ GeV protons})$ was found $\sim 3 \pm 1$.

Bowen *et al.* [28] observed that, the rate of production of V^0's as compared with the number of observed nuclear interactions produced by π^- of 1.9 GeV was the same in C, Fe and Pb and corresponded to $\sim 6 \cdot 10^{-3}$ V^0's per interaction. Preliminary results indicated that using a 2.8 GeV proton beam, (in Fe) this ratio was considerably lower (*).

18.2. – Cosmic ray experiments.

A comparatively large number of data on the production of new particles in cosmic rays has been accumulated since their discovery and some are reported in the Tables I to IV, and **I-21**·6.

Unfortunately in most cases the information which can be obtained with regard to the nature and the energy of the primary particle, responsible for the observed event, is very scarce and not much use can be made of the data which have been collected.

The brief summary of the results given in the following pages may be of some use, howewer, to workers engaged in cosmic ray research.

18·2.1. *Relative rates of production of* $V^{\pm}$ *and* S *particles*. – To our knowledge the absolute rate of occurrence of new particles in cosmic rays has been rarely estimated and the available data are affected by a large degree of uncertainty. Far more significant are—in our opinion—the data concerning he rate of occurrence with respect to shower particles.

In Tables I to VI, V-particles are intended to indicate those particles (heavier than π-mesons) observed to decay in flight in a cloud chamber;

(*) An attempt aiming at interpreting theoretically the results of these experiments was made by Jastrow [29] and Block and Jastrow [30]. They tried to get information on the primary interaction responsible for the production of unstable particles and on their subsequent interactions in traversing nuclear matter, from the observed numbers of hyperons and K-particles in various materials at various primary energies. Assuming a production reaction

$$\pi^- + p \to \Lambda^0 + K^0 ,$$

they traced the Λ^0 and K^0 in nuclear matter — by the Montecarlo method — until they emerged or were absorbed. The comparison with the experimental data available at the time ([26], [31], [32], [33]) was however inconclusive, owing to the scarcity of data.

TABLE I-18·2. – *Relative rates of production of V-particles in Cosmic Rays (cloud chamber data).*

Ref.	Target nucleus	E_p (GeV)	$\overline{n}_s$	$N(V^\pm)/n_s$ (1)	$N(V^0)/n_s$ (2)	$N(V^0)/\nu_s$ (3)
[34]	Pb	—	—	—	—	$3\cdot10^{-2}$
[35]	Pb	$5\div10$	—	—	$(2\div3)\cdot10^{-2}$	—
[36]	Pb	$10\div30$	—	$10^{-1}\div10^{-2}$	—	—
[37]	C and Pb	—	3	—	$3\cdot10^{-2}$	0.12 ± 0.02
[38]	Pb	~50	—	—	$3\cdot10^{-2}$	—
[39]	Cu	$\geqslant3$	—	—	—	$5\cdot10^{-2}$
[32]	Pb	—	4.9 ± 1.2	—	$(1.2\pm0.3)\cdot10^{-2}$	$(5.6\pm1)\cdot10^{-2}$
[40]	C	—	—	10^{-3}	—	$0.8\cdot10^{-2}$
[41]	Pb	$5\div10$	—	—	$(3\div4)\cdot10^{-2}$	—
[42]	Pb	—	—	—	—	$3.7\cdot10^{-2}$

E_p = primary interaction energy;
n_s = number of shower particles;
$\overline{n}_s$ = average number of shower particles originating from the primary interaction;
ν_s = number of showers;
$N(V^0)$ = number of neutral V-particles;
$N(V^\pm)$ = number of charged V-particles.

S particles are those which stop in the chamber. Both groups include, in different proportions, K's and hyperons. « Shower particles » (n_s) are in general intended to be fast penetrating particles, namely a mixture of π and K mesons, protons and hyperons—the π mesons being on the average the most frequent component ($\sim$ 70-80%).

Most of the data given in these tables refer to observations on locally produced showers. This is clearly seen from Fig. 1. where two examples of cloud chamber arrangements used in this type of research are shown: in both cases the production takes place predominantly in the « generating block » placed above the chambers which are triggered by a convenient system of GM-counters, in general pre-arranged to select events of a definite energy, type and multiplicity.

In Fig. 2 the design of the counter exper-

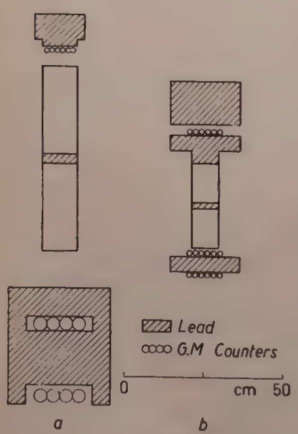


Fig. 1-18·2. – Typical cloud chamber arrangements used to study the production of heavy mesons and hyperons. *a*) The magnet chamber installed at the Jungfraujoch by the Manchester group and used by BUCHANAN *et al.* [59]; *b*) the chamber used by BARKER *et al.* at the Pic du Midi [36].

iment of MEZZETTI and KEUFFEL [43] is shown. In many ways it is very similar to some of the cloud chamber arrangements: they recorded K particles emitted in disintegrations produced in 20 cm Pb stopping and decaying in a large Čerenkov counter (see Sect. 1·5) and let the recording system be triggered when at least two of the counters placed under the lead were discharged. In general this required at least two penetrating particles although one, emerging at a convenient angle, could occasionally have started the trigger. Since K^- particles are known to interact strongly with matter, it is certain that they observed only K_L^+ which can safely be identified with the S-particles.

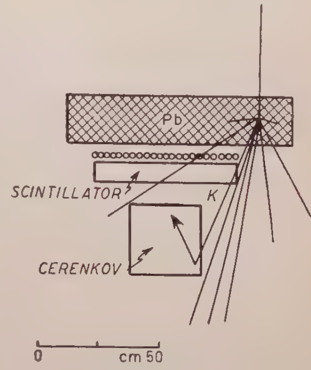


Fig. 2-18·2.

In emulsion experiments the selection criterion is far more precise and better defined. The works of DANIEL *et al.* [44], DAHANAYAKE *et al.* [45], MULVEY [46], make reference only to particles selected on the basis of direct mass measurements. The data of HERTZ *et al.* [47], AMALDI *et al.* [48, 49] and CRUSSARD *et al.* [50] refer to particles stopping in the emulsion. In HERZ *et al.*'s experiment the stacks were exposed under ice and most of the particles observed came from the surrounding material, while the data of [48] and [49] have been obtained exposing the emulsions to high altitude radiation and most of the particles were produced inside the same block in which they have been observed, the surrounding matter being not more than 0.1 g/cm².

TABLE II-18·2. – *Relative frequency of production of* Λ^0 *and* K^0 *particles by cosmic rays (cloud chamber data).*

Ref.	Generating material	Primary energy (GeV)	$N(\Lambda^0)/N(\pi^\pm)$	$N(K^0)/N(\pi^\pm)$	$N(\Lambda^0)/N(K^0)$
[53]	Pb	—	—	—	1.6 ± 0.5
[54, 41]	Pb	$5\div10$	$3\cdot10^{-2}$	$1.3\cdot10^{-2}$	1.7 ± 0.7
[55]	Pb	—	—	—	1.3 ± 0.3
[42]	Pb	—	—	—	$0.5^{+0.9}_{-0.2}$
[56]	C	—	—	—	0.5 ± 0.3

It was pointed out by MEZZETTI and KEUFFEL that a certain amount of disagreement exists among the results obtained by various workers with regard to the relative frequency of production of positive K particles. In particular they found the ratio $N(K^+)/N(\pi^+)$ obtained in their experiment to be larger than those obtained in other laboratories (see Tables III and IV). In most

cases the discrepancy is probably a consequence of optimistic estimates of the detection efficiencies.

In fact, so far as the emulsion work is concerned, the best way to estimate the relative proportion is not that of comparing directly the number of

TABLE III-18·2. – *Relative frequency of production of S-particles by cosmic rays (cloud chamber and counter experiments).*

Ref.	Detector	Generating material	n_s	$N(\mathrm{S})/N(\pi^+)$	$\overline{N}(\mathrm{S}) = N(\mathrm{S})/\Delta M\,\Delta t$ (kg^{-1}, day^{-1})
[51]	Cl. Ch.	Pb	—	0.02	—
[43]	Counters	Pb	$\geqslant 2$	0.062	14

n_s = minimum number of particles required to emerge from the generating layer to trigger the system;
$N(\mathrm{S})$ = number of S-particles;
$N(\pi^+)$ = number of π^+;
$\overline{N}(\mathrm{S})$ = average $N(\mathrm{S})$ per kg of absorber placed above the detector, per day of observation.

TABLE IV-18·2. – *Relative frequency of production of K-particles by cosmic rays (emulsion data)*

Ref.	Primary energy (GeV)	n_s	$N(\mathrm{K}^\pm)/N(\pi^\pm)$			$N(\mathrm{K}^\pm)/N(s)$	$E(\mathrm{K}^\pm)/E(\pi^\pm)$
			$0.25 \leqslant \beta \leqslant 0.56$	$0.5 \leqslant \beta \leqslant 0.84$	$330 \leqslant p \leqslant 950$ (*) (MeV/c)	$g/g_0 \geqslant 1.4$ $\beta \leqslant 0.7$	$330 \leqslant p \leqslant 950$ (*) (MeV/c)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
[57]	$1.5 \div 8$	1, 2, 3	—	0.13 ± 0.05	0.09 ± 0.04	0.015 ± 0.006	0.17 ± 0.06
	$6 \div 40$	4	—	0.28 ± 0.08	0.20 ± 0.06	0.018 ± 0.004	0.36 ± 0.1
	50	« jets »	—	—	0.50 ± 0.2	0.08	1.0 ± 0.3
[45]	—	—	0.08	—	—	—	—
[50]	—	—	$N(\mathrm{K}^\pm)/N(\pi^\pm) \sim 0.09$ (for particles created and stopped in emulsion; energy range unspecified);				
[46]	$2 \cdot 10$	« jets »	$N(\mathrm{K}^\pm)N(\pi^\pm) \sim 0.08$ (all particles emitted in one very highly energetic jet observed in emulsion);				
[47]	—	—	$N(\mathrm{K}^+_{\pi 3})/N(\pi^\pm) \sim 0.006$ (particles stopping in emulsion exposed under 3 m ice);				
[48]	—	—	$N(\mathrm{K}^+_{\pi 3})/N(\pi^\pm) \sim 0.003$ (particles stopping in emulsions and created in the same emulsion block);				
[49]	—	—	$N(\mathrm{K}^+_{\pi 3})/N(\pi^+) \sim 0.002$ (id. id.);				
[58]	$50 \div 250$	—	$N(\mathrm{K}^\pm) + N(\mathrm{Y}^\pm)/N(\pi^\pm) \leqslant 0.25$ (energy range unspecified).				

Code: n_s – number of shower particles associated with the primary interaction;
N_s = total number of shower particles in a certain velocity or momentum interval;
$E(\mathrm{K})$, $E(\pi)$ – total energy associated with observed K or π particles.

(*) The values of p and E for K-particles have been calculated by DANIEL and PERKINS on a mass value $m_{\mathrm{K}} = 1200$, which was generally accepted when their experiment was carried out.

observed slow K^+ and π^+ but rather that of deducing it from the observed $K^+_{\pi 3}/\pi^+$ ratio and from the now well established $K^+_{\pi 3}/K^+$ ratio—as given by the recent machine experiments (*).

Then, one can easily see that—in so far as the emulsion work is concerned—no inconsistency really exists. The value given by AMALDI *et al.* [48] relative to $K_{\pi 3}$ mesons stopping in emulsion is $= 3.3 \cdot 10^{-3}$. Considering that $K_{\pi 3}$ mesons are known to be but a small fraction of the K^+ particles ($\sim 5.6\%$) the corresponding value of the ratio $N(K^+)/N(\pi^\pm)$ would be $\sim 5.9 \cdot 10^{-2}$ and that of $N(K^+)/N(\pi^+) \sim 12 \cdot 10^{-2}$. Assuming that MEZZETTI and KEUFFEL observed only $K^+_{\mu 2}$ ($\sim 60\%$ of the K^+'s) the corresponding emulsion value of $\varrho = N(K^+)/N(\pi^+)$ to be compared with, is

$$\varrho_{em} \cong 0.07 \,,$$

which is in very good agreement with theirs ($\varrho_{MK} \sim 0.062$; See Table III).

Less easy to explain is the discrepancy between the counter and cloud chamber experiments [51].

Additional reasons for being cautious in making direct comparison of the results quoted in this section can be seen from the following considerations. The value of ϱ just deduced refers to particles generated, almost totally, in the same emulsion block in which they have been observed to stop. In doing so AMALDI *et al.* have selected a range interval (depending on the size of the emulsions) which roughly corresponds to $R \lesssim 80$ g/cm², i.e. to K particles having a momentum $\lesssim 550$ MeV/c. In the counter experiment [43] the particles were produced in a block of lead 20 cm thick. Assuming that they were produced uniformly throughout the whole thickness (which—of course—is not rigorous), the average range of K particles capable of being detected would be of the order of 165 g/cm² of lead, which corresponds to particles of ~ 660 MeV/c.

How sensitive is ϱ to the choice of the momentum (or range or velocity) interval, in which both K and π mesons are selected, cannot be stated at present, but it is certainly not independent of it and is known to depend also on the energy E_p of the disintegrations in which they are created (see Subsect. **18**.2.3). Presumably in both counter and cloud chamber experiments the majority of the events were associated with values of $E_p \leqslant 10$ GeV. To our knowledge the only measurements of ϱ, made in emulsion at comparable values of E_p, are those of DANIEL *et al.* [57], reported in Table IV which are slightly larger than those obtained in counter experiments.

(*) Thanks are due to prof. L. MEZZETTI for an enlightening discussion on this question.

TABLE V-**18**·2. – *The ratio* V^+/V^- *in cosmic rays.*

Ref.	Detector	Low momenta: Selecting criteria	Low momenta: Result	High momenta: Selecting criteria	High momenta: Result	Remark
Cal. Tech (*)	Cl. Ch.	heavily ionizing	19/5 = 3.8	light ionizing	22/38 = 0.58	
Princeton (*)	»	$p_V < 1$ GeV/c	15/10 = 1.5	$p_V > 1$ GeV/c	6/11 = 0.55	
Ecole Pol. (*)	»	$p_V < 1$ GeV/c	25/6 = 4.2	$p_V > 1$ GeV/c	0/2	includin identifie $K_{\pi 3}$-meso
»	»	S-events ($p <$ 0.5 GeV/c)	22/2 = 11	—	—	
CERN-group (*)	»	$p_V < 0.6$ GeV/c	31/12 = 2.6	$p_V > 1$ GeV	25/23 = 1.1	
	»	$p_V < 1$ GeV/c	6/0	—	—	
	Total		118/35 = 3.4		53/74 = 0.72	
				Grand Total	171/109 = 1.57	
[62]	Emulsion	K-particles ending in emulsion		$K_L^+/K^- = 10$; $K_{\pi 3}^+/K^- = 1$		

(*) Taken from COOPER *et al.* [61].

18·2.2. *The ratio* V^+/V^-. – The existence of a positive excess among V particles has been—for some time—an uncertain question [59, 60]. For particles of low momentum (~ 1 GeV/c) it has been recently established by a number of experiments (see Table V-**18**·2). For higher momenta however negative V's seem to be more frequent than positive ones.

According to COOPER *et al.* [61] the high momentum negative excess could be due to an experimental bias. They point out that the low momentum events listed in Table V-**18**·2 are mostly K particles while amongst the high momenta, a large number of fast Σ hyperons is present. Positive Σ's can decay into a $\pi^+ + n$ or alternatively into a $\pi^0 + p$ and Σ^- only into $\pi^- + n$. The protonic decay of a fast Σ^+ is often difficult to detect owing to the small mass difference between the Σ^+ and the proton, and it would not be surprising to discover that these decays are often missed, thus altering the final results in favour of the negative component.

A large positive excess has been observed among the K particles in emulsion: about 10 positive to 1 negative [62]. Considering that the K mesons detected in emulsion have not in general a momentum larger than 800 MeV/c (see Subsect. **18**·2.1) and that their average momentum probably does not exceed 500 MeV/c, it seems reasonable to compare this result with the positive excess found for S particles. An inspection of Table V-**18**·2 shows that it agrees remarkably well with the value found by the Ecole Polytechnique group (11:1) but not so well with that of the CERN group (2.6:1). The

uncertainty associated with the individual data is too large to consider the discrepancy as substantial.

A definite difference exists between cosmic ray and machine data, the latter indicating a ratio $K^+/K^- \sim 100$ (see Table I-**18**.1, II-**18**.1, III-**18**.1). The whole situation will be better understood when something more is known about the dependence of the K^+/K^- ratio on the energy of both the primary and secondary radiation.

TABLE VI-**18**.2. – *Frequency of production of V^0-particles in cosmic rays as a function of the primary energy (cloud chamber data).*

Ref.	n_s	$N(V^0)/\nu_s$	$N(\Lambda^0)/\nu_s$	$N(K^0)/\nu_s$
[40]	4	$(3\pm1)\cdot10^{-2}$	$(2.1\pm1)\cdot10^{-2}$	$(0.9\pm0.3)\cdot10^{-2}$
	4—8	$(17\pm6)\cdot10^{-2}$	$(8\pm3)\cdot10^{-2}$	$(8\pm4)\cdot10^{-2}$
	8	$(11\pm6)\cdot10^{-2}$	$(10\pm4)\cdot10^{-2}$	$\sim 10^{-2}$

Code: n_s = number of shower particles ejected in the primary interaction;
$N(V^0)/\nu_s$ = number of neutral V-particles per shower;
$N(\Lambda^0)/\nu_s$ = number of Λ^0-particles per shower;
$N(K^0)/\nu_s$ = number of K^0-particles per shower.

18.2.3. *Dependence of the frequency of production on the energy of the primary interaction E_p.* – A marked dependence of the yield of V or K particles on the primary energy E_p is shown by the data reported in Tables IV and VI. The emulsion data require some comment. The values of $N(K)/N(\pi)$ quoted in Table IV, columns 4, 5 and 7 refer to particles selected within certain limits of velocity whilst column 6 refers to those in a certain momentum interval. The different meaning of the results is easily derived from an inspection on Fig. 3 where the familiar g/g_0-$1/p\beta$ plot is shown (see Sect. **1**.4). The particles related to the numbers given in column 5 of Table IV are those between the horizontal lines β_1 and β_2, those of column 6 are between p_1 and p_2; while a selection based on residual range measurement—as often done in counter experiments—would take only events between certain R_1 and R_2 lines. It may be worth noting that columns 5 and 6 associate practically the same part of the K momentum or velocity spectrum with two different parts of the π spectrum. Thus only a comparison between numbers in the same column has a precise meaning: these indicate that when the primary energy is increased from 5 to ~ 20 GeV the ratio $\varrho = N(K^\pm)/N(\pi^\pm)$ is more than doubled and so is the total energy associated with the two types of radiation. It also appears that at energies above 50 GeV almost equal amounts of energy go to π's and K's.

A similar trend in the energy dependence of the production of V^0 particles is shown by the results of GAYTHER and BUTLER [41] in Table VI. The energy

of the primary interaction was not given by these authors, but it can be estimated by the number of shower particles associated with it. The comparatively low rate of observed V^0 in showers having $n_s \geq 8$ is to be taken with caution in view of the obvious bias against detecting these particles in showers of high multiplicity.

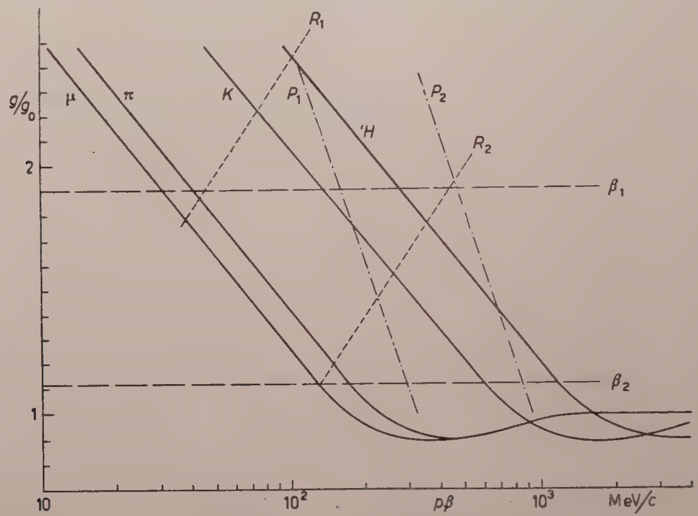


Fig. 3-**18**·2. - Lines of equal velocity, momentum and range superimposed on the (g/g_0 - versus $p\beta$)-diagrams for μ, π, K-mesons and protons. The values of the parameters are: $\beta_1=0.5$; $\beta_2=0.84$; $p_1=330$ MeV/c; $p_2=950$ MeV/c; $R_1=1$ cm; $R_2=10$ cm (compare with Table IV-**18**·2).

CASTAGNOLI *et al.* [63] and FRIEDLANDER *et al.* [62] have studied the characteristics of the stars from which at least one charged K or Y particle is emitted. Compared to ordinary stars, those associated with a strange particle have been found—on the average—more energetic than those not associated; those in which K^- are created include a higher proportion of low N_h disintegrations (*)—a result which can be taken as an indication that K^- are so strongly absorbed by nuclear matter than only those produced in light elements have a strong probability of emerging from their parent nucleons.

18·2.4. *Associated production.* – Subsidiary evidence in favour of the hypothesis of associated production has been obtained from cosmic ray experiments. This evidence is based on a number of disintegrations, both in cloud chamber and emulsion, which appear to be associated with two or more new

(*) I.e. disintegrations associated with a small number of slow particles.

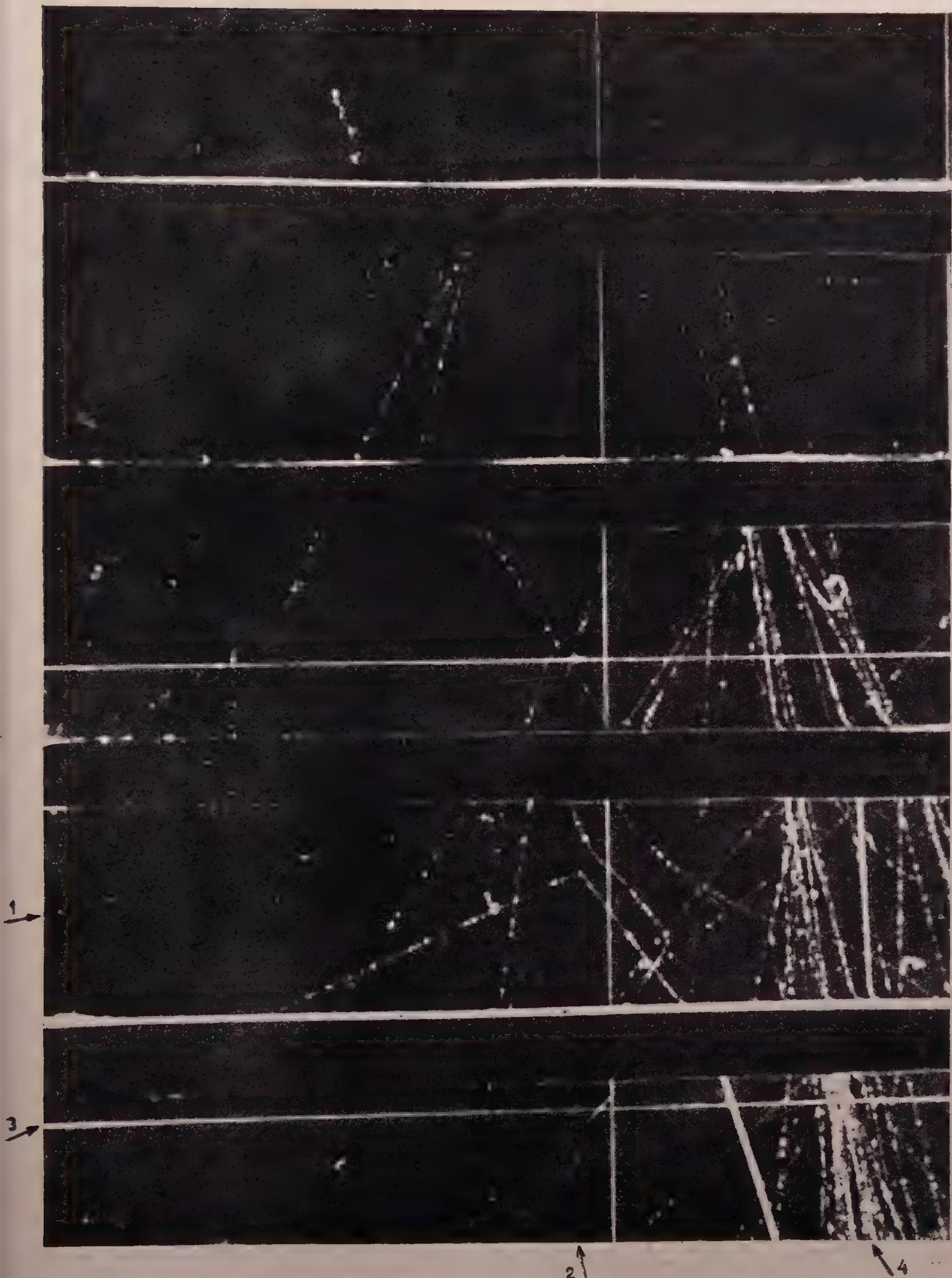


Fig. 4-18·2. – Pair of new particles probably produced in the same interaction. (Courtesy of Prof. B. BRUNELLI).

particles (compare e.g. Fig. 4). A list of those events in which the identity of both components was well established—is given in Table VII-18·2. A much larger number of not-so-well-identified cases is reported in the literature quoted at the end of this chapter.

Since the production of the particles observed took place in a heavy nucleus any correlation between particles created simultaneously is, in most cases, destroyed by scattering in nuclear matter, and no proof can be given that the observed « pairs » were not produced in two subsequent acts in the same nucleus. However, even if the dynamical analysis of the observed events has failed, in most cases, to give any conclusive evidence (see [64]), we cannot overlook the fact that none of the identified pairs (see Table VII) contrasts Gell-Mann and Nishijima's predictions. It seems unlikely that this would happen if the two components of the pairs were produced in separate collisions.

TABLE VII-18.2. – *Examples of associated production (identified events).*

Ref.	Parent star	Identity of the associated particles	Remarks
		Emulsion Data	
[67]	—	K^+ and K^0 or Λ^0	
[68]	—	$K^+_{\pi 3}$ and Y^- or K^- (*)	(*) The sign determined from the presence of a terminal capture star.
[69]	19+ 3n	K^+_L and $Y^\pm$	
[69]	13+ 3p	K^+_L and $\Sigma^\pm$	
[70]	28+29p	$K^+_{\pi 3}$ and $\Sigma^\pm$	
[71]	18+ 4p	K^+ and K^- or Y^- (*)	(*) The sign deduced from the presence of a terminal capture star.
[72]	22+36α	$K^+_{\mu 2}$ and $\Sigma^\pm$	
[73]	18+20α	$K^+_{\pi 2}$ and $\Sigma^\pm$	
[74]	1+ 2p	$K^+_{\mu 3}$ and $\Sigma^\pm$	
[75]	22+ 3p	K and ${}^4_\Lambda H$	
[76]	6+ 5p	K and K^- (*)	(*) The sign deduced from the presence of a terminal capture star.
[77]	26+ 9p	$K^+ + K + K + Y^\pm$	
[78]	2+ 0p	$K^+_{\pi 2}$ and $\Sigma^\pm$	
		Cloud Chamber Data	
[65]	—	$K^0 + K^0$ and Ξ	See the text
[78]	—	K^0 and Λ^0	
[79]	—	K^0 and Λ^0	10 similar cases identified.
[79]	—	K^+_L and Λ^0	
[79]	—	K^+_L and K^0	
[56]	1+0n	K^0 and K^0	
[56]	1+0n	K^0 and K^+	

Most pertinent to this question is the observation by SORRELS *et al.* [65] of a Ξ^- particle ejected in a highly energetic shower also associated with two V^0. The analysis of the event strongly favours the interpretation in terms of an associated production satisfying the conservation of strangeness $(\Xi^- + K^0 + \bar{K}^0)$.

18·2.5. *Polarization effect in* V^0 *decays.* – DEUTSCHMANN *et al.* [66] have found that when only V^0's are selected which are produced in interactions associated with a small number of ionizing particles, then the angles between the production and the decay planes generally tend to be small. No correlation has been found by GAYTHER and BUTLER [41] who also searched for such an effect among neutral V's emitted in cosmic ray disintegrations in heavy materials.

REFERENCES (CH. 18)

[1] S. S. T. RIDGWAY, D. BERLEY and G. B. COLLINS: mimeogr. preprint, in press on the *Phys. Rev.*
[2] J. E. OSHER, B. J. MOYER and S. I. PARKER: UCRL 3300 (Berkeley, 1956).
[3] M. P. BALANDIN, B. D. BALASHOV, W. A. ZHUKOV, B. M. PONTECORVO and G. I. SELIVANOV: *Journ. Exp. Theor. Phys. USSR*, **29**, 265 (1955).
[4] G. BERNARDINI and E. SEGRÉ: *Proc. Roy. Soc.*, A **221**, 413 (1954).
[5] C. COCCONI and A. SILVERMAN: *Phys. Rev.*, **84**, 1062 (1951).
[6] R. L. GARWIN: *Phys. Rev.*, **90**, 274 (1953).
[7] A. H. ROSENFELD and S. B. TREIMAN: *Phys. Rev.*, **92**, 727 (1953).
[8] R. W. BIRGE, R. P. HADDOCK, L. T. KERTH, J. R. PETERSON, J. SANDWEISS, D. H. STORK and M. N. WHITEHEAD: *Suppl. Nuovo Cimento*, **4**, 351 (1956).
[9] L. W. ALVAREZ, F. S. CRAWFORD, M. L. GOOD and M. L. STEVENSON: UCRL 3165 (1955).
[10] W. W. CHUPP and S. GOLDHABER: UCRL 3326 (Nov.-Dec. 1955 - Jan. 1956).
[11] S. C. FREDEN and H. K. TICHO: *Phys. Rev.*, **99**, 1057 (1955).
[12] V. F. FRY, J. SCHNEPS, G. M. SNOW and M. S. SWAMI: *Phys. Rev.*, **100**, 1448 (1955).
[13] E. P. GEORGE, A. J. HERZ, J. H. NOON and N. SOLNTSEFF: *Nuovo Cimento*, **3**, 94 (1956).
[14] J. HORNBOSTEL and E. O. SALANT: *Phys. Rev.*, **102**, 502 (1956).
[15] D. M. RITSON, A. PEVSNER, S. C. FUNG, M. WIDGOFF, G. T. ZORN, G. GOLDHABER and S. GOLDHABER: *Phys. Rev.*, **101**, 1081 (1956).
[16] R. D. HILL, E. O. SALANT, M. WIDGOFF: *Phys. Rev.*, **99**, 229 (1955).
[17] M. SCHEIN, D. M. HASKIN and M. WIDGOFF: *Nuovo Cimento*, **3**, 131 (1956).
[18] G. HARRIS, J. OREAR and S. TAYLOR: *Phys. Rev.*, **100**, 932 (1955).
[19] G. HARRIS, J. OREAR and S. TAYLOR: *Phys. Rev.*, **101**, 1214 (1956).
[20] V. FITCH and R. MOTLEY: *Phys. Rev.*, **101**, 496 (1956).
[21] W. W. CHUPP, S. GOLDHABER, G. GOLDHABER, W. R. JOHNSON and F. WEBB: *Suppl. Nuovo Cimento*, **4**, 359 (1956).
[22] M. CECCARELLI, M. GRILLI, M. MERLIN, G. SALANDIN and B. SECHI: *Nuovo Cimento*, **2**, 828 (1955).

[23] M. WIDGOFF, A. M. SHAPIRO, R. SCHLUTER, D. M. RITSON, A. PEVSNER and V. HENRI: *Phys. Rev.*, **104**, 811 (1956).
[24] J. HORNBOSTEL and E. O. SALANT: *Phys. Rev.*, **98**, 218 (1955).
[25] M. M. BLOCK, E. M. HARTH and R. M. STERNHEIMER: *Phys. Rev.*, **100**, 324 (1955).
[26] H. BLUMENFELD, E. T. BOOTH, L. M. LEDERMAN and W. CHINOWSKY: *Phys. Rev.*, **98**, 1203 (1955); **102**, 1184 (1956).
[27] H. BLUMENFELD, E. BOLDT, H. S. BRIDGE, D. O. CALDWELL, YASH PAL and and C. P. LEAVITT: *Bull. Am. Phys. Soc.*, Ser. II, **2**, 19 (1957).
[28] T. BOWEN, F. COOKSON, G. TAGLIAFERRI, A. WERBROUCH and W. H. MOORE: *Bull. Am. Phys. Soc.*, Ser. II, **2**, 19 (1957).
[29] R. JASTROW: *Phys. Rev.*, **97**, 181 (1955).
[30] M. M. BLOCK and R. JASTROW: *Nuovo Cimento*, **2**, 865 (1955).
[31] J. BALLAM, H. HARRIS, H. L. HODSON, R. R. RAU, G. T. REYNOLDS, S. B. TREIMAN and M. VIDALE: *Phys. Rev.*, **91**, 1019 (1953).
[32] W. B. FRETTER, M. M. MAY and M. P. NAKADA: *Phys. Rev.*, **89**, 168 (1953).
[33] R. B. LEIGHTON, S. D. WANLASS and C. D. ANDERSON: *Phys. Rev.*, **89**, 148 (1953).
[34] A. J. SERIFF, R. B. LEIGHTON, C. HSIAO, E. W. COWAN and C. D. ANDERSON: *Phys. Rev.*, **78**, 290 (1950).
[35] K. H. BARKER: *Congrès sur le Rayonnement Cosmique* (1953), Bagnères de Bigorre, mimeogr. rep., p. 35.
[36] K. H. BARKER, C. C. BUTLER, M. G. SOWERLY and C. M. YORK: *Phil. Mag.*, **43**, 1201 (1952).
[37] M. DEUTHSCHMAN: *Congrès sur le Rayonnement Cosmique* (1953), Bagnères de Bigorre, mimeogr. rep.
[38] J. A. NEWTH: *Congrès sur le Rayonnement Cosmique* (1953), Bagnères de Bigorre, mimeogr. rep.
[39] G. T. REYNOLDS: *Congrès sur le Rayonnement Cosmique* (1953), Bagnères de Bigorre (1953), mimeogr. rep.
[40] W. D. WALKER and N. M. DULLER: *Phys. Rev.*, **90**, 320 (1953).
[41] D. B. GAYTHER and C. C. BUTLER: *Phil. Mag.*, **46**, 467 (1955).
[42] I. C. GUPTA, W. Y. CHANG and A. L. SNYDER: circulated preprint.
[43] L. MEZZETTI and J. W. KEUFFEL: *Nuovo Cimento*, **4**, 1096 (1956).
[44] R. R. DANIEL, J. H. DAVIES, J. H. MULVEY and D. H. PERKINS: *Phil. Mag.*, **43**, 753 (1952).
[45] C. DAHANAYAKE, P. E. FRANCOIS, Y. FUJIMOTO, P. IREDALE, C. J. WADDINGTON and M. YASIN: *Nuovo Cimento*, *1*, 888 (1955).
[46] J. H. MULVEY: *Proc. Roy. Soc.*, A **221**, 367 (1954).
[47] A. J. HERZ, P. E. HODGSON and R. M. TENNENT: *Phil. Mag.*, **44**, 85 (1953).
[48] E. AMALDI, G. BARONI, G. CORTINI, C. FRANZINETTI and A. MANFREDINI: *Suppl. Nuovo Cimento*, **12**, 181 (1954).
[49] E. AMALDI, E. FABRI, T. F. HOANG, O. W. LOCK, L. SCARSI. B. TOUSCHEK and B. VITALE: *Suppl. Nuovo Cimento*, **12**, 419 (1954).
[50] J. CRUSSARD, L. JAUNEAU, G. KAYAS, L. LEPRINCE RINGUET, C. MABBOUX, D. MORELLET, A. ORKIN-LECOURTOIS and J. TREMBLEY: *Congrès sur le Rayonnement Cosmique* (1953), Bagnères de Bigorre, mimeogr. rep., p. 128.
[51] H. S. BRIDGE, CH. PEYROU, B. ROSSI and R. SAFFORD: *Phys. Rev.*, **90**, 921 (1953).
[52] C. CASTAGNOLI, G. CORTINI and A. MANFREDINI: *Nuovo Cimento*, **2**, 565 (1955).
[53] R. ARMENTEROS, K. H. BARKER, C. C. BUTLER and A. CACHON: *Phil. Mag.*, **42**, 1113 (1951).
[54] D. B. GAYTHER: *Phil. Mag.*, **46**, 1362 (1955).
[55] C. C. BUTLER: *Suppl. Nuovo Cimento*, **4**, 260 (1955).

[56] W. A. Cooper, H. Filthuth, J. A. Newth, G. Petrucci, R. A. Salmeron and A. Zichichi: *Nuovo Cimento*, in press.
[57] R. R. Daniel and D. H. Perkins: *Proc. Roy. Soc.*, A **221**, 351 (1954).
[58] D. Lal, Yash Pal and Rama: *Suppl. Nuovo Cimento*, **12**, 347 (1954).
[59] J. S. Buchanan, W. A. Cooper, D. D. Miller and J. A. Newth: *Phil. Mag.*, **45**, 1025 (1954).
[60] C. C. Butler: *Congrès sur le Rayonnement Cosmique* (1953), Bagnères de Bigorre mimeogr. rep., p. 90.
[61] W. A. Cooper, H. Filthuth, J. A. Newth and R. A. Salmeron: *Nuovo Cimento*, **4**, 390 (1956).
[62] M. W. Friedlander, Y. Fujimoto, D. Keefe, M. G. K. Menon, M. Ceccarelli, M. Grilli, M. Merlin, A. Salandin and B. Sechi: *Suppl. Nuovo Cimento*, **4**, 428 (1956).
[63] C. Castagnoli, G. Cortini and C. Franzinetti: *Nuovo Cimento*, **2**, 550 (1955).
[64] G. D. James and R. A. Salmeron: *Phil. Mag.*, **46**, 571 (1955).
[65] J. D. Sorrels, R. B. Leighton and C. D. Anderson: *Phys. Rev.*, **100**, 1457 (1955).
[66] M. Deutschmann, M. Cresti, W. D. B. Greening, L. Guerriero, A. Loria and G. Zago: *Nuovo Cimento*, **3**, 566 (1956).
[67] A. Debenedetti, C. M. Garelli, L. Tallone and M. Vigone: *Nuovo Cimento*, **12**, 369 (1954).
[68] D. Lal, Yash Pal and B. Peters: *Proc. Ind. Acad. Sci.*, **38**, 398 (1953).
[69] C. Dahanayake, P. E. Francois, Y. Fujimoto, P. Iredale, C. J. Waddington and M. Yasin: *Phil. Mag.*, **45**, 855 (1954).
[70] K. Gottstein: *Nuovo Cimento*, **1**, 284 (1955).
[71] M. W. Friedlander, D. Keefe and M. G. K. Menon: *Nuovo Cimento*, **1**, 694 (1955).
[72] P. H. Fowler and W. C. G. Ortel: *Nuovo Cimento*, **2**, 864 (1955).
[73] P. H. Fowler and D. H. Perkins: *Nuovo Cimento*, **2**, 874 (1955).
[74] M. W. Friedlander, D. Keefe and M. G. K. Menon: *Nuovo Cimento*, **1**, 694 (1955).
[75] M. W. Friedlander, D. Keefe and M. G. K. Menon: *Nuovo Cimento*, **2**, 663 (1955).
[76] M. W. Friedlander, D. Keefe and M. G. K. Menon: *Nuovo Cimento*, **2**, 666 (1955).
[77] D. F. Falla, M. W. Friedlander, W. D. B. Greening, S. Limentani, B. Sechi, C. Cernigoi, G. Iernetti and G. Poiani: private communication.
[78] R. W. Thompson, J. R. Burwell, R. W. Huggett and C. J. Karzmark: *Phys. Rev.*, **95**, 661 (1954).
[79] G. H. Trilling and R. B. Leighton: in press on the *Phys. Rev.*

CHAPTER 19.

Interactions of K^+ with nucleons.

19·1. – The possible reactions and the difference in behaviour between K^+ and K^-.

According to the theoretical scheme the interactions of negative and positive K particles with ordinary particles should be very different [1], due to the fact that (Table I-11·5 and Fig. 1-11·7) the K^- has strangeness -1 and the K^+ has strangeness $+1$.

A K^-, interacting with a nucleon may give rise to the following strong absorption reactions:

$$(1\text{-}19{\cdot}1)\qquad \begin{cases} K^- + p \to \Lambda^0 + \pi^0 \\ \qquad\quad\;\; \to \Sigma^0 + \pi^0 \\ \qquad\quad\;\; \to \Sigma^- + \pi^+ \\ \qquad\quad\;\; \to \Sigma^+ + \pi^- \end{cases}$$

$$(2\text{-}19{\cdot}1)\qquad \begin{cases} K^- + n \to \Lambda^0 + \pi^- \\ \qquad\quad\;\; \to \Sigma^0 + \pi^- \\ \qquad\quad\;\; \to \Sigma^- + \pi^0 \end{cases}$$

in addition to the normal scattering processes:

$$(3\text{-}19{\cdot}1)\qquad \begin{cases} K^- + p \to K^- + p \\ \qquad\quad\;\; \to K^0 + n \qquad \text{(charge exchange process)} \end{cases}$$

$$(4\text{-}19{\cdot}1)\qquad K^- + n \to K^- + n\,.$$

All these reactions conserve the strangeness. We also write down the as yet unobserved, strong electromagnetic processes which should take place at a rate $\sim 1/137$ of that of the corresponding processes with pions,

$$(5\text{-}19{\cdot}1)\qquad \begin{cases} K^- + p \to \Sigma^0 + \gamma \\ \qquad\quad\; \to \Lambda^0 + \gamma \\ K^- + n \to \Sigma^- + \gamma \,. \end{cases}$$

For K^+ particles the analogues of the reaction (1) (2) (5) are not possible, because they do not conserve the strangeness and only the analogues of the scattering processes (3) and (4) are strong

$$(6\text{-}19{\cdot}1)\qquad \begin{cases} K^+ + p \to K^+ + p \\ K^+ + n \to K^+ + n \end{cases}$$

$$(7\text{-}19{\cdot}1)\qquad \to K^0 + p \quad \text{(charge exchange process)}.$$

All the reactions above have been written for collisions between K and single nucleons but the same processes may take place of course when the nucleon is bound in a nucleus. In such case however real or virtual secondary processes may happen and the resulting reaction may finally take place with the partecipation of more than one nucleon in the nucleus; the difference between the K^+ and K^- possible reactions however remains. Such difference is indeed confirmed by the experiments which will be discussed in the next sections. They will also allow us to determine the relevant cross-sections and the branching ratios between the processes listed above.

19·2. – Composition of a K^+ beam after scattering.

Before passing to the discussion of the experiments in question a further point has to be considered. If, as discussed in Sects. **12**·2, **12**·3, **12**·4 there is more than one K, one may expect the interactions of these various K with the nucleon to be different; for instance, in the notations already used in Sect. **12**·2, K^+_τ may interact with the nucleons in a different way as K^+_θ and similarly K^-_τ may interact differently from K^-_θ.

There are at present two experiments which throw some light on this question; they have already been mentioned in Sect. **12**·2 and their result is negative in the sense that if a K^+_τ and a K^+_θ exist, they seem to interact in the same way with the nuclear matter. One experiment [2] is based on the

following idea: consider among the K^+ produced in an uranium nucleus those which have been surely scattered in the same production nucleus; kinematic considerations allow to choose an angle (120° with respect to the forward direction) and a momentum (340 MeV/c) for which this is most probably the case. Now if for the various kinds of K^+ the scattering cross-sections from nucleons were different, the relative abundances of the various decay modes among these scattered K^+ should be different from the relative abundances among those of the normal beam. Over a total of about 600 K^+ scattered as described above the ratio between the number of $K^+_{\pi 3}$ and the number of K^+_L's turned out to be the same as in the normal beam; the same result applies also to the ratio between the numbers of $K^+_{\pi 2}$ and the number of the other K^+_L; to this purpose however only 72 K^+_L have been examined.

The statistical errors over these figures are about 30%.

The weaker point in this experiment is the assumption that the K^+ considered have been effectively scattered inside the parent nucleus; we refer for this to the discussion in [2], where it is concluded that the fraction of scattered K^+ contained in their beam is at least 75%.

Another experiment in the same line has been performed by BISWAS *et al.* [3] in which the K^+ are not scattered inside the same production nucleus but from a nucleus of the same emulsion stack in which they are detected and arrested. A total of 146 K^+ scattering events with an energy between 50 and 110 MeV and a scattering angle larger than 40° have been considered and an examination of the decay modes of the K^+ which have undergone such a scattering has been performed; for the criteria followed in such an examination we refer to the quoted paper. Again the ratio between $K^+_{\pi 3}$ decays and other decay modes was the same as in the normal beam; and also for the other decay modes, inside the error, no variation is noticed with respect to a non scattered beam, although the statistics are poor. Though more definite experimental evidence is needed, we shall now proceed to discuss the scattering and absorption experiments of K from nucleons without further reference to the possible differences in the properties of the various K's.

19·3. – Scattering of K^+ from hydrogen.

To the present date, the number of observed K^+ scatterings by free protons is small but it will certainly increase rapidly in the future due to the ever increasing use of the hydrogen bubble chambers. So far the individuation was based on the selection of those scattering events in an emulsion which, by momentum energy conservation may be attributed to a collision of a K^+ with an H nucleus of the emulsion. The criteria for accepting a given event as due to a K-p collision are the tests of coplanarity and the equality of

the K and p transverse momentum; such tests are all well satisfied in the following events and the proton track is always sufficiently long to make the above tests accurate enough. The first two events of this kind have been observed by CHUPP and coll. [4] and a larger collection (14 events) has been assembled by BISWAS *et al.* [5]. A recent compilation by BISWAS *et al.* [6] includes 27 events observed by several groups. In the Fig. 1-**19**·3 we have reported the lengths of K^+ track followed in the various energy intervals. The total length of track followed is ~ 600 m up to an energy of about 150 MeV. In the same figure the shaded squares correspond to the primary energies of the 27 K^+-p collisions. For all of them the scattering angle in the centre of mass system is larger than 16° and for the most part of them both the K^+ and the struck proton stop in the emulsion.

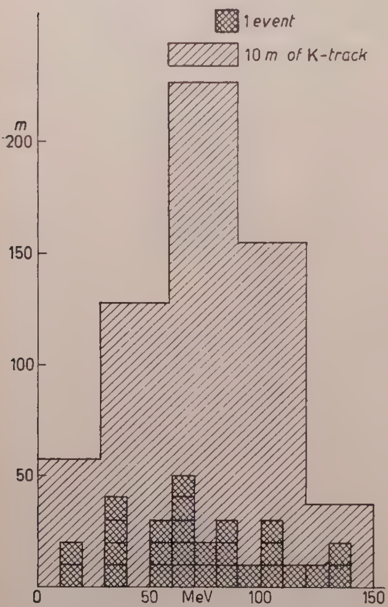


Fig. 1-**19**·3. – (From [6]). Distribution of the primary energy of 27 K^+-p collisions and of the K-track length observed in various intervals.

The mean free path for K^+-p scattering ($\chi_{c.m.} \geqslant 16°$) results to be $600/27 \simeq 22$ m. The corresponding total cross-section is found to be (14 ± 3) mb. In Fig. 2-**19**·3 the angular distribution of the 27 events is reported in the centre of mass system (*).

Agreement with the angular distribution of Fig. 2-**19**·3, is obtained by assuming in addition to the Coulomb interaction, an S wave nuclear repulsive interaction; in this way have been obtained the hystograms reported in Fig. 2-**19**·3 and calculated as the mean of a number of distributions corresponding to the energies over which the collisions were observed.

Let us close this section with a few theoretical remarks:

1) Call $(K^+p|K^+p)$, $(K^+n|K^+n)$, $(K^+n|K^+p)$ the amplitudes of the processes (3-**19**·1) and (4-**19**·1) in which we are interested. In view of the invariance of the theory with respect to rotations in the isotopic spin space the three amplitudes above may be expressed in terms of two scattering amplitudes, R_0 and R_1, respectively in $T=0$ and $T=1$ state.

(*) Graphs for the transformation laboratory-centre of mass system may be found in [7].

We have:

$$(1\text{-}19.3)\quad \begin{cases} (K^+p\,|\,K^+p) = R_1\,, \\ (K^+n\,|\,K^+n) = \tfrac{1}{2}(R_1 - R_0)\,, \\ (K^+n\,|\,K^0p) = \tfrac{1}{2}(R_1 + R_0)\,. \end{cases}$$

An inequality which, in particular, may be derived from (1) at each angle is

$$(2\text{-}19.3)\qquad d\sigma(K^+n\,|\,K^+n) + d\sigma(K^+n\,|\,K^0p) \geqslant \geqslant \tfrac{1}{2}\, d\sigma(K^+p\,|\,K^+p)\,.$$

In words: The cross-section for interaction of a K^+ with a neutron should be (at a given energy) equal or larger than one half that of a K^+ with a proton; the equality corresponds to a vanishing amplitude in the $T=0$ state.

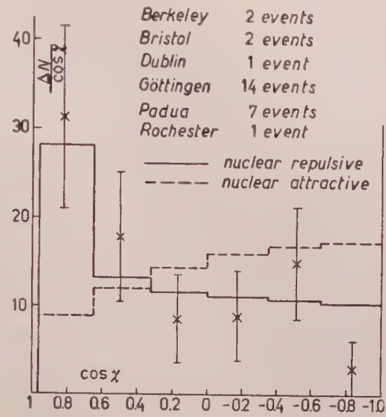


Fig. 2-19.3. – (From [6]). χ=centre of mass scattering angle. Distribution of the angles χ for 27 K^+-p collisions. The curves correspond to the cases of interference between the Coulomb field and: ——— a repulsive nuclear potential; --------- an attractive nuclear potential.

2) Ceolin and Taffara [8] have performed a perturbative calculation of the K^+ nucleon cross-section, that is of R_0 and R_1, using an interaction of the form (Sect. **11.10**)

$$(3\text{-}19.3)\qquad g_{\Lambda_0}\Phi^+_{K^+}(\psi^+_{\Lambda^0}\Gamma\psi_{\text{Nucleon}}) + g_\Sigma\Phi^+_{K^+}(\psi^+_{\Sigma_i}\tau_i\Gamma\psi_{\text{Nucleon}}) + \text{h. c.}$$

A summation over i is understood. Γ stands for β if the parity of the K is the same as that of the Λ^0-nucleon or Σ nucleon pair and for $\alpha_1\alpha_2\alpha_3\beta = i\gamma_5\gamma_4$ if the opposite is true.

Both the cases have been considered; not the case in which the parity of the Λ^0 is opposite to that of the Σ.

They obtain for R_1 and R_0 in (1) the expressions:

$$(4\text{-}19.3)\qquad \begin{cases} R_1 = g^2_\Lambda M(\Lambda) + g^2_\Sigma M(\Sigma)\,, \\ R_0 = g^2_\Lambda M(\Lambda) - 3g^2_\Sigma M(\Sigma)\,. \end{cases}$$

In (4) $M(\Lambda)$ is the matrix element of (3) corresponding to the sum of the processes

$$\begin{cases} K^+ + p \to K^+ + K^{+\prime} + \Lambda^0 \to K^{+\prime} + p'\,, \\ K^+ + p \to p + p' + \overline{\Lambda}^0 \to K^{+\prime} + p' \end{cases}$$

and $M(\Sigma)$ is the similar expression with Λ^0 replaced by Σ^0. From (2) and (4) one derives:

$$(5\text{-}\mathbf{19}{\cdot}3)\qquad \begin{cases} (\mathrm{K^+p}\,|\,\mathrm{K^+p}) = \;\; g_\Lambda^2 M(\Lambda) + g_\Sigma^2 M(\Lambda) \\ (\mathrm{K^+n}\,|\,\mathrm{K^+n}) = 2g_\Sigma^2 M(\Sigma)\,, \\ (\mathrm{K^+n}\,|\,\mathrm{K^0\,p}) = \;\; g_\Lambda^2 M(\Lambda) - g_\Sigma^2 (M\Sigma)\,. \end{cases}$$

The cross-sections for the corresponding processes may be easily derived from (5) inserting the expressions for $M(\Lambda)$ and $M(\Sigma)$, squaring, and multiplying by $2\pi\varrho/v$, where ϱ is the density of the final states and v the velocity of the incident K's.

The angular and energy behaviour for the $(\mathrm{K^+p}\,|\,\mathrm{K^-p})$ process assuming $g_\Lambda^2 = g_\Sigma^2$ (such behaviour is however almost independent from the g_Λ^2/g_Σ^2 ratio depending almost entirely only on $g_\Lambda^2 + g_\Sigma^2$) are reported graphically in the laboratory and centre of mass systems in Fig. 3-**19**·3 and 4-**19**·3 both for the scalar and pseudoscalar case; the angular distributions were calculated at a kinetic energy of 80 MeV. The angular distributions are compatible with the Coulomb corrected experimental ones. Nothing can be said on the energy distributions.

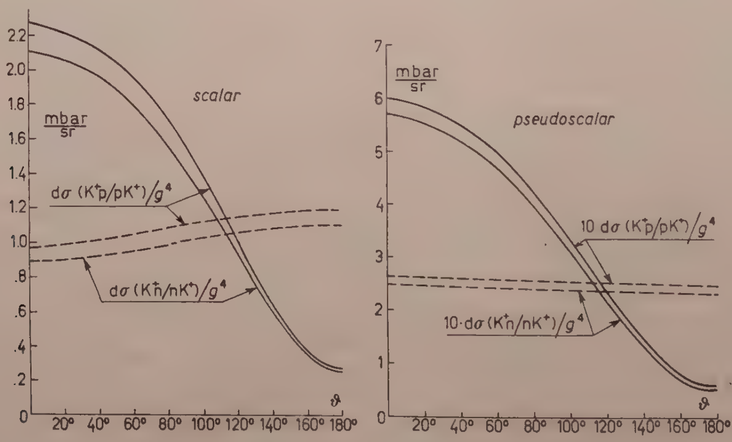


Fig. 3-**19**·3. – (From [8]).

---- differential cross-section in centre of mass system;
——— differential cross-section in laboratory system.

-- ·- differential cross-sectino in centre of mass system;
——— differential cross-section in laboratory system.

It is important to notice that the order of magnitude of the scalar cross-section is essentially determined by the square of the K Compton wave length while that of the pseudoscalar is determined by the square of the Λ^0 (or Σ^0)

Compton wave length. This is apparent from the following formulas which give the total cross-section in the scalar and pseudoscalar case at zero kinetic energy for the case $g^2_\Sigma=0$ ($\hbar=c=1$),

$$\text{(6-19.3)}\quad \begin{cases} \sigma_{sc} = 4\pi \dfrac{g^2_{\Lambda^0 sc}}{16\pi^2}\dfrac{M^2}{(M+\mu)^2}\dfrac{1}{(M-\mu-M_{\Lambda^0})^2}\,, \\[2ex] \sigma_{ps} = 4\pi \dfrac{g^2_{\Lambda^0 ps}}{16\pi^2}\dfrac{M^2}{(M+\mu)^2}\dfrac{1}{(M-\mu+M_{\Lambda^0})^2}\,. \end{cases}$$

Here M, μ, M_{Λ^0} are respectively the masses of nucleon, K and Λ^0.

A determination of $g^2_{\Lambda^0}/4\pi$ and of $g^2_\Sigma/4\pi$ at $E_{kin} = 80$ MeV gives, assuming the two to be equal and comparing σ with the experimental value of 10 mb for the total K^+ p cross-section with Coulomb scattering subtracted:

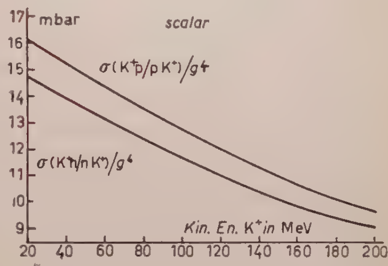
$$\text{(7-19.3)}\qquad \frac{g^2_{\Sigma sc}}{4\pi} = \frac{g^2_{\Lambda sc}}{4\pi} \simeq 0.8\,,$$

and

$$\frac{g^2_{\Sigma ps}}{4\pi} = \frac{g^2_{\Lambda ps}}{4\pi} \simeq 1.9\,.$$

The coupling constants are therefore larger than the pion-nucleon ones in the scalar case and smaller in the pseudoscalar.

The order of magnitude of the scalar coupling constant is similar to that derived [9] from the production cross-section of $K^0\Lambda^0$ pairs and from the binding energy of hyperfragments.



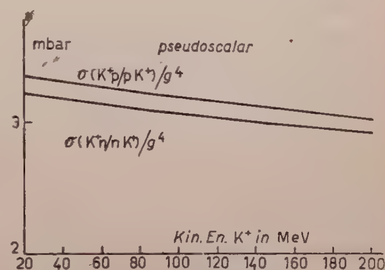


Fig. 4-19.3. – Energy dependence of the cross-sections for K^+ n and K^+ p scattering in the laboratory system (from [8]). (Scalar and pseudoscalar case).

We finally add that also the ratios:

$$\alpha = \frac{\text{no. of processes with ch. exch.}}{\text{no. of processes without ch. exch.}} \qquad \text{and} \qquad \alpha' = \frac{\sigma(K^+ n)}{\sigma(K^+ p)}\,,$$

have been calculated in [8]; they are of course strongly dependent on g^2_Λ/g^2_Σ and almost independent from the energy. The experimental data are as yet not sufficient to discuss these expressions.

19·4. – Interaction of K^+ with complex nuclei.

The interactions of K^+ with complex nuclei have been mainly (*) studied using stacks of nuclear emulsions exposed to the K^+ beam of the bevatron at a distance corresponding to a time of flight of the order 10^{-8} s.

The method used is the scanning along the track.

In the most part of the interactions observed the K^+ is simply scattered losing possibly a fraction of its original energy and giving rise, in part of the cases, to a small star. In a much smaller fraction of the interactions the K^+ disappears producing or not a small star; however *in no case the visible energy of the star which is sometimes produced does exceed the kinetic energy of the impinging* K^+, in agreement with the theoretical rules; in particular the disappearance of a K^+ is consistent with an exchange reaction (like (7-**19**·1)), and not with an absorption which would be contrary to the theoretical scheme.

The first results on the K^+ nucleus interactions were published by the Berkeley group [11]; the energy of the impinging K^+ varied from 30 to 120 MeV. Other experiments with K^+ in similar energy intervals were performed by the Bologna [12], Bristol [14], Dublin [14], Padua [15] and Göttingen [6] groups. These last two groups have collected the largest statistics published up to now and the following discussion will be largely based on their results, and on a paper [13] by the Bologna group.

The following Table I-**19**·4 reports some data of the Padua and Göttingen events.

TABLE I-**19**·4.

Lab.	Energy interval (MeV)	Total track length (m)	*A* scattering events (+)	*B* scattering events	Charge exchange events	Cut off angle used
Padua	40 ÷ 160	110	345 (213)	46	10	12°
Göttingen	60 ÷ 100	∼ 105 (*)	208 (154)		not given	12° (20°)

(+) The numbers in brackets are those of scattering events in fact observed; the other figures are corrected to take into account the geometrical losses. Use of the corrected figures will always be made in the following. H scattering events are not included.

(*) This number may be still subject to some slight variation, according to ref. [6], as a consequence of the difficulties in estimating the contamination by background tracks.

Some explanations are necessary in connection with the table.

(*) For two scattering events observed from cosmic rays, compare e.g. [10].

The A scattering events are those in which no detectable star is formed at the struck nucleus. Those events have been collected for which the projection of the scattering angle in the plane of the emulsion is larger that the cut-off angle defined in the last column. Moreover, in the Göttingen events, the scattering angle itself had to be larger than 20°. The B scattering events are those in which a detectable star (having at least a charged prong longer than 5 μm) is formed by the struck nucleus. The Göttingen group does not report separately those two kinds of events.

The following points will now be discussed.

1) Separation of the events into elastic and inelastic scatterings; the relevant differential and total cross-section.

2) The charge exchange events.

1) *Elastic and inelastic events.* – The natural definition of elastic and inelastic events is of course the following; a scattering event is called elastic if the energy ΔE transferred from the K^+ to the collided nucleus is just the recoil energy of the nucleus $E_{\text{rec.}}$ corresponding to the initial energy E_1 of the K^+ and to the observed scattering angle of the K^+: $\Delta E = E_{\text{rec}}$; an event is called inelastic if $\Delta E > E_{\text{rec}}$; it is clear in particular that all the scattering events of class B are inelastic. But the situation is not so simple for the class A events, mainly on account of the experimental errors in the measurements of ΔE.

Let us plot, following the Padua group, the observed events of class A as a function of the measured value of $\Delta E/E_1$, the relative energy loss of the K^+ (Fig. 1-19·4).

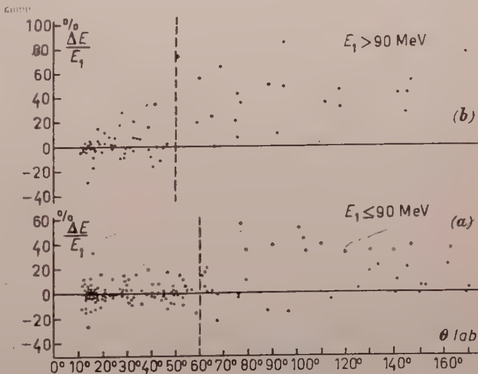


Fig. 1-19·4. – (From [15]). Plot of relative energy loss ($\Delta E/E_1$) vs. angle of scattering for two energy intervals.

It is apparent that values of $\Delta E/E_1 \simeq -0.2$ are by no means improbable. Since it is impossible that a K^+ acquires energy in the scattering process, this simply means that we have to deal with experimental errors in the determination of $\Delta E/E_1$ which may be as large as 20%.

With such errors, all what one can reasonably do is perhaps to call *inelastic* all the events for which $\Delta E/E_1 \geqslant 0.2$ and *elastic* all the ones for which $\Delta E/E_1 \leqslant 0.2$.

With this definition, which will be always used from now on, the inelastic events are almost certainly inelastic in the proper sense of the word, but the

elastic ones may contain a fraction of slightly inelastic events, that is of events with nuclear excitation less or equal, on the average, to ~ 20 MeV.

The numbers of the so defined inelastic events is 67 for the Göttingen group (this is the figure which may be derived from the Table I, p. 126 of [6]). It includes also the scattering events giving rise to a star and satisfying $\Delta E/E_1 \geqslant 0.2$) and 83 for the Padua group (this is the figure which may be obtained adding to the number of A events with $\Delta E/E_1 \geqslant 0.2$, that part of the 46 events of class B having $\Delta E/E_1 \geqslant 0.2$ (in fact all but one)).

The remaining events are elastic events. The mean free path for inelastic collisions in emulsion (always with the meaning specified above of the word inelastic) are reported in the following Table II-**19**·4 both for the Göttingen and Padua events.

TABLE II-**19**·4. – *Inelastic events.*

Laboratory	Number of inelastic scatterings	Inelastic mean free path in emulsion (+)	Corresponding inelastic cross section per average nucleus of the emulsion (+) (*)
Göttingen	67	105/67 = 1.58 m	—
Padua $40 < E < 90$	29 $(17A + 12B)$	53/29 = 1.81 m	110 mb
Padua $90 < E < 160$	54 $(21A + 33B)$	57/54 = 1.05 m	220 mb

(+) Excluding charge exchange events.
(*) The average nucleus of the emulsion is defined as having $Z = 22$, $A = 49$. The cross section is calculated according to $\sigma = (\lambda N)^{-1}$ where λ is the mean free path and N is the number of (average) nuclei per cm³.

Here for the Padua events, we have divided the total energy interval into two parts and taken into account that the total track length in the first $(40 < E < 90$ MeV) has been 53 m and in the other $(90 < E < 160$ MeV) has been 57 m. This subdivision shows a considerable decrease of the inelastic mean free path with increasing energy, as expected.

In the last column of the table we have also reported the average inelastic cross-section $\bar{\sigma}$ per nucleus of the emulsion (*), $\bar{\sigma}$ being defined as $(\lambda N)^{-1}$, where N is the number of average nuclei per cm². (Density of the emulsion $\varrho \simeq 3.9$ g/cm²). Charge exchanges are not included.

(*) The Göttingen angular distribution for inelastic scattering is plotted in Fig. 2-**19**·4.

The following Table III-**19**·4 contains instead the number of elastic events, the relevant mean free path and the corresponding cross-section per average nucleus of the emulsion (the emulsion composition is taken as 43% of the nuclei with $A = 95$, $Z = 42$ and 57% with $A = 14$, $Z = 7$; the average nucleus of the emulsion is defined as having $A = 49$, $Z = 22$).

TABLE III-**19**·4. – *Elastic scattering mean free path for the Padua events* ($\theta > 12°$).

	Number of elastic scattering events	Mean free path	Elastic cross section per average nucleus
$40 < E < 90$	209	0.25 m	(848 ± 78 mb)
$90 < E < 160$	98	0.58 m	(366 ± 51 mb)

Of course the cross-sections here depends critically on the cut-off angle due to Coulomb scattering.

The important point about the values of the cross-sections reported in Table III-**19**·4 is that they are more than three times larger than the ones for pure Coulomb scattering showing the existence of a nuclear potential acting on the K^+. Moreover from the value of the total elastic scattering cross-section and from the angular distribution of such scattering (the region $7° < \theta < 12°$ was explored by the Padua group in an additional set of measurements) the interference may be shown to be constructive (compare the next section).

The Padua angular distributions of the elastic events are reproduced in Fig. 3-**19**·4; on this point there is some discrepancy with the results of the Göttingen group, which have a smaller number of elastic events at large angles.

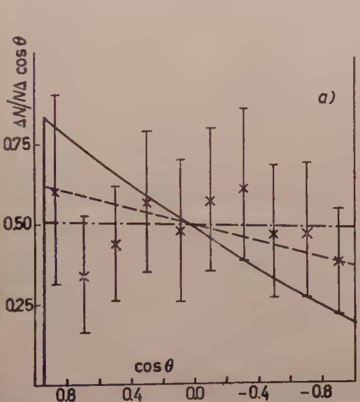


Fig. 2-19.4. – Angular distribution of inelastic scattering events (from [6]). The curves correspond to the Monte-Carlo calculations under various assumptions (compare ref. [6]).

2) *Charge exchange events.* – As is apparent from Table I-**19**·4, the total number of the charge exchange interactions observed by the Padua group is 10 over a total of 84 inelastic interactions (with our definition of inelastic). Experimentally the identification of a charge exchange interaction event is not straightforward, for the following reasons: charge exchange events may take place in two ways: *a*) a K^+ gives rise to a small star from which no prong which is identified as a K^+ appears to come out; 6 events (among the 10 above) belong to this class; in this case one

must pay attention to the short prongs for which the distinction between K$^+$ and protons is not possible and to the fact that not always the secondaries from the K$^+$ decay are visible; *b*) an identified K$^+$ stops in flight; events of

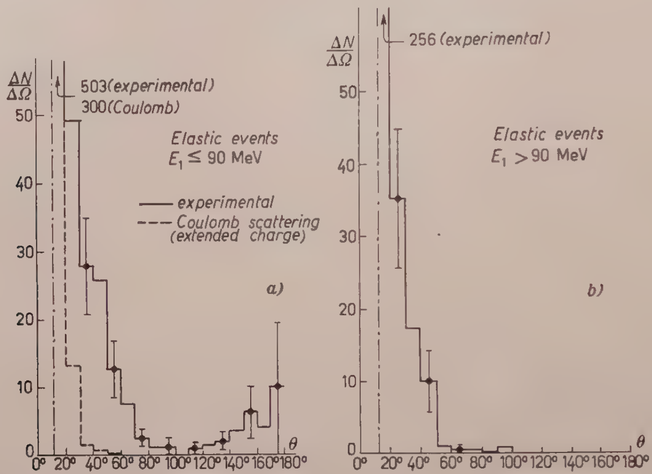


Fig. 3-19·4. – (From [15]). – Angular distributions of elastic events for two energy intervals.

this kind may be either charge exchanges in some nucleus without the emission of charged secondaries from the nucleus, or decays in flight with a non visible secondary. Among 13 events of this kind, according to ref [15], 9 are decays in flight and 4 are charge exchanges.

19·5. – Interpretation of the results on K$^+$-nucleus scattering.

Essentially two kinds of information one may try to derive from the results of K$^+$-nucleus scattering: 1) information on the K$^+$-neutron scattering (from inelastic scattering data); 2) information on the potential between nucleus and K$^+$, apart from the Coulomb potential (mainly from elastic scattering).

We first examine the second point, and show that there is evidence for a nucleus-K$^+$ repulsive potential.

We simply report the facts, which, according to [16], support the above conclusion (*); the calculations are made with an optical model of the nucleus and only the real part of this potential essentially intervenes in the elastic

(*) According to [17] the (preliminary) data were in favour of an attractive potential, but a later paper [18] is in agreement with the conclusions of [16].

scattering. This potential is the sum of a Coulomb part (where account is taken of the extension of the nucleus) and of a purely nuclear potential V_n. Now, according to [16] the value of the total elastic cross-section from 20° on determines the nuclear potential V_N as $V_N = +14$ MeV or -19 MeV (the extension of this potential well is supposed to be that of the nucleus, with a radius $R = 1.36\,A^{\frac{1}{3}}\cdot 10^{-13}$ cm).

But when account is taken of the small angle scattering events, which are in large excess of the ones expected for a pure Coulomb scattering (Fig. 3-**19**·4), the need for a constructive interference, hence for a repulsive V_N, appears.

Hence the first choice of V_N (+14 MeV) seems indicated: the small angle elastic scattering events would, themselves alone, give a somewhat higher value for V_N, namely (19 ± 7) MeV.

Another argument which both COSTA and PATERGNANI [16] and the Bologna and Göttingen groups report as a *check* for a repulsive nuclearpot ential is the behaviour of the distribution of $\Delta E/E_1$ for the inelastic events, with increasing E_1. They show in particular that the fact that there is a lack of large energy losses for backward scattered K^+ is explained by a repulsive potential with the above order of magnitude, if one wants to maintain the model of collisions against individual nucleons.

We now come to the first point, the evidence on the K^+-n cross-section. Assuming the inelastic collisions to be collisions between the K^+ and single nucleons in the nucleus, one may try to derive, taking into account of course the very important (at these energies) role of the Pauli principle, from the observed K^+-nucle*us* inelastic cross-section, the average cross-section with a free nucle*on*. Since we know, on the other hand, the cross-section with a free proton from the K^+-H scattering data (Coulomb scattering subtracted) it is possible to determine the cross-section with a free neutron (+).

If this is done, according to the Padua group (compare [15] for a detailed discussion, noticing however that the footnote (17) of [15] can hardly be reconciled with the general relation (2-**19**·3)) the K^+ n cross-section (charge exchange included) turns out to be much smaller than the K^+ p one; in other words the observed total inelastic K^+ nucleus cross-section appears to be explainable, according to the Padua group, almost without invoking the participation

(+) The cross section for K^+-H scattering (Coulomb scattering subtracted) is of the order $10 \div 11$ mb. The average cross section for scattering by a free nucleon, determined from the K^+ nucleus scattering, taking into account the effect of the Pauli principle is of the order, charge exch. included, of 8 mb [13]. It turns out that the average cross section for scattering by a neutron must be $5 \div 6$ mb, or about $\frac{1}{2}$ of that for proton scattering; according to (2-**19**·3) this means scattering in the pure $T = 1$ state; however the above figures are still affected by rather large errors and have been quoted here just to make the argument more concrete.

of neutrons. This would point to the conclusion (at least at the lower energies) that the dominant state is the $T=1$ state, in which case the inequality (2-**19**·3) may be satisfied with the sign of equal.

In our opinion however, in view of the still large experimental errors and of the uncertainties of the theoretical treatment in the derivation of the elementary cross-section from the K^+-nucleus one, this conclusion has to be taken with much caution.

It is true that it is perhaps supported also by the smallness of the K^+ nucleus exchange scattering; with a pure $T=1$ state one would expect, in collisions against a gas of neutrons and protons, in equal numbers, a ratio $\frac{1}{6}$ for the exchange scattering to the total scattering (compare again the formulas (1-**19**·3)). But also here the situation is not so simple, both theoretically and experimentally.

Added note (April 15, 1957).

Results from the Bristol-Dublin groups (B. Bhowmik, D. Evans, S. Nillson, D. Prowse, F. Anderson, D. Keefe, A. Kernan, J. Losty - to be published) at energies from 0 to 130 MeV have arrived when this review was ready for publication. They are generally consistent with the above data.

In particular: *a*) The average ($60 < E < 130$ MeV) inelastic cross-section per average nucleus of the emulsion is ~ 150 mb. *b*) The ratio of charge exchange to non charge exchange events is 0.20 ± 0.07 (18 events have been interpreted as charge exchanges). *c*) The interpretation of the Coulomb scattering leads to essentially the same values of the nuclear potential as already reported. *d*) A detailed analysis made on the same lines as that in Sect. **19**·5 leads to a value 0.1 ± 0.7 for the ratio between the square of the scattering amplitude in the $T=0$ and that in the $T=1$ state. The above results refer to 148 m of K^+ track scanned.

REFERENCES (Ch. 19)

[1] M. Gell-Mann and A. Pais: *Proceed. of the Glasgow Conf.* (1954).

[2] W. Widgoff, A. Shapiro, R. Schluter, D. M. Ritson, A. Pevsner and V. Henri: *Phys. Rev.*, **104**, 811 (1956).

[3] N. N. Biswas, L. Ceccarelli Fabbrichesi, M. Ceccarelli, K. Gottstein, N. C. Varshneya and P. Waloschek: *Nuovo Cimento*, **4**, 631 (1956).

[4] W. W. Chupp, G. Goldhaber, S. Goldhaber, Johnson and J. E. Lannutti: *Phys. Rev.*, **99**, 1042 (1955).

[5] N. N. Biswas, L. Ceccarelli-Fabbrichesi, M. Ceccarelli, M. Cresti, K. Gottstein and N. C. Varshneya: *Nuovo Cimento*, **3**, 1481 (1956).

[6] N. N. Biswas, L. Ceccarelli-Fabbrichesi, M. Ceccarelli, K. Gottstein, N. C. Varshneya and P. Waloschek: *Nuovo Cimento*, **5**, 123 (1957).
[7] E. Pedretti: *Nuovo Cimento* **2**, 956, (1956).
[8] C. Ceolin and L. Taffara: *Nuovo Cimento*, **5**, 123 (1957).
[9] S. Iwao: *Progr. of Theor. Phys.* (*Japan*), **13**, 111 (1955).
[10] M. W. Friedlander, D. Keefe and M. G. K. Menon: *Nuovo Cimento*, **1**, 694 (1955).
[11] J. E. Lannutti, W. E. Chupp, G. Goldhaber, S. Goldhaber, E. Helmy, E. L. Iloff, A. Pevsner and D. M. Ritson: *Phys. Rev.*, **101**, 1617 (1956).
[12] C. Marchi, E. Pedretti and S. Stantic: *Nuovo Cimento*, **4**, 940 (1956).
[13] G. Cocconi, G. Puppi, G. Quareni and A. Stanghellini: *Nuovo Cimento*, **5**, 172 (1957).
[14] N. Dallaporta: *Rochester Conference* (1956).
[15] M. Baldo Ceolin, M. Cresti, N. Dallaporta, M. Grilli, L. Guerriero, M. Merlin, G. A. Salandin and G. Zago: *Nuovo Cimento*, **5**, 393 (1957).
[16] G. Costa and G. Patergnani: *Nuovo Cimento*, **5**, 448 (1957).
[17] L. S. Osborne: *Phys. Rev.*, **102**, 296 (1956).
[18] D. Fournet Davis: to be published.

Chapter 20.

Interactions of K^- and negative hyperons with nucleons.

20.1. – K^- interactions in flight with complex nuclei.

In discussing the K^- interactions it is convenient to distinguish between the interactions with H and with complex nuclei; we shall here follow the historical order and first describe the interactions with complex nuclei which have been mainly obtained through the use of nuclear emulsions and artificially produced K^-. Later (Sect. **20**·6) we shall report on the K^--H interactions which furnish much more detailed information, of course, and on which we have now, by means of the hydrogen bubble chamber rather abundant data [1].

Dealing with the K^- complex nucleus interactions we shall first discuss the interactions in flight, and later the interactions at rest. This section will consider the interactions in flight.

Again the method followed in finding the interactions in flight is generally the same used for the K^+, that of « scanning along the track », which does not introduce any bias in connection with the type of interaction found.

Now which is the appearence of these interactions in flight? The first point to be noted is that almost without exception they lead to the disappearance of the incident K^- and to a rather larger star; the reason is that contrary to the K^+ case, the reactions (1-**19**·1) and (2-**19**·1) may now take place; what the data show is that at least in complex nuclei they are overwhelmingly dominant over the elastic (or inelastic) nuclear scattering reaction (of course small angle Coulomb scattering is always present). Also the charge-exchange reaction seems very unfrequent. These conclusions appear clearly from the following table I in which 9 events of interaction in flight found by Hornbostel and Salant [2] are described as an example.

The Table I-**20**·1 is self explanatory as far as columns 1 to 5 are concerned; the contents of column 6 will be clear after the discussion of the K^- interactions at rest. The point which is important for the moment is that out of 9 interactions only one (the last) is possibly a nuclear elastic scattering, one (the eighth)

is possibly a charge exchange and all the other 7 show visible energies which necessarily imply the absorption of the initial K^-; in 4 of these 7 events, the emission of a hyperon is observed in agreement with the theoretical scheme; clearly such an emission, though undetected, is consistent with all of them.

TABLE I-20·1.

Kinetic energy of K^- (MeV)	Kinetic energies (MeV) and identity of star particles			Visible star energy (MeV)	Presumed reaction
	Hyperon	Pion	Stable heavy prongs		
74 ± 6	$58\ \Sigma^-$	100^{+60}_{-30}		558^{+60}_{-30}	$K^-+p\to\Sigma^-+\pi^+$
40 ± 12	$31\ {}^4_\Lambda H$		5p; 13α; $\sim$45p	327 ± 27	$K^-+n\to Y+\pi^0$
50 ± 8	hyperfrag.		11p; 66p	273	$K^-+n\to Y+\pi^0$
70	hyperfrag.	100^{+40}_{-20}	$\sim$40p; (13p)	465^{+40}_{-20}	$K^-+n\to Y+\pi^-$
72 ± 6		66 ± 2		205 ± 2	$K^-+n\to\Sigma^0+\pi^-$
45 ± 5		100^{+40}_{-20}	20f; 45p	314^{+40}_{-20}	$K^-+n\to\Lambda^0+\pi^-$
58 ± 4			11α; 10α; 45p; 5p	99 ± 14	$K^-+p\to Y^0+\pi^0$
56 ± 3			14p	24 ± 5	$K^-+p\to K^0+n$ or $K^-+p\to Y^0+\pi^0$
76					elastic nuclear scattering

The same features appear from a larger statistics (24 interactions in flight excluding Coulomb scatterings) collected by FOURNET and WIDGOFF [3] at energies up to 100 MeV. Of these 24 interactions 16 are surely absorptions, 1 is an elastic nuclear scattering, 7 could possibly be charge exchange scatterings. Among the stars corresponding to the 16 certain absorptions, 3 show the emission of a charged pion, 5 of a hyperon or hyperfragment, 1 of both; all are consistent (energetically) with the emission of a hyperon.

Passing now to the total cross-section for the nuclear interaction of K^- in flight, it is to be expected that on account of the higher number of final states involved it should be larger than that for K^+ interaction; that this is so is shown by the following Table II-20·1 [5] which contains the results of the Authors said above and also the ones of the Berkeley groups [4, 5]. Elastic nuclear scattering is excluded.

TABLE II-20·1. – (from S. GOLDHABER [5]).

Author	No. of interactions in flight ($30 < E < 100$ MeV)	Track length explored (m)	Mean free path (cm)
[2]	8	1.36	17 ± 5
[4, 5]	21	4.88	23 ± 5
	6	2.77	46 ±19
[3]	23	8.68	36 ± 9
Total	58	17.69 (*)	30.5 ± 4 cm

(*) A figure 27.2 ± 2.3 cm in emulsion has been obtained by [6] from along the track scanning of 1 224 K^- at energies from 30 to 90 MeV.

It is apparent that the mean free path λ is more than four times smaller than the corresponding one for the inelastic scattering plus charge exchange of K^+ in the same energy interval. The above value of λ corresponds to a geometrical cross-section.

A feature of the above data which is worth stressing is the smallness of the elastic and also of the inelastic nuclear scattering; a possible reason for this, pointed out by HORNBOSTEL and SALANT [2], may be the fact that the K^- interacts with more than one nucleon in the same nucleus, so that it is finally absorbed.

Another point on which further investigation is needed is the frequency of the charge exchange events (reaction (3-19·1)). In principle any interaction star in which the K^- disappears and the visible energy release is less than the initial kinetic energy of the K^- may belong to this class; but it also may not. If the exchange reaction is frequent one should observe that the percentage of 0 and 1 prong stars due to K^- absorption in flight are larger than the corresponding percentages when the K^- is absorbed at rest, because in

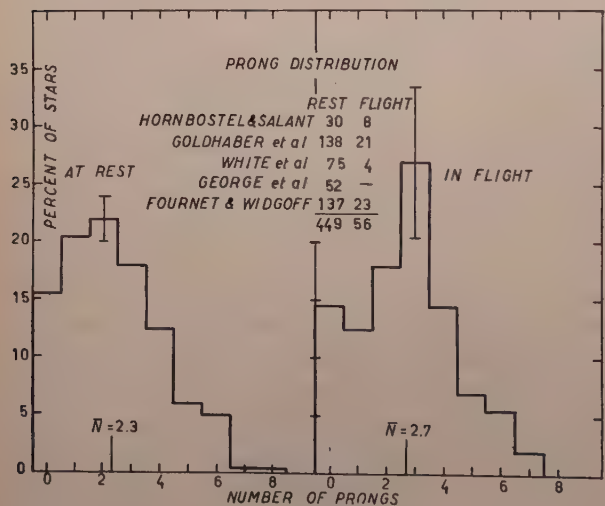


Fig. 1-20·1. – (From [5]).

the latter case the charge exchange process presumably cannot take place. This is not observed, as it is apparent from the preceding graphs (1-**20**·1) taken from S. GOLDHABER's compilation [5] (*).

20·2. – Interactions of K^- at rest. Historical introduction.

The first examples of stars produced in the nuclear absorption of K^- at rest were detected in nuclear emulsions (compare for instance [7-9]) and are summarized in [10]; it was immediately noticed that the visible energies of such stars were less than the mass of the K^-. Some time elapsed before the emission of neutral hyperons was first detected by the cloud chamber observations of BARKER and of DE STAEBLER [11, 12]; later the Genoa-Milan group [13] gave the first example of absorption in nuclear emulsion of a K^- at rest with emission of a charged hyperon. Thus examples of K^- absorption reactions in agreement with the theoretical scheme (compare reactions 1, 2-**19**·1)) were available.

Referring from now on to the work on nuclear emulsions with which the most part of the observations have been made, the question is then: are also those absorption events, in which no emission of hyperons is actually seen, consistent with the emission of a neutral hyperon in agreement with the theoretical scheme?

An argument presented by TOUSCHEK [14] even before knowing that the actual emission of a hyperon in the absorption events was observed, shows that that this is the case; that is, the visible energy release in those K^- absorption stars in which no hyperon is apparently emitted, may be simply interpreted only if the assumption is made that a neutral hyperon is emitted in agreement with one of the reactions (1-**19**·1) and (2-**19**·1).

The point is simply that in those events in which no hyperon is seen but a pion is emitted, such a pion has a relatively low energy; in the events discussed by TOUSCHEK it never exceeded 54 MeV (now there are examples with pions up to 160 MeV). If we then consider the various processes which might a priori be possible such as:

$$K^- + N \to N + \pi \tag{1-20·2}$$

$$K^- + N \to N + \pi + \gamma \tag{2-20·2}$$

one realizes that the π energy might be generally much larger than the above

(*) This compilation has been extensively used throughout this chapter.

figure, being for example around 300 MeV in case (1) and extending up to a similar value in case (2). One may then try to explain the absorption assuming the reaction to take place according to a scheme like:

$$\text{(3-20.2)} \qquad K^- + N \to Y^0 + \pi ,$$

where Y^0 is a new particle and ask what the mass of Y^0 must be in order that the pion energy does not exceed some given value E_π; for $E_\pi = 54$ MeV TOUSCHEK found $m_Y = 1240$ MeV in rough agreement with the mass of a Σ^0; this provides evidence that in fact the absorption process is consistent with the emission of a hyperon.

At the time in which the above considerations were made only about ten events of K^- absorption at rest were available; now (summer 1956) ~ 750 events have been reported. The main features of such events will be described in the next section; in Sect. **20.4** a more detailed discussion is given.

20·3. – The main features of the K^- absorption events at rest.

The largest part of the above 750 cases of K^- absorption have been observed with artificially produced K^-; only a small fraction are cosmic ray events, and in part (45) these have been reviewed [15] at the Pisa Conference.

Not all the authors have published all the information in their possession so that part of the discussion below will have to be based on very partial data.

The main set of data from artificial K^- are due to HORNBOSTEL and SALANT [2, 16], GOLDHABER *et al.* [17], GEORGE *et al.* [18], FOURNET and WIDGOFF [3], SCHEIN *et al.* [19], FRY *et al.* [20], WHITE *et al.* [21].

The proper identification of an event as a K^- capture is made very easy by the artificial production of K^-, so we shall not insist upon this part of the problem.

The methods used are two: the area scanning and the scanning along the track. In the first method small stars may be undetected while the second method is not affected by biasses of such kind; therefore for making statistics of the stars as a function of their number of prongs, only the second method is reasonable, at least if one wants to include stars with a small number of prongs.

The stars produced by the K^- absorption may be divided into several classes:

1) zero-prongs stars, that is events in which a K^- stops at some point of the emulsion and no visible prong comes out. Events of such kind are also called K^-_ϱ;

2) stars from which only stable charged prongs are emitted;

3) stars which contain one charged pion, but not a charged hyperon;

4) stars which contain one charged hyperon but not a charged pion;

5) stars which contain both a charged hyperon and a charged pion.

In the following Table I the subdivision of the stars into the above categories, observed by the groups who have used the scanning along the track method are reported.

It appears from the Table I that about 20% of the K^- terminate without giving rise to a visible star, a figure to be compared with 30% for pions. The cases in which a hyperon is seen are $\sim 18\%$ and the cases in which a pion is seen (not accompanied by a hyperon) are $\sim 22\%$. The rest 42% are stars containing only visible stable prongs (p, α, H, D, etc.). Some of these figures (more precisely the one referring to the fraction of pions and to the fraction of hyperons accompanied by pions) may be found also in the compilation by GOLDHABER referring to 410 K^- [5].

TABLE I-20'3.

Author	Total number of stars	Percentage of stars of class					
		1° K^-_ϱ	2° Stable prongs only	3° π but not Y	4° Y but not π	5° Y and π	
[2]	30	5	8	8	5	4	along the track scanning
[3]	137	24	55	32	16	10	
[18]	37	11	20	4	1	1	
[18]	22	1	12	6	1	2	
Total	224	41 = 18%	95 = 42%	50 = 22%	23 = 10%	17 = 8%	
[19] (*)	67		37 = 45%	16 = 20%	8 = 10%	6 = 8%	area scanning
[20] (+)	30		16 = 45%	8 = 24%	5 = 14%	1 = 3%	

(*) The percentages refer to $67(1 + 18/100) = 80$ events so as to make them comparable to the above ones.

(+) The percentages refer to $30(1 + 18/100) = 35$ events so as to make them comparable to the above ones. The data given in a paper referring to 207 captures are insufficient to compile this table.

20·4. – Discussion of the various kinds of events.

The absorption events belonging to the various groups 1 to 5 of the past section may now be separately discussed (compare also [2, 22]). The purpose of such a discussion will be to amplify that of Sect. **20·3** and show that all the events are consistent with the reactions allowed by the theoretical scheme.

To facilitate the following discussion it may be useful to give [2] the expected energy releases for the various reactions which may take place, in order to compare them with the observed ones.

1) Reaction $K^- + «N» \to \Sigma + \pi$. The Q of this reaction, taking into account the fact that the nucleon which absorbs the K^- is bound as indicated by the dashes, is 94 MeV. If the Σ and π escape from the nucleus without interacting their kinetic energies will be (assuming the nucleon which absorbs the K^- initially at rest):

$$\text{(1-20·4)} \qquad \left\{ \begin{aligned} T_\pi &= Q\,\frac{(m_\Sigma + Q/2)}{m_K + m_{«N»}} \simeq 82 \text{ MeV}\,, \\ T_\Sigma &= Q\,\frac{(m_\pi + Q/2)}{m_K + m_{«N»}} \simeq 12 \text{ MeV}\,. \end{aligned} \right.$$

When account is taken of the Fermi motion of the absorbing nucleon inside the nucleus, the kinetic energies are no more univocally determined; T_π may vary from 94 to 49 MeV and T_Σ correspondingly from 0 to 45 MeV, the above values being for a Fermi momentum of 200 MeV/c.

2) Reaction $K^- + «N» \to \Lambda^0 + \pi$. The Q of this reaction is 168 MeV; for absorption by a nucleon at rest this energy is shared between the Λ^0 and the pion in the following way: $T_\pi = 142$ MeV, $T_{\Lambda^0} = 26$ MeV; when account is taken of the Fermi motion of the absorbing nucleon, T_π may range between 99 to 166 MeV and correspondingly T_{Λ^0} between 69 to 2 MeV.

We may now proceed to see whether the observed reactions do, in some way contradict the above figures. We consider separately each class of events and begin with the:

a) K_ϱ *events*. Not much can be said about them; they are presumably capture events in which the most part of the energy is taken away by a neutral hyperon and a π^0. The Q being as low as 94 and 168 MeV for the two reactions discussed above it is not improbable that the most part of the energy is carried away as kinetic energy of the outgoing particles and the remainder is given to evaporation neutrons. One may recall that the energy release in π^- absorption has the same order of magnitude as the one which comes into play here and

there 30% of the absorptions are of the π_ϱ type. The absorption mechanism is however rather different in the two cases.

b) We next discuss the absorption events of class 2 in which only stable prongs are present. Such events may again be attributed to reactions in which a larger fraction of the available energy is spent to give rise to a star. If this is the case the visible energy carried by the stable prongs should not exceed 168 or 94 MeV according to whether a Λ^0 or a Σ^0 is emitted.

This is indeed seen to be the case in the most part of the events; however there are also a few others in which the energy exceeds 168 MeV. They are however not unexpected; indeed it may well happen that the pion (charged or neutral) emitted together with a neutral hyperon in one of the two fun- reactions (1, 2-**19**·1) is absorbed (really or virtually) by another nucleon in the same nucleus (an estimate of the probability of this process has been given in [23]). In this case the energy release in the star may be as high as $\sim 168 + 140$ MeV $\simeq 308$ MeV.

The process may then be written:

$$\text{K}^- + «\text{N}» + «\text{N}» \rightarrow \text{Y} + \pi \,. \tag{2-20·4}$$

The experimental situation is reproduced in the figure below (Fig. 1-**20**·4). It must be added that to obtain the visible energies of the stars one has to add to the kinetic energies of the emitted particles 8 MeV for each emitted proton and $\sim 2 \div 3$ MeV for each α particle to take account of the binding energy. It also has to be noticed that it is often difficult to distinguish a proton from a deuteron or from a triton when they have a range too short and most of the analyses are carried out under the assumption that all the singly charged particles are protons. Finally it is obvious that the visible energy *is in general much less than the energy release* of the star due to the energy taken away by neutrons.

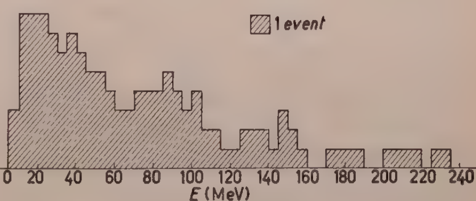


Fig. 1-**20**·4. – Visible energies of 60 K⁻ stars with stable prongs only.

The graph shows that in the great majority of the cases the visible energy is much less than 168 MeV and that in no case it exceeds 308 MeV, in agreement with what one had to expect.

The distributions in the number of stable heavy prongs in this kind of events have been discussed in [18] and compared with the similar quantity for stars from the absorption of pions.

c) *Class* 3 *events* (*a pion but no hyperon visible*). Considering again the visible energies of the stable prongs in this class, these should never exceed here 168 MeV. This is indeed the case as it is apparent from the Fig. 2.

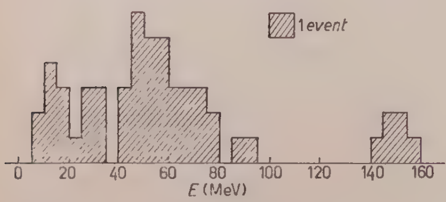


Fig. 2-20·4. – Visible energies of the stable prongs in class 3 events (includes 27 cases).

Considering next the energy of the pion it must be comprised between 99 and 166 MeV if a Λ^0 is emitted in the primary absorption act and between 49 and 94 MeV if a Σ is emitted. It is obvious however that a pion of an energy less than 94 MeV is not a proof of a Σ process because of the possible inelastic collisions of pions inside the nucleus.

In any case the pion energy cannot exceed 166 MeV and the experimental spectrum, shown below (Fig. 3) confirms this prediction. It has already been pointed out that much larger pion energies (up to ~ 300 MeV) should be present if a reaction like (1), contradicting the conservation of strangeness, might take place.

Very little can be said presently concerning the signs of the emitted pions. The present situation, including the events of class 5, as summarized in [24], from various groups is 29 negative pions to 3 positive pions. It may be pointed out, as observed by KOSHIBA, that a bias is present in these figures tending to select low energy pions, namely those which can stop in the emulsion.

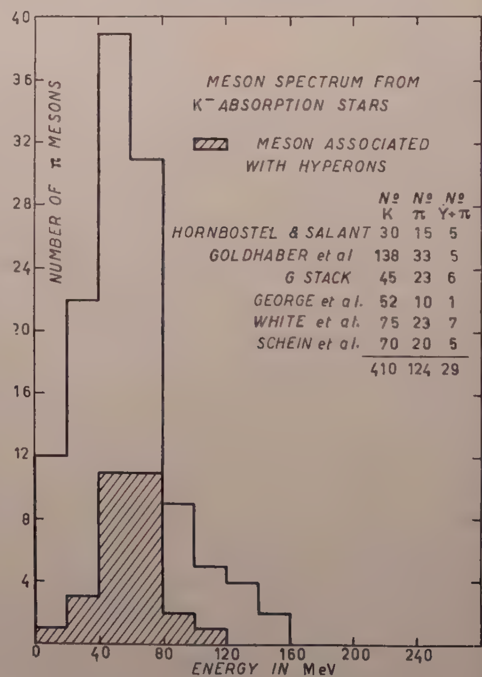


Fig. 3-20·4. – (From [5]).

d) *Class* 4 *events* (*a hyperon but no pion visible*). We have included in this class both events with emission of a $\Sigma^\pm$ and with emission of a hyperfragment. Concerning the first kind of events the energy spectrum of the $\Sigma^\pm$ is presented in Fig. 4. The Σ's produced according to the reaction $K^- + p \rightarrow \Sigma^\pm + \pi^\mp$ cannot have a kinetic energy larger than 45 MeV. Fig. 4 shows that only in few cases it is not so. Such cases may be attributed to a « two nucleon » absorption

process (compare (2)) in which case the energy may be as large as 180 MeV. However it may be pointed out that perhaps a bias exists against the detection of too energetic Σ's. They may in fact escape from the plates and, their decay being unobserved be confused with protons.

A question of some interest is the relative numbers of Σ^+ and Σ^- (in this consideration we include also the events belonging to class 5).

The experimental ratio between the numbers of Σ^- and Σ^+ observed by various investigators is [24]: $\Sigma^-/\Sigma^+ = 2.7$. The results are probably slightly biassed against the detection of Σ^-'s; in fact Σ^-'s which, being absorbed at the end of their range (Sect. **20**·7) give rise to zero prong stars [25] may be confused with protons. FRY has observed Auger electrons arising from the ends of some Σ^- tracks and has suggested to look for such electrons as a way to correct for this bias; the assumption being that the fraction of cases in which an Auger electron is observed at the end of a track giving rise to a zero prong star is the same as for the μ meson absorption, in which case it is 25%. It is not easy to know how good such an estimate can be.

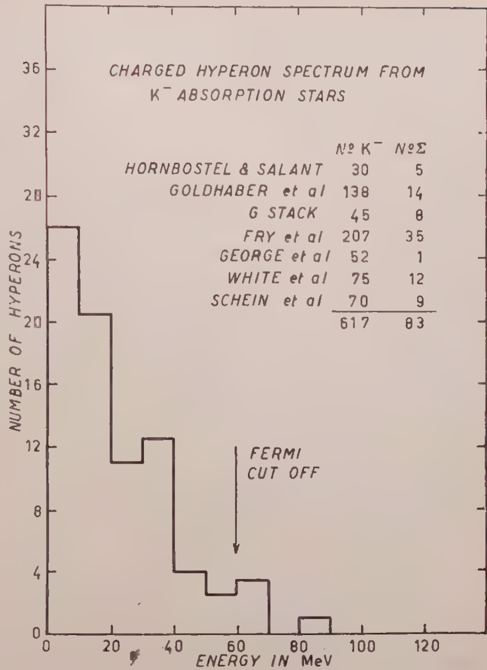


Fig. 4-**20**·4. – (From [5]).

Finally the question of the hyperfragments produced in K⁻ absorption may be mentioned. The numbers of such hyperfragments observed will be discussed in Ch. **21**; the identification of the hyperfragments as such is often uncertain because they frequently have a shtor range.

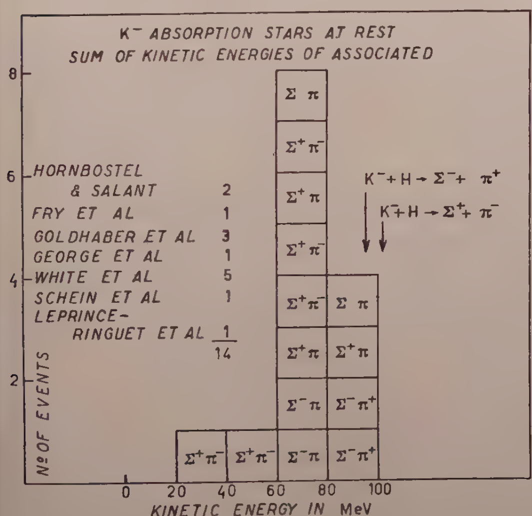


Fig. 5-**20**·4. – (From [5]).

e) Events in which both a hyperon and a pion are emitted. – We consider only the events in which a $\Sigma^{\pm}$ and a pion are observed (not a bound Λ^0). They constitute the most direct confirmation of the reaction (1-**19**·1). For most of such events the sum of the kinetic energies of the Σ and of the pion is near the theoretical limit of 93 MeV (compare Fig. 5) showing that the absorption act has not implied a large excitation of the whole nucleus. On the other hand the absorption has, in the cases reported below, surely taken place in a complex nucleus and not in hydrogen, because considering the momenta of the pion and of the $\Sigma^{\pm}$, there is a momentum unbalance.

Two or three cases in which the absorption has been in hydrogen were found in emulsion and used for a computation of the masses of the Σ^- and of the K^- (Sects. **5**·3 and **2**·2).

20·5. – Summary of the situation of K⁻ absorption from complex nuclei.

We may at this point summarize the discussion above:

a) the observations are perfectly consistent with the reactions predicted by the theoretical scheme; by the way it may be remarked that there is no evidence for Ξ^- coming out from a K^- absorption star; although the reaction $K^-+p \to \Xi^-+\pi^-$ is not energetically possible, the reaction K^-+« N »$+$« N »$\to$ $\to \Xi^-+N$ would be possible for a K^- of strangeness -2; this is an indication (but not a proof because of the difficulties of discriminating between Σ^- and Ξ^-) that K^- having strangeness -2 either do not exist, or are not easily produced, or live a time shorter than the one needed for falling into the lowest Bohr orbit;

b) that often it is possible, by energetic considerations, to establish if a Σ^0 or a Λ^0 has been produced in the absorption event even if it is not actually observed. It is however difficult to establish a quantitative value for the ratio between the numbers of Σ and Λ^0 produced in the primary absorption act. The reasons for this are two:

1) it is difficult to establish with some precision the numbers of Λ^0, Σ^+, Σ^- and Σ^0 which come out from the stars;

2) even if we knew such numbers there is no way to estimate the number of hyperons produced as Σ but which come out from the star as Λ^0 due to the strong secondary process:

$$\Sigma^- + p \to \Lambda^0 + n$$
$$\Sigma^+ + n \to \Lambda^0 + p \quad \text{etc.}$$

We shall not insist on this question but simply refer to the papers quoted, in many of which a discussion (not always convincing) of this point is given.

The best way to give a definite answer to this question probably starts from the absorption experiments in H and in D by means of a bubble chamber; the experiments in H, recently performed [1] will now be discussed.

20·6. – K^- absorption events in H.

A remarkable experiment, which clearly illustrates the advantages of the H bubble chamber technique, has been recently accomplished by the Berkeley group [1] on K^--H interactions. The K^- interacts in a H bubble chamber 10 inch in diameter and 6.4 inch deep with a magnetic field of 11 kG and exposed at the Bevatron's repetition rate (10 times per minute). The results below refer to 35000 pictures examined out of 10^5 taken; the most part of the data obtained from this experiment have been already inserted at the appropriate places: spin of the Σ and parity doublet question (Sect. **15**·6); lifetimes and masses of the Λ^0, Σ^+, Σ^- (Chs. **4**, **5**); proof of the existence of the Σ^0 (Ch. **6**); Σ^--proton interaction (Sect. **20**·7); Σ^-, Σ^+ decay and $|\Delta T| = \frac{1}{2}$ rule (Sect. **16**·11).

Here we shall simply *a*) present the statistics obtained and *b*) discuss the information which may be obtained from the production branching ratio of the various kinds of hyperons regarding the participation of the various isotopic spin states to the processes in question.

As far as *a*) is concerned the Table I-20·6 is self explanatory: 137 K^- interaction events have been observed, including 26 events of the kind $K^-\varrho$, that is in which a K^- stops at some point withouth giving rise to visible interactions or decay products.

As far as the 111 remaining events are concerned one may see from the Table I that the relative production rates of Σ^-, Σ^+, Σ^0, Λ^0, are in the ratios of about 4:2:2:1 including both the interactions at rest and those in flight. Pictures of some events are shown below (Fig. 1, 2, 3). No event contradicting the Gell-Mann and Nishijima scheme has been found.

As far as point *b*) is concerned, consider first the production processes of Σ^-, Σ^+, Σ^0. The initial K^-+p state is a coherent superposition 1:1 of states with $T=1$ and $T=0$; if T is conserved the state with $T=1$ will give rise to a final state $T=1$ with some amplitude J_1 and the state with $T=0$ will give a final $T=0$ state with some amplitude J_0. The final state will therefore be, with notations of obvious meaning:

$$J_1 \frac{1}{\sqrt{2}} (\pi^+\Sigma^- - \pi^-\Sigma^+) + J_0 \frac{1}{\sqrt{3}} (\pi^+\Sigma^- + \pi^-\Sigma^+ - \pi^0\Sigma^0) .$$

From this expression it is straightforward to derive that:

$$\text{(1-20·6)} \qquad \frac{N(\Sigma^-)}{N(\Sigma^+)} = \frac{|r \exp[i\varphi] + \sqrt{\frac{2}{3}}\,|^2}{|r \exp[i\varphi] - \sqrt{\frac{2}{3}}\,|^2} ,$$

TABLE I-20·6. – *Distribution of Events* (*) (from ref. [1]).

A) K^- interactions.

$K^- + p \rightarrow$	Circumstances of K^- interactions			
	In flight		At rest	Total
I. $K^- + p$ (elastic scatter)	3		—	3
II. $K^0 + n$ $\left\{ K_1 \longrightarrow \begin{cases} \pi^+ + \pi^- \\ \pi^0 + \pi^0 \end{cases} \right.$	(1) ([a])	$K\varrho$ ([b])	(2) ([a])	3
K_2 decay		None observed		
III. $\Sigma^+ + \pi^-$; $\Sigma^+ \longrightarrow n + \pi^+$	0		14	28
$\ p + \pi^0$	1		13	
IV $\Sigma^- + \pi^+$ $\Sigma^- \longrightarrow n + \pi^-$	4		44	55
Σ^- interacts	0		7	
V. $\Sigma^0 + \pi^0$; $\Sigma^0 \rightarrow \gamma + \Lambda$, $\Lambda \rightarrow \begin{cases} p + \pi^- \\ n + \pi^0 \end{cases}$	0	$K\varrho$ ([b])	14	14
VI. $\Lambda + \pi^0$; $\Lambda \rightarrow \begin{cases} p + \pi^- \\ n + \pi^0 \end{cases}$	1	$K\varrho$ ([b])	7	8
VII. $\Lambda + \pi^0 + \pi^0$		None observed		
VIII. $\Lambda + \pi^+ + \pi^-$	0		0	0
Total excluding $K\varrho$	10		101	111
$K\varrho$ ([b])				26 ([b])
Total Interactions				137

B) Σ^- Interactions.

$\Sigma^- + p \rightarrow$	Circumstances of Σ^- interactions			
	In flight		At rest	Total
IX. $\Sigma^- + p$ (elastic scatter)	0		—	0
X. $\Sigma^0 + n$; $\Sigma^0 \rightarrow \gamma + \Lambda$; $\Lambda \rightarrow \begin{cases} p + \pi^- \\ n + \pi^0 \end{cases}$	0	$\Sigma\varrho$ ([b])	2	2
XI. $\Lambda + n$; $\Lambda \rightarrow \begin{cases} p + \pi \\ n + \pi^0 \end{cases}$	0	$\Sigma\varrho$ ([b])	2	2
Σ^-_ϱ events ([b])	1		2	3
Total	1		6	7

([a]) Indicates identification uncertain.

([b]) Between 20 and 40 K's disappear in the chamber, yielding no visible interaction or decay products. We call these « $K\varrho$ » endings, and assume that there are 26 of them. $\Sigma\varrho$ are defined analogously.

(*) *Note added in proof.* – In the continuation of the experiment, the number of analyzed hyperons has been doubled. All the qualitative conclusions suggested by the earlier events are now strengthened, with the following important exception. Preliminary analysis of the present angular distribution, which now includes 155 Σ-decays, shows that it has become consistent with isotropy.

Fig. 1·20·6. – (From [1]). A K^- scatters elastically on a proton before coming to rest and forming a K-mesic atom. Then $K^- + p \to \pi^- + \Sigma^+$; $\Sigma^+ \to \pi^+ + n$.

Fig. 2-20·6 – (From [1]). A K-mesic atom disintegrates into $\Sigma^- + \pi^+$. The Σ^- goes its full range, comes to rest, and is captured by a proton, whereupon it produces $\Sigma^0 \to \Lambda + \gamma$. The π^+ is scattered by a proton.

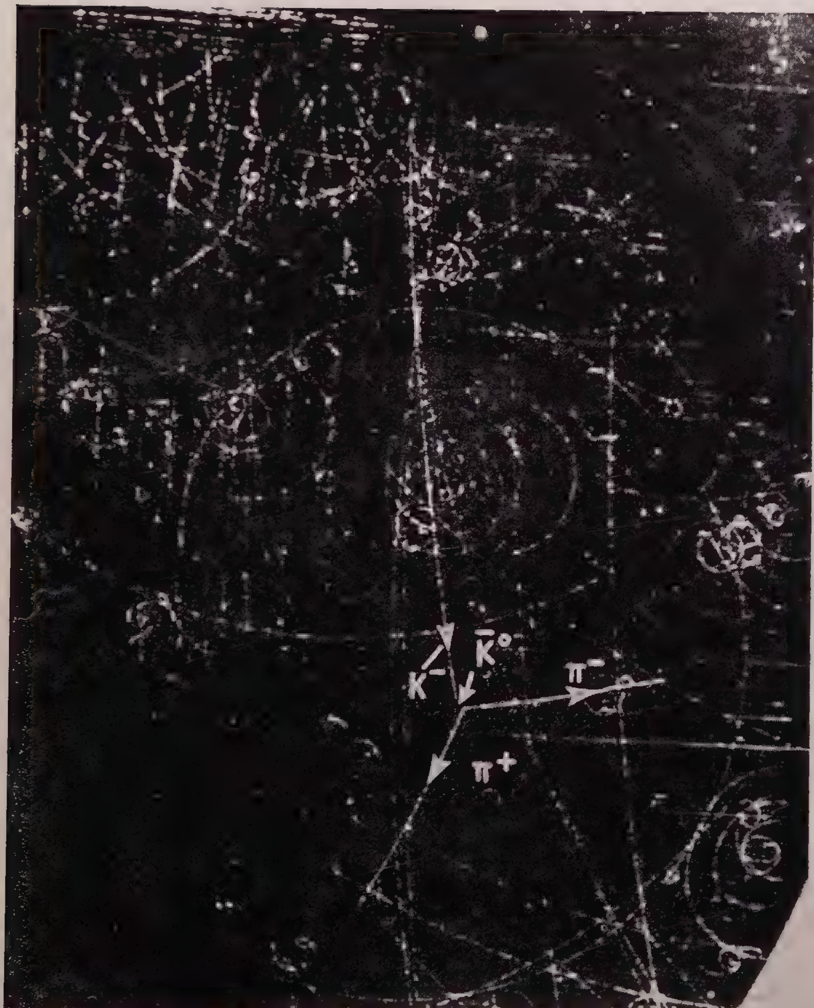


Fig. 3-20·6 – (From [1]). Production of a $\overline{K}^0$.

and

$$\frac{N(\Sigma^+)+N(\Sigma^-)}{2N(\Sigma^0)}=\frac{3}{2}r^2+1\,, \tag{2-20.6}$$

where we have put $J_1/J_0=r\exp i\varphi$, and $N(\Sigma)$ means: number of Σ's.

The ratio $r=|J_1|/|J_0|$ may be derived from (2-20.6) taking into account that the number of Σ^0 observed is not equal to the number of effectively produced Σ^0, because a fraction of the Λ^0 may decay into neutral products; in other words, $N(\Sigma^0)$ to be inserted in (2) is equal to $(1/\alpha)N_{\text{observed}}(\Sigma^0)$, where α is the fraction of Λ^0 which decay in the charged way. Using the value of α from the Brookhaven Columbia experiments ($\alpha=\frac{2}{3}$) we get from (2) inserting the values for $N_{\text{observed}}(\Sigma^0)$ given in the table

$$r\simeq 0.8\,,$$

φ may then be determined from (1); one gets $\varphi\simeq 70°$; it so appears that there is no predominance of an isotopic spin state over the other in this process.

As far as the ratio between the matrix element J_1 (for Σ production in the $T=1$ state) and the matrix element G_1 for Λ^0 production (necessarily only in the $T=1$ state) is concerned, also this ratio depends on α: for $\alpha=\frac{2}{3}$ one gets $|G_1/J_1|^2\simeq\frac{1}{3}$.

It should be realized that all these figures depend in a very critical way from the input figures: it is therefore better to wait for larger statistics to draw definite conclusions.

Notice finally that the K^- absorption experiments in D may provide a check of the isotopic spin conservation and of the value of r and φ found from H.

K^--H *interaction cross-section*. – All what is presently known on the above cross-section is contained in the following Table II-20.6 which summarizes

TABLE II-20.6.

Author	K^- energy	Method	Absorption cross-section	Scattering cross-section
[1]	30 MeV (at the entrance in the chamber)	9 interactions in flight in the H bubble chamber	(210 ± 100) mb	(45 ± 30) mb
[6]	K^- beam energy ~ 160 MeV	6 interactions in flight with protons in emulsions	—	$(52^{+31}_{-2?})$ mb
[26]	~ 530 MeV	transmission	(52 ± 9) mb (total cross section)	

the data obtained by different authors with various methods at different energies.

These values are still affected by very large errors except the last one, which refers to a rather higher energy than the first two.

20·7. – Interactions of Σ^- and Ξ^- with nucleons.

There is very little evidence on the interactions of these particles with nucleons and nuclei. This is essentially due to the shorter lifetime of the Σ with respect to the $K^\pm$, which prevents, at the moment, to perform with the Σ's the same kind of experiments which have been done with the K's. One or two events have been reported which represent probably captures in flight of a $\Sigma^\pm$, in a complex nucleus [27, 28], and a similar number in H [1].

A few more events on the contrary are available on the capture of Σ^- at rest. The number of Σ interaction events will increase with the exploration of K^- capture events, because, as it appears from the results of the last sections, such stars are a sufficiently rich source of Σ's.

The fast processes which may take place in the interaction of a Σ with a nucleon are listed below

$$\text{(1-20·7)} \qquad \left\{\begin{array}{l} \Sigma^+ + p \to \Sigma^+ + p \\ \qquad\quad\;\; \to \Sigma^0 + n \\ \Sigma^+ + n \to \Lambda^0 + p \end{array}\right.$$

$$\text{(2-20·7)} \qquad \begin{array}{r} a) \\ b) \\ c) \end{array} \left\{\begin{array}{l} \Sigma^- + n \to \Sigma^- + n \\ \Sigma^- + p \to \Lambda^0 + n \\ \Sigma^- + p \to \Sigma^0 + n \end{array}\right.$$

The processes which may intervene in the case of Σ^- interaction at rest are (2*b*) and (2*c*); we shall now show that there is some evidence that such processes actually take place in these events. Here again we shall first discuss interactions in complex nuclei and next in H.

The first event which was interpreted as a Σ^- capture at rest was observed by JOHNSTON and O'CEALLAIGH [29]; the difficulty, in establishing the nature of such kind of events, lies of course in a proper identification of the presumed Σ^-. The mass measurements are often insufficient to discriminate among a hyperon, a deuteron and a proton, so that additional criteria must be invoked; furthermore one has to be sure that the particle was brought to rest before being captured.

In the following Fig. 2 we give the visible energies of the stars from the Σ^- absorption events at rest which have been reported up to now; the 12 cases reported in [20] and the 8 cases reported by FOURNET and WIDGOFF [3] have not yet been described by the Authors and are therefore not included in the figure.

A feature shared by all these Σ^- absorption events is that they give rise to very small energy releases. This is in fact expected if the absorption process takes place according to (2). Two cases may happen [30] [22];

1) The Λ^0 and the n both come out from the absorbing nucleus; then the energy of the star cannot be larger than ~ 70 MeV, and the visible energy is probably much less; to this class belong also the events in which the Λ^0 which comes out from the absorbing nucleus is surrounded by some nucleons, forming a hyperfragment.

2) The Λ^0 remains in the nucleus and disintegrates: in this case there are still ~ 177 MeV available from the Λ^0 disintegration; if the disintegration is non-mesonic this may often result in the ejection of an energetic (~ 90 MeV) proton; if mesonic, a pion with 30 MeV or less may come out. Apparently among the events described so far no case with the emission of a charged pion is present; this is consistent with the predominance of the non-mesonic decay mode of the Λ^0 in the heavy nuclei of the emulsion. Also if the process (2c) takes place most of the events which are produced through it may be expected to belong to this class.

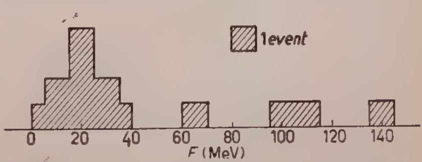


Fig. 1-20·7. – Energies of stars from absorption of Σ^- at rest. The events are those of [29-37]. Of the original compilation by [30] Br_2 and Bo_1 have not been included here because they have been reinterpreted as hfs. Also three cases in [38] have not been included because the mass of the Σ^- is too uncertain.

The few events reported in Fig. 1 probably represent events from the two classes.

Seven *interactions of* Σ^- *in* H have also been recently observed by the Berkeley group in the H bubble chamber. They are easily identified on the basis of the kinematics and have been already reported in Table I-20·6. It is particularly interesting to have in two cases a direct confirmation of the process (2c); these events allow to determine the Σ^-, Σ^0 mass difference as $1.7 < M_{\Sigma^-} - M_{\Sigma^0} < 22.7$ MeV with a most probable value for M_{Σ^0} somewhere in between M_{Σ^-} and M_{Σ^+} (See Ch. **6**).

Before closing this section we may ask what could be said concerning the absorption of Ξ^- hyperons. The only strong reaction which may take place according to the theoretical scheme is then:

$$\Xi^- + p \rightarrow \Lambda^0 + \Lambda^0 , \tag{3-20·7}$$

or possibly:

$$\Xi^- + p \to \Xi^0 + n\,. \tag{4-20·7}$$

The two extreme cases which (3) implies are either very small and probably invisible stars (if the two Λ^0's come out) or stars with an energy release up to ~ 340 MeV, if the two Λ^0 are captured and decay in the same nucleus. Such a situation seems however improbable so that most of the Ξ^- stars would probably look exactly as Σ^- absorption stars and a discrimination between the two should be made either by a very careful determination of the mass of the primary or by using experimental conditions in which one may be sure that only either Σ or Ξ are present (using possibly their difference in lifetime).

REFERENCES (Ch. 20)

[1] L. W. Alvarez, H. Bradner, P. Falk-Vairant, J. D. Gow, A. H. Rosenfeld, F. T. Solmitz and R. D. Tripp: UCRL 3583 (to be published).
[2] J. Hornbostel and E. O. Salant: *Phys. Rev.*, **102**, 552 (1956).
[3] D. M. Fournet and M. Widgoff: *Phys. Rev.*, **102**, 929 (1956).
[4] G. Goldhaber, S. Goldhaber, E. Iloff, J. Lannutti, F. Webb, M. Widgoff, A. Pevsner and D. Ritson: *Suppl. Nuovo Cimento*, **4**, 367 (1956).
[5] G. Goldhaber: *Proceed. Rochester Conference* (1956).
[6] W. H. Barkas, W. F. Dudziack, P. Giles, H. Heckman, F. Inman, C. Mason, N. Nichols and F. Smith: UCRL 3627, to be published.
[7] D. Lal, Yash Pal and B. Peters: *Phys. Rev.*, **92**, 438 (1953).
[8] W. F. Fry and J. Lord: *Phys. Rev.*, **87**, 533 (1952).
[9] J. Naugle, H. P. Ney, P. S. Freier and W. B. Cheston: *Phys. Rev.*, **96**, 1383 (1954).
[10] B. Rossi: *Suppl. Nuovo Cimento*, **12**, 161 (1955).
[11] K. H. Barker: *Proc. of the Duke Conf.* (1953) pp. 11-18.
[12] H. DeStaebler: *Phys. Rev.*, **95**, 1110 (1954).
[13] M. Di Corato, B. Locatelli, G. Mignone and G. Tomasini: *Suppl. Nuovo Cimento*, **12**, 270 (1954).
[14] B. F. Touschek: *Suppl. Nuovo Cimento*, **12**, 281 (1955).
[15] M. W. Friedlander, Y. Fujimoto, D. Keefe, M. G. K. Menon, M. Ceccarelli, M. Grilli, M. Merlin, A. Salandin and B. Sechi: *Suppl. Nuovo Cimento*, **4**, 428 (1956).
[16] J. Hornbostel and E. O. Salant: *Phys. Rev.*, **99**, 338 (1955).
[17] W. W. Chupp, G. Goldhaber, S. Goldhaber and F. B. Webb: *Suppl. Nuovo Cimento*, **4**, 382 (1956).
[18] E. P. George, A. Herz, J. Noon and N. Solnsteff: *Nuovo Cimento*, **3**, 94 (1956). A. Herz, R. May, J. Noon, B. O. Brien and N. Solnsteff: *Nuovo Cimento*, **3**, 1491 (1956).
[19] D. M. Haskin, T. Bowen and M. Schein: *Phys. Rev.*, **103**, 1512 (1956).

[20] W. F. FRY, J. SCHNEPS, G. A. SNOW and M. S. SWAMI: *Phys. Rev.*, **100**, 1448, 950 (1955).
[21] R. WHITE *et al.* (to be published).
[22] M. W. FRIEDLANDER, Y. FUJIMOTO, D. KEEFE and M. G. K. MENON: *Nuovo Cimento*, **2**, 90 (1955).
[23] W. B. CHESTON: *Nuovo Cimento*, **3**, 645 (1956).
[24] M. KOSHIBA: *Nuovo Cimento*, **4**, 357 (1956).
[25] C. CASTAGNOLI, C. FRANZINETTI and A. MANFREDINI: *Nuovo Cimento*, **3**, 1159 (1956).
[26] B. CORK, G. LAMBERTSON, O. PICCIONI and W. WENZEL: UCRL 3654 (to be published).
[27] R. D. HILL, F. T. GARDNER and J. E. CREW: *Nuovo Cimento*, **2**, 824 (1955).
[28] W. F. FRY, J. SCHNEPS, G. A. SNOW and M. S. SWAMI: *Phys. Rev.*, **100**, 939 (1955).
[29] R. JOHNSTON and C. O'CEALLAIGH: *Phil. Mag.*, **45**, 424 (1954); *Nuovo Cimento*, **1**, 468 (1955).
[30] M. CECCARELLI, N. DALLAPORTA, M. GRILLI, M. MERLIN, G. SALANDIN, B. SECHI and M. LADU: *Nuovo Cimento*, **2**, 542 (1955).
[31] L. BACCHELLA, M. DI CORATO, M. LADU, R. LEVI SETTI, L. SCARSI and G. TOMASINI: *Suppl. Nuovo Cimento*, **4**, 428 (1956).
[32] M. M. BLOCK and D. T. KING: *Phys. Rev.*, **97**, 1415 (1955).
[33] M. CECCARELLI, M. GRILLI, M. MERLIN, A. SALANDIN and B. SECHI: *Suppl. Nuovo Cimento*, **4**, 428 (1956).
[34] PH. ROSSELET, R. WEILL and M. GAILLOUD: *Nuovo Cimento*, **3**, 505 (1956).
[35] M. TEUCHER: *Suppl. Nuovo Cimento*, **4**, 482 (1956).
[36] TSAI-CHU: *Nuovo Cimento*, **3**, 921 (1956).
[37] M. SCHEIN, D. M. HASKIN and D. LEENOV: *Phys. Rev.*, **100**, 1455 (1955)
[38] S. J. GOLDSACK and W. O. LOCK: *Nuovo Cimento*, **3**, 600 (1956).

CHAPTER 21.

Binding of the new particles to nucleons - Hyperfragments.

Introduction.

In this chapter the basic facts and theories concerning the binding of new particles to old ones are discussed. An example of aggregates involving both types is presented by the « Λ^0-excited fragments » often called simply « excited fragments » or « hyperfragments » (*). They are in fact nuclear splinters, emitted in disintegrations not infrequently of moderate energy ($2\div3$ GeV), in which a neutron is replaced by a Λ^0-hyperon. Such a system will eventually disintegrate—as a consequence of the decay of the Λ^0 which it contains—giving rise to a star.

Nuclear systems involving unstable particles other than Λ^0's, have not as yet been proved to exist. Three « anomalous » cases have been reported which may be interpreted as spontaneous disintegrations of fragments releasing more energy than expected from a bound Λ^0.

21·1. – Experimental evidence for the existence of hyperfragments. Early work (+).

In 1953 DANYSZ and PNIEWSKY [1] observed in a plate exposed to cosmic rays the event which is shown in Fig. 1-**21**·1. It consists of a nuclear fragment f of charge between 4 and 6, coming out from a 21+18p star A reaching the end of its range at a point B ($\sim 90\ \mu$m distant from A) where another star B with four prongs, is observed. The first of the secondary disintegration products has a range of 9 μm, the second a range of 123 μm and the fourth a range of 2 μm, the range of the third prong being undetermined since it does

(*) Following a suggestion of Prof. M. GOLDHABER.
(+) Compare for the early work also a review article by POWELL [2].

not terminate in the emulsion. The identification of the four tracks is doubtful: 1, 2, may be p, d, t or α; 3 may be a proton, a deuteron, a triton or a π; 4 is probably a heavy splinter. Excluding the hypothesis that the whole event is a casual coincidence between the end of the fragment f emitted from A and the centre of the star B (the probability for such a coincidence is of the order 10^{-4} and the observation of several other events of the same kind excludes this possibility) one has to explain the remarkable fact, noted by DANYSZ and PNIEWSKY, that the nuclear fragment explodes at the end of its range after such a long time as that which is necessary to cover 90 μm in emulsion (10^{-12} s). The fact that the fragment was at rest, or practically at rest, at the moment of its disintegration rules out the possibility that B was due to a collision between f and a nucleus of the emulsion; and the long time spent between the creation and the disintegration also excludes the possibility that the fragment f be simply an ordinary nucleus in a highly excited state (apart from energy considerations); if such were the case its life could not be much longer than 10^{-20} s.

Fig. 1-21·1 – (From [1]).

The following possibilities remained: *a*) that the disintegration was due to the absorption by the nuclear fragment of a π^- meson produced in A and bound in a Coulomb orbit; *b*) that it was due to the absorption of some other heavy meson bound in a Coulomb orbit; *c*) that it was due to the disintegration of some kind of new particle contained *inside* the nucleus. If *a*) were the case, the energy of the star B should be equal to the rest energy of the π meson (neglecting its small binding energy). But a similar event found some months later by CRUSSARD and MORELLET [3], proved that at least in the Crussard and Morellet event this was not the case: in fact one of the secondary particles was a π having a kinetic energy of 25^{+4}_{-3} MeV so that the energy liberated in it was certainly higher than that which could be liberated in the absorption of a π bound in a Coulomb orbit. Possibilities *b*) and *c*) remained to be examined. An event observed by BONETTI *et al.* [4, 5] was particularly useful in this con-

nection. In such an event the mass of the fragment, as well as its charge could be directly determined, and turned out to be: $M_f = (5\,150 \pm 1\,000)\,\mathrm{m_e}$ and $Z_f = 1$; the secondary star S_2 (see Fig. 2-21·1) consisted of two prongs, having opposite directions, of which one was certainly due to a meson because it gave rise to a σ star at the end of its range and the other appeared as a short heavy track.

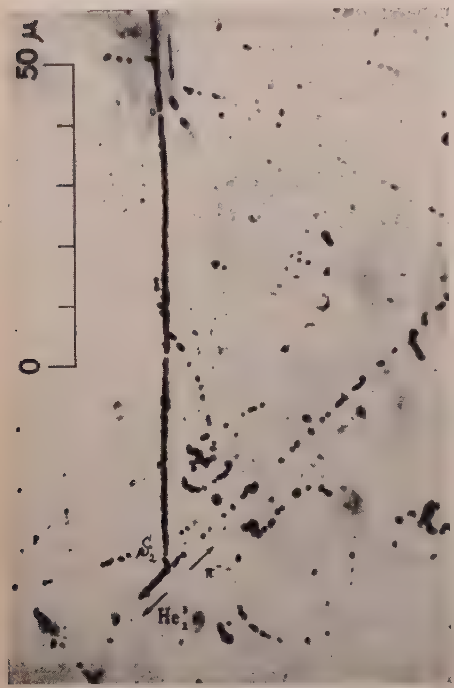


Fig. 2-21·1. – (From [4])

From the above data one was entitled to assume the fragment to be an excited triton ${}^{3}_{\Lambda}\mathrm{H}$ (*), and everything was found to be consistent with a two body decay of the form:

$$ {}^{3}_{\Lambda}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + \pi^- + Q \,. $$

From the energy and momentum balance the values of Q and of the mass of the fragment could be calculated; the latter turned out to be $M_f = 5\,860\ \mathrm{m_e}$, in agreement with the directly measured value; the Q resulted (41.7 ± 1) MeV.

First, we observe that this value is much too small to be compatible with the absorption of a K^- meson—thus excluding the possibility *b*). Examining now the possibility *c*) we see that the assumption of a bound Λ^0 is perfectly consistent with the data and leads to a value of the binding energy of the Λ^0 in the ${}^{3}_{\Lambda}\mathrm{H}$ nucleus of the order of 1 MeV.

We may conclude that the interpretation of the event as a nuclear fragment having a neutron replaced by a Λ^0 is the correct one, at least if we assume the events to be of the same nature in the three cases considered above.

Both in the Crussard-Morellet and Bonetti *et al.* events one of the prongs of the secondary star is due to a π meson; the same is observed in a large number of other events, but *not in all*—on the contrary for heavier excited fragments the emission of a π seldom happens. In some of these cases, one may think, π^0's are emitted which escape detection: however, if all hyperfragments are assumed to be Λ^0-hyperfragments the total release of energy cannot exceed 176 MeV and if a π^0 is emitted the sum of the kinetic energies of

(*) A hyperfragment consisting of A nucleons plus a Λ^0 will be indicated as ${}^{A+1}_{\Lambda}X$.

the disintegration products cannot exceed ~ 37 MeV. Experiments show that in many cases it does: we are then forced to assume that hyperfragments may disintegrate in a *non mesic way*.

As an example of the latter type, let us discuss the event observed by FRY and WHITE [6]. Two charged secondaries (1 and 2 in Fig. 3-21·1) are emitted as a result of the decay of a hyperfragment f created in a 17+5? star; they are both heavier than protons, most probably due to doubly charged particles (^{3}He or ^{4}He), their kinetic energies totalling ~ 100 MeV (if they are both ^{3}He) and having a resulting momentum of ~ 370 MeV/c. Since the disintegration took place at rest, this momentum has to be balanced by some particles which escape detection—either neutral or so massive that they are stopped before they cover any appreciable path in emulsion. The latter hypothesis can be discarded immediately since the total charge and mass of the unstable fragment could be estimated and were found to be $Z \leqslant 4$ and $A = 9$ respectively: even taking the most pessimistic view, i.e. assuming that both visible products are less massive than α particles, what remains would still be light enough to produce a visible track if it had a momentum of 370 MeV/c. On the other hand if a neutron is assumed to be responsible for the balance, its energy would be about 74 MeV—this, added to the energy of the α's gives a total of ~ 174 MeV, remarkably close to that expected from the non mesic decay of a Λ^0 hyperfragment. The most probable disintegration scheme would be

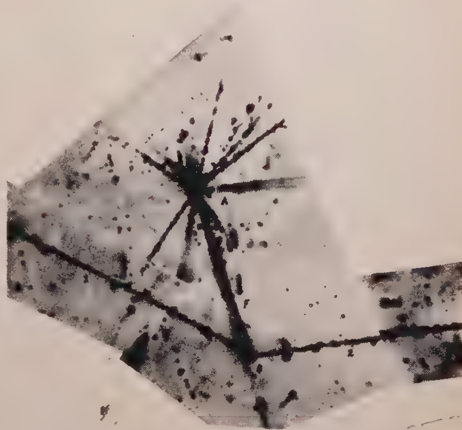


Fig. 3-21·1. (From [6]).

$$ {}^{8}_{\Lambda}\mathrm{Be} \to {}^{3}\mathrm{He} + {}^{4}\mathrm{He} + \mathrm{n} + Q \; ; $$

The other scheme:

$$ {}^{9}_{\Lambda}\mathrm{Be} \to {}^{4}\mathrm{He} + {}^{4}\mathrm{He} + \mathrm{n} + Q' $$

could not be discarded, though it appears less probable as it would imply a very low binding energy for the Λ^0 in the Be nucleus (*). Consequently, to classify an event as a hyperfragment we shall require that adding together the kinetic energies of the visible decay products and of the neutral ones which, are assumed to balance the total momentum, one should get ~ 37 MeV for mesic and ~ 176 MeV for non mesic decays.

(*) B_Λ (${}^{9}_{\Lambda}\mathrm{Be}$) $= (-0.7 \pm 3)$ MeV. Compare with Fig. 1-21·7 or Table I-21·7. However, the errors are too large to allow a definite conclusion.

These values refer to disintegrations at rest. For disintegrations in flight the interpretation is somewhat more difficult and some expedients must be used to determine the velocity of the hyperfragment: the reader will find an ample discussion of the experimental methods to be used to this purpose in SÖRENSEN's paper quoted in the reference list [7]. Comparatively few cases of decays in flight have been reported so far (compare Sect. **21**·5).

21·2. – Identified hyperfragment disintegrations.

To the present date about 300 cases of probable hyperfragments have been observed, almost all in nuclear emulsion. Only for 53 of them the identity of the interconnecting track (i.e. the charge and mass of the hyperfragment) and the characteristics of the terminal disintegration have been established. Their most relevant data are given in Tables I, II, III (+): in column 2 the classification of the parent star, using the universally accepted code originally proposed by the Bristol group [8]; in column 3 the range R_{H} of the track interpreted as a hyperfragment; in column 4 its time of flight; in column 5 the identity (and the kinetic energy in brackets) of the charged disintegration products; in column 6 the total kinetic energy associated with both charged and uncharged products (the latter determined from the momentum conservation law); in column 7 the decay scheme; in column 8 the binding energy B_Λ of the Λ to the residual fragment defined in Sect. **21**·7 and, for comparison, the binding energy B_n which a neutron would have, taking the place of the Λ in the same fragment.

It must be added that the data of these tables *cannot* be used to discuss either the relative abundances of hyperfragments of different charge or mass, or of their modes of decay. The events listed in Tables I and II are those which were consistent with only one decay scheme—or at least one existed which was highy favoured with respect to other possible ones (*). In doing so, strong discriminations have been introduced. Mesic decays have been favoured with respect to non-mesic ones. For example, when the interconnecting track is too short to allow a direct determination of Z, the presence of a meson among the decay products immediately rules out the alternative explanation —true in many cases—that the event is due to a π^- capture star. Moreover when the π escapes unhurt from the fragment the energy available for the disintegration of the fragment is ~ 140 MeV smaller: as a consequence the number of secondaries is—on the average—smaller and the scheme of the dis-

(+) The data available in 1954 were summarized by GRILLI and LEVI SETTI [9, 10].

(*) In compiling Table III the selection criterion has been somewhat relaxed and some events have been included in which the mass of the hyperfragment was uncertain by not more than one unit, but the charge was known exactly.

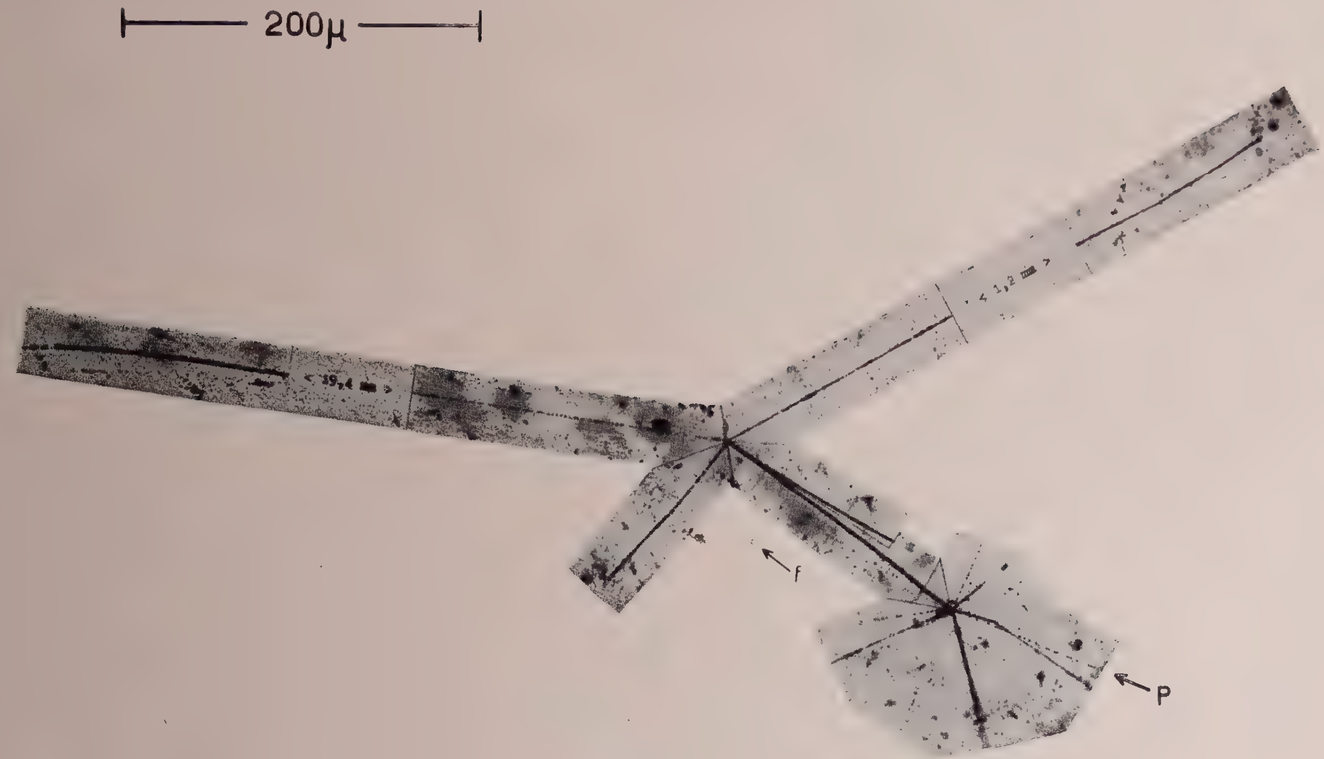


Fig. 4.-21·1. – $_{\Lambda}$Be emitted from a 10 + 7p star produced by a cosmic ray charged primary. (After GRILLI *et al.* [59]. Courtesy of Prof. M. MERLIN).

TABLE I-21'2. – *Details on identified singly charged hyperfragment disintegrations.*

Ref.	Parent star	R_F (μm)	Time of flight (10^{-11} s)	Visible disintegration products (in brackets their kinetic energy expressed in MeV)	Q MeV	Disintegration scheme	B_Λ (MeV)	B_n (MeV)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
[11]	16+2?	2 200	6.5	π^- (39.3±0.8) ^{3}He (2.6±0.6)	41.6±1	$^3_\Lambda$H → ^{3}He + π^-	0.8±1.1	6.2	
[4, 5]	22+3n	13 000	25	π^- (39.4±1.0) ^{3}He (2.7±0.6)	41.7±1.0	»	0.7±1.1	»	
[12]	6+3n	5 800	12.6	π^- (38.8±4.5) ^{3}He (2.6±0.7)	41.2±4.7	»	1.7±4.7	»	
[13]	21+7p	1 490	5	π^- (39.8±4) ^{3}He (2.66±0.7)	42 ±4.2	»	0.3±4.2	»	(1)
[14]	4+0π^-	635	2.8	π^- (31.8) ^{1}H (1.1) ^{2}H (3.5)	36.4	$^3_\Lambda$H → ^{1}H + ^{2}H + π^-	0.5±0.6	»	(2)
[14]	5+1π^-	324	1.7	π^- (23.4±0.5) ^{1}H (12.7) ^{2}H (1.3)	37.4±0.5	»	−0.5±0.6	»	
[15]	23+3p	44	0.1	π^- (22.8) ^{1}H (26.3) ^{2}H (24.8)	35.7±1.0	»	1.4±1	»	(3)
[14]	—	100	0.7	π^- (27.04) ^{1}H (2.97) ^{1}H (0.52)	34.3	$^3_\Lambda$H → ^{1}H + ^{1}H + n + π^-	0.4±0.7	»	
[16]	10+2π^-	152	1.0	π^- (14.5) ^{1}H (4.8) ^{1}H (0.3)	33.3	»	1.4±0.6	»	
[16]	—	603	5	π^- (—) ^{1}H (—) ^{1}H (—)	33.5	»	3.3±2	»	
[16]	5+0K	368	2	π^- (26.8±2) ^{1}H (—) ^{1}H (—)	35.6	»	1.3±0.6	»	
[17, 18]	6+0n	384	2.1	π^- (54.5±2.4) ^{4}He (2.6±0.2)	57.1±2.5	$^4_\Lambda$H → ^{4}He + π^-	0.4±2.4	—	
[17]	22+2p	22 500	40	π^- (53.6±0.9) ^{4}He (2.5)	56.1±1	»	1.4±1	—	
[19, 20, 21]	22+3p	268	1.7	π^- (53.1±1) ^{4}He (2.55±1)	55.6±1	»	1.2±1	—	(4)
[22]	6+0p	50	< 0.5	π^- (52.1) ^{4}He (2.4)	54.5	»	2.2	—	
[23]	4+0K	—	5.4	π^- (52.8) ^{4}He (2.9)	55.7	»	1.6±3	—	
[24]	3+0Σ^-	255	1.6	π^- (51) ^{4}He (2.4)	53.4	»	3.3±1	—	
[25]	—	—	—	π^- (52.99) ^{4}He (—)	55.38	»	1.32±1	—	
[25]	—	—	—	π^- (53.43) ^{4}He (—)	55.82	»	0.88±1	—	
[25]	—	—	—	π^- (53.74) ^{4}He (—)	56.13	»	0.57±1	—	
[25]	—	—	—	π^- (54.16) ^{4}He (—)	56.55	»	0.15±1	—	
[26]	3+0Σ	3.75	2.1	π^- (30) ^{3}He (0.9)	34.9±0.5	$^4_\Lambda$H → ^{3}He + n + π^-	1.2±0.7	—	
[25]	—	—	—	π^- (29.91) ^{1}H (—) ^{3}H (—)	35.94	$^4_\Lambda$H → ^{1}H + ^{3}H + π^-	0.96±.7	—	
[25]	—	—	—	π^- (31.94) ^{1}H (—) ^{3}H (—)	35.43	»	1.47±.55	—	
[14]	—	13.2	—	π^- (25.5±2) ^{1}H (0.76) ^{2}H (2.19)	—	$^4_\Lambda$H → ^{1}H + ^{2}H + n + π^-	1.9±2	—	

(1) Produced in association with $K^+_{\pi 3}$.
(2) The three secondaries are coplanar.
(3) Decay in flight ($\beta \sim 0.16$).
(4) Produced in association with K^+.

TABLE II-21·2. – *Details on identified doubly charged hyperfragment disintegrations.*

Ref.	Parent star	R_F (μm)	Time of flight (10^{-11} s)	Visible disintegration products (in brackets their kinetic energy expressed in MeV)	Q (MeV)	Disintegration scheme	B_Λ (MeV)	B_n (MeV)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
[27]	$11+7\alpha$	633	—	^{1}H (114.7 ± 2.7) ^{2}H (36.3 ± 0.2)	$167.6\ \pm2.9$	$^4_\Lambda\mathrm{He}\to{}^1\mathrm{H}+{}^2\mathrm{H}+\mathrm{n}$	$1.9\ \pm2.9$	20.6	
[28]	14+3p	354	—	^{1}H (—) ^{3}H (—)	—	$^4_\Lambda\mathrm{He}\to{}^1\mathrm{H}+{}^3\mathrm{H}$	$4.3\ \pm1.6$	»	
[14]	—	171	0.8	π^- (23.8 ± 0.5) ^{1}H (10.7 ± 0.1) ^{3}He (0.6 ± 0.1)	$35.1\ \pm\ .6$	$^4_\Lambda\mathrm{He}\to{}^1\mathrm{H}+{}^3\mathrm{He}+\pi^-$	$1.8\ \pm0.6$	»	
[14]	—	2.6	< 0.5	π^- (34.5 ± 2) ^{1}H (1.2) ^{3}He (1.2)	$36.9\ \pm2$	»	$0\ \ \pm2$	»	
[29]	$6+0\mathrm{K}^-$	97.2	0.6	π^- (28.1 ± 0.5) ^{1}H (2.91 ± 0.1) ^{3}He (4.43 ± 0.1)	$35.5\ \pm\ .6$	»	$1.5\ \pm0.6$	»	
[25]	—	—	—	π^- (30.77) ^{1}H (—) ^{3}He (—)	35.51	»	$1.4\ \pm0.6$	»	
[25]	—	—	—	π^- (30.56) ^{1}H (—) ^{3}He (—)	34.81	»	3.09 ± 0.6	»	
[25]	—	—	—	π^- (29.53) ^{1}H (—) ^{3}He (—)	35.63	»	1.27 ± 0.6	»	
[7]	—	—	54	π^- (30.9) ^{1}H (—) ^{3}He (—)	$38.9^{+3.7}_{-3.4}$	»	$-\ 1.9^{+3.4}_{-3.7}$	»	(1)
[30]	4+2n	5180	7.7	π^- (23.6 ± 0.5) ^{1}H (13.4 ± 0.1) / ^{1}H (10.2 ± 0.1) ^{2}H (5.7 ± 0.05)	$29.6\ \pm0.6$	$^4_\Lambda\mathrm{He}\to{}^1\mathrm{H}+{}^1\mathrm{H}+{}^2\mathrm{H}+\pi^-$	$1.9\ \pm0.6$	»	(2)
[17,18]	26+2p	36	< 0.5	π^- (27.6 ± 1) ^{1}H (4.4 ± 0.1) ^{4}He (3 ± 0.4)	$35.0\ \pm1.5$	$^5_\Lambda\mathrm{He}\to{}^4\mathrm{He}+{}^1\mathrm{H}+\pi^-$	$1.9\ \pm1.0$	—	
[30]	19+3p	122	0.7	π^- (22.4 ± 0.4) ^{1}H (6.7 ± 0.1) ^{4}He (5.3 ± 0.2)	$34.0\ \pm0.5$	»	$2.9\ \pm0.5$	—	
[14]	—	31	< 0.5	π^- (29.5) ^{1}H (3.4) ^{4}He (2.0)	$34.9\ \pm0.5$	»	$2.0\ \pm0.6$	—	
[31]	$4+2\pi^-$	262	—	π^- (25.4 ± 0.7)d^{1}H (5.48) ^{4}He (—)	$34.4\ \pm1$	»	$2.5\ \pm1.3$	—	
[25]	—	—	—	π^- (27.31) ^{1}H (—) ^{4}He (—)	$34.02\pm\ .6$	»	$2.9\ \pm0.6$	—	
[25]	—	—	—	π^- (15.12) ^{1}H (—) ^{4}He (—)	$34.7\ \pm\ .6$	»	$2.2\ \pm0.6$	—	
[25]	—	—	—	π^- (23.19) ^{1}H (—) ^{4}He (—)	$34.2\ \pm\ .6$	»	$2.7\ \pm0.6$	—	
[25]	—	—	—	π (22.67) ^{1}H (—) ^{4}He (—)	$33.4\ \pm\ .6$	»	$3.5\ \pm0.6$	—	
[25]	—	—	—	π (28.01) ^{1}H (—) ^{4}He (—)	$34.9\ \pm\ .6$	»	$2.0\ \pm0.6$	—	

(1) Observed in cloud chamber decaying in flight. (2) Decaying in flight.

TABLE III-21·2. - *Details on disintegrations of identified hyperfragments having charge higher than 2e.*

Ref.	Parent star	R_F (μm)	Time of flight (10^{-11} s)	Visible disintegration products (in brackets their kinetic energy expressed in MeV)	Q (MeV)	Disintegration scheme	B_Λ (MeV)	B_n (MeV)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
[29]	4+0K	6.3	—	π^- (37.2±0.7) ^{7}Be (0.89±0.02)	38.1±1	${}^7_\Lambda$Li → ^{7}Be+π^-	4.4±0.7	7.17	
[25]	—	—	—	π^- (46.7) ^{4}He (—) ^{4}He (—)	49.8	${}^8_\Lambda$Li → ^{4}He+^{4}He+π^-	4.5±1	1.99	
[6]	17+5?	55	—	^{3}He (42) ^{4}He (57)	170 ±3	${}^8_\Lambda$Be → ^{4}He+^{3}He+π^-	4 ±5	18.9	
[25]	—	—	—	π^- (20.89) ^{1}H (—) ^{3}He (—) ^{4}He (—)	29.5±0.5	${}^8_\Lambda$Be → ^{4}He+^{3}He+^{1}H+π^-	5.8±0.5	»	
[14]	3+0K	13.2	—	π^- (26.6) ^{1}H (2.38) [^{4}He+^{4}He] (1.28)	30.5±0.5	${}^9_\Lambda$Be → ^{4}He+^{4}He+^{1}H+π^-	6.5±0.6	1.7	
[25]	—	—	—	π^- (19.92) ^{1}H (—) ^{4}He (—) ^{4}He (—)	31.3±0.5	${}^9_\Lambda$Be → ^{4}He+^{4}He+^{1}H+π^-	5.9±0.5	»	
[25]	—	—	—	π^- (29.25) ^{1}H (—) ^{4}He (—) ^{4}He (—)	30.4±0.6	»	6.6±0.6	»	
[32]	20+1n	19	—	^{1}H (104±6) ^{3}He (26) ^{7}Li (17.5)	147 ±6	${}^{11}_\Lambda$C → ^{7}Li+^{3}He+^{1}H	13±6	12.1	(¹)

(¹) The three secondary tracks are coplanar.

integration easier to establish. It follows that light hyperfragments have been favoured by the selecting criterion with respect to heavier ones, since the latter rarely decay mesically, while the former do. On the other hand, regardless of whether a meson is emitted or not, the difficulty in interpreting the observations increases as the size of the fragment increases owing to the larger variety of schemes which become acceptable within the limits of experimental uncertainty.

21·3. – Relative frequency of hyperfragments as a function of Z.

The data presented in this section have been based on all the events reported as hyperfragments, regardless of whether the decay scheme was established or not. It was only required that Z be known with a precision of at least ± 1 unit. The experimental distributions which have been obtained (see Fig. 1-21·3 and 2-21·3) gathering together all available information give probably a grossly distorted picture of the true facts and require some comment.

We have found it impossible to apply any correction since different experiments are affected by different biasses: this appears clearly from Fig. 1-21·3 where *a*) shows the distribution obtained assembling together individual events dispersed in the literature of the last few years, *b*) the results obtained by the Wisconsin group, *c*) is the sum of *a*) and *b*); and from Fig. 2-21·3, which refers only to hyperfragments ejected from K^- capture stars. The difference among the three graphs need not be emphasized.

To simplify the discussion the data

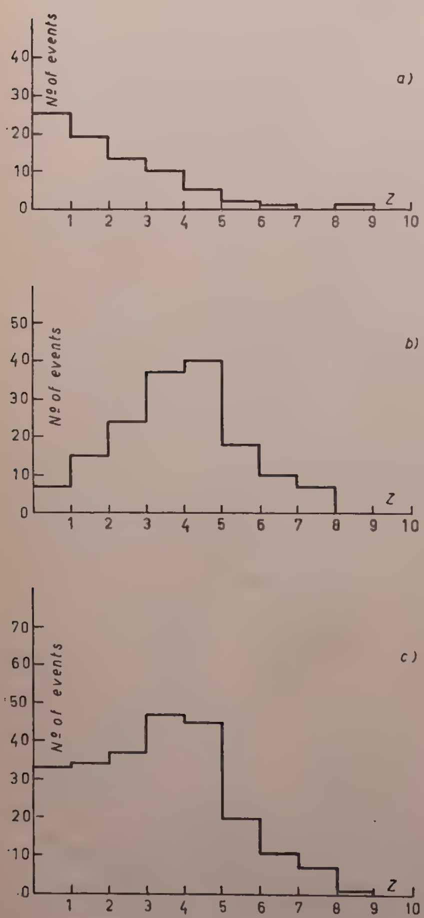


Fig 1-21·3.

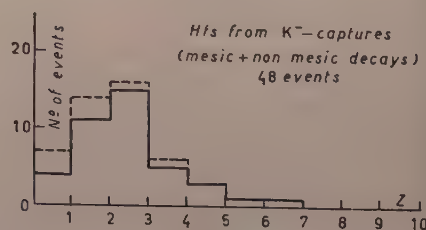


Fig. 2-21·3.

have been replotted in two separate graphs (Fig. 3-**21**·3 *a,b*) regarding respectively π^- mesic and non-mesic π^0 decays. The first one (3-**21**·3*a*) is probably the least distorted, for the reasons mentioned in the previous section. Greater care should be taken in interpreting the second graph 3*b*: it may well be possible that its shape is *entirely* determined by experimental biasses. This can be seen on the basis of the following considerations:

1) very massive and highly charged hyperfragments ($Z \geqslant 5$) are usually emitted at low velocity and have such a short residual range not to allow direct charge estimates.

Unless their mode of decay is known—and we have seen how rarely that happens—their charge and their identity cannot be established (*).

2) The efficiency in detecting non-mesic and π^0-mesic light hyperfragment decays is extremely low. For instance, we have seen in Sect. **21**·2 that singly charged hyperfragments decay π^- mesically producing little stars often consisting only of a light meson track and a short recoil a few micron long. One would expect a comparable number of cases in which a π^0 instead of its charged counterpart is emitted, but *not one* has been individuated with certainty (+). Most probably this type of events is confused by the observers with others of different nature such as

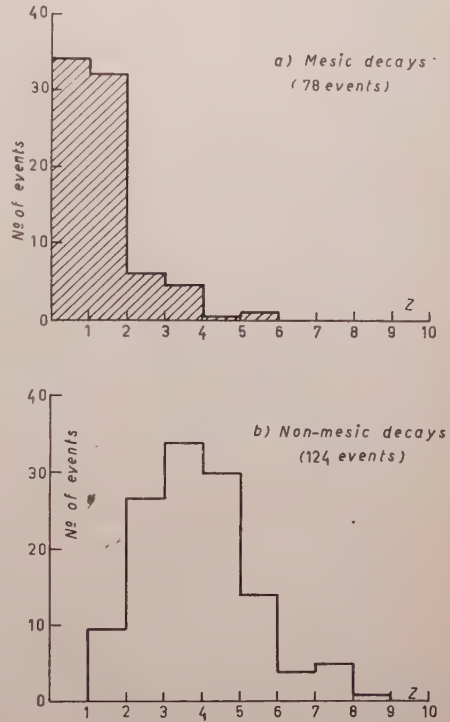


Fig. 3-**21**·3.

(*) Double stars connected by a short ($< 15\ \mu$m) black track may be due to slow π^-, K^-, or Σ^- interactions apart from slow hyperfragments. The «visible» energy associated with the terminal disintegrations produced by such particles does not in general differ greatly from that expected for a Λ^0-hyperfragment. An attempt to separate the different species was made by ALVIAL *et al.* [33] who did so by measuring very precisely the «thickness» of the interconnecting track even when only a few microns long. Their method requires careful calibration and depends critically on the intensity of the development. According to SILVERSTEIN *et al.* [25] K^- and Σ^- contribute in negligible number to the production of these short «pseudo-hyperfragments» but the π^- may be responsible for as much as 70%.

(+) A possible case of non-mesic triton hfg-decay has been reported by BLAU [34], but it was not a clear cut case.

single Coulomb or nuclear scatterings of slow particles. Genuine interactions of this type are far most frequent in emulsion and the discrimination between these and hyperfragments would require an enormous amount of work. As an example let us consider the following decay mode

$$ {}^{3}_{\Lambda}\mathrm{H} \to \pi^0 + {}^{3}\mathrm{H} \, , $$

which is analogous to the other ${}^{3}_{\Lambda}\mathrm{He} \to \pi^- + {}^{3}\mathrm{H}$ observed in a number of cases. The residual range (see Table I-21·3) of the recoiling ${}^{3}\mathrm{H}$ is expected to be $\sim 25\ \mu\mathrm{m}$ and it would appear as black as the terminal part of the parent ${}^{3}_{\Lambda}\mathrm{H}$, just as a particle suffering a single scattering at $25\ \mu\mathrm{m}$ from the end of its range.

TABLE I-21·3. – *Details on hyperfragment decays which may simulate scattered tracks in Ilford-G5 emulsions.* (*Calculated by* M. FERRO-LUZZI *and* M. MUCHNIK).

Type of event	Residual range of the charged secondary particle (μm)	Grain density of the charged secondary particle in G5-Ilford	Type of event	Residual range of the charged secondary particle (μm)	Grain density of the charged secondary particle in G5-Ilford
${}^{3}_{\Lambda}\mathrm{H} \to \pi^0 + {}^{3}\mathrm{H}$	26	black	${}^{4}_{\Lambda}\mathrm{H} \to {}^{2}\mathrm{H} + 2\mathrm{n}$	$< 13\,330$	$5 \times$ min
$\to \pi^0 + {}^{2}\mathrm{H} + \mathrm{n}$	< 627	»	$\to {}^{1}\mathrm{H} + 3\mathrm{n}$	$< 45\,000$	$2 \times$ min
$\pi^0 + {}^{1}\mathrm{H} + 2\mathrm{n}$	$< 2\,800$	»	${}^{3}_{\Lambda}\mathrm{He} \to \pi^0 + {}^{3}\mathrm{He}$	6	black
${}^{2}\mathrm{H} + \mathrm{n}$	7070	»	${}^{4}_{\Lambda}\mathrm{He} \to \pi^0 + {}^{4}\mathrm{He}$	6	»
${}^{1}\mathrm{H} + 2\mathrm{n}$	$< 40\,000$	$3 \times$ min	$\to \pi^0 + {}^{3}\mathrm{He} + \mathrm{n}$	< 73	»
			$\to {}^{3}\mathrm{He} + \mathrm{n}$	769	»
${}^{4}_{\Lambda}\mathrm{H} \to \pi^0 + {}^{3}\mathrm{H} + \mathrm{n}$	< 278	black	${}^{5}_{\Lambda}\mathrm{He} \to {}^{4}\mathrm{He} + \mathrm{n}$	435	»
$\pi^0 + {}^{2}\mathrm{H} + 2\mathrm{n}$	< 806	»	$\to {}^{3}\mathrm{He} + 2\mathrm{n}$	$< 1\,369$	»
$\pi^0 + {}^{1}\mathrm{H} + 3\mathrm{n}$	$< 2\,175$	$5 \times$ min	$\to \pi^0 + {}^{4}\mathrm{He} + \mathrm{n}$	< 43	»
${}^{3}\mathrm{H} + \mathrm{n}$	3050	$6.5 \times$ min	$\to \pi^0 + {}^{3}\mathrm{He} + \mathrm{n}$	< 39	»

From Table I-21·3 one can see that the same situation is verified for many other cases: for example, in almost all the π^0-mesic decays of singly charged hyperfragments the charged products produce a track in G5 Ilford emulsion which is « black » and not different in appearance from that of its parent hyperfragment.

Since the secondary is emitted isotropically and scatterings due to Coulomb or nuclear interactions are strongly peaked forward, by selecting large angle deflections one would easily separate the two types of events. In ordinary energetic disintegrations however the relative frequency of emission of hyperfragments in comparison to other particles is so low that little advantage is got from it. A search on this line has been attempted at the Rome Laboratory, but out of 82 « possible » hyperfragments of charge 1 and 2e, not one could be proved inconsistent with other explanations. Similar attempts might be successful if one dispose of a large number of K^- stars, since about 2 to 5 over 100 of them are associated with hyperfragments (see Sect. **21**.6).

In conclusion, hyperfragments of intermediate size ($2 < Z \leqslant 5$) are strongly favoured by the selection imposed by our means of detection and that is why we believe that the existence of a peak in the curves shown in Fig. 1-**21**.3, 2-**21**.3 just around the value $Z \sim 2$ to 4 is not very significant.

The distribution based on hyperfragments produced in K^- capture stars deserves special attention. One would expect these to be the least biassed since K^- stars have a high probability of being associated with new particles and consequently are very closely examined. At present, however, statistics are too poor to serve any purpose.

21.4. – Mesic versus non-mesic decays.

Despite the large biasses, discussed in the previous section, the experimental results indicate clearly that the ratio

$$R = \frac{\text{mesic decays}}{\text{non-mesic decays}},$$

is a rapidly decreasing function of Z. This is in agreement with theory [34]. Ruderman and Karplus [36] have also shown how a knowledge of the ratio mesic/non-mesic is important in order to determine the spin of the Λ^0 (this was discussed in detail in Sect. **15**.7, 8).

The exact dependence of R on Z is at the moment difficult to establish. Out of 156 tracks of any range—interpreted as hyperfragments—the Wisconsin group [37] finds

$$\begin{array}{lcccc} Z = & 1 & 2 & 3 & >3 \\ R = & 6/0 & 6/10 & 6/21 & 2/109\,. \end{array}$$

21·5. – Mean life of hyperfragments.

Since the great majority of the hyperfragments decay at rest (*) it is not possible to estimate their lifetimes. It can be seen however that a considerable number of them are brought to rest in the emulsion in a time (moderation time) (+) which exceeds 10^{-11} s and a few in at least 10^{-10} s. This shows that the lifetime of Λ^0 in a nucleus is not appreciably different from its value when free ($t = 3 \cdot 10^{-10}$ s).

If one takes into account the fact that a bound Λ^0 may decay also non mesonically, this provides (GATTO - ref. [10] Ch. **10**) an argument against the explanation of the stability of the Λ^0 as due to a very high spin value.

21·6. – Production of hyperfragments in nuclear disintegrations.

Hyperfragments are a comparatively rare phenomenon in nuclear disintegrations. In the range of energy explored (3 GeV π^-; 3 ÷ 6 GeV protons) they occur with a frequency of about one every thousand stars (see Table I-**21**·6): at cosmic ray energies—which are distributed on a continuous spectrum rapidly

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(*) The following hyperfragments have been observed to decay in flight

TABLE I-**21**·5. – *Observed lifetimes of hyperfragments decaying in flight.*

Event	Lifetime (10^{-10} s)	Reference
^{3}H (?)	3.3	[38]
^{3}H	1.3	[39]
^{3}H	0.01	[15]
^{4}He	0.8	[30]
^{3,4}He	0.1	[40]
^{4}He	5.4	[7]
^{7}Li	0.1	[40]
B (?)	0.2	[41]

The query indicates uncertainty with regard to the disintegration scheme.

(+) What is intended for moderation time or time of flight must be specified. As in the last part of their range heavy fragments begin to pick up electrons (and loose them) their effective charge varies and the velocity is no longer a well defined function of the residual range. In most of the work published it was not stated how the time of flight was calculated: CASTAGNOLI *et al.* [12] bracketed the true time of flight of a fragment of initial charge Z between the extreme values calculated respectively assuming that the fragment conserved its effective charge Z throughout its motion until it stopped and assuming that its effective charge was 1 e all the time.

TABLE I-21·6. – *Frequency of production of hyperfragments.*

Reference	Primary radiation	No. of hyper-fragments observed	No. of stars examined	Frequency per star	
[34]	3 GeV	14	14480	$1 \cdot 10^{-3}$	
[12]	Cosmic rays	6	24000	$0.25 \cdot 10^{-3}$	
[41]	»	5	25000	$0.2 \cdot 10^{-3}$	
[37, 32, 14]	3 GeV	72	80000	$0.9 \cdot 10^{-3}$	
	3 GeV	21	20000	$1 \cdot 10^{-3}$	
	6 GeV	7	10000	$0.7 \cdot 10^{-3}$	
	Cosmic rays	53	110000	$0.5 \cdot 10^{-3}$	
[43]	K^--capture	28	1152	$24 \cdot 10^{-3}$	Average frequency $31 \cdot 10^{-3}$
[14]	»	11	206	$53 \cdot 10^{-3}$	
[44]	»	2	52	$26 \cdot 10^{-3}$	
[45]	»	4	70	$57 \cdot 10^{-3}$	

falling as the energy increases—the frequency per star is reduced owing to the large number of disintegrations below the threshold for production of new particles. At least in cosmic ray events stable fragments are 200 to 400 times more frequent. Considering the losses discussed in the previous sections, these values are not inconsistent with those predicted by JASTROW [46] ($\sim 2 \cdot 10^{-3}$).

The last four lines of Table I, referring to K^- capture stars, indicate that hyperfragments emerge from K^- stars far more frequently than from disintegrations due to collisions, a fact which is in agreement with theoretical predictions.

Data on the characteristics of the parent stars are scarce and not very informative. CASTAGNOLI *et al.* [12] showed that the mean energy of the cosmic ray stars associated with the emission of hyperfragments (or other types of new particles) is higher than ordinary cosmic ray stars. In Fig. 1 we have reproduced the energy distribution of hyperfragments, observed by FRY *et al.* [32], and in Fig. 2 their angular distribution with respect to the primary

Fig. 1-21.6.

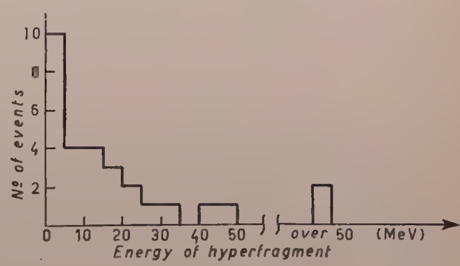

Fig. 2-21.6.

particle. Worth noting in the latter is the definite tendency of hyperfragments to be projected in the forward direction.

21·7. – Binding energies of the hyperfragments.

The quantity B_Λ, defined by the equation

$$B_\Lambda(A+1) = M_\Lambda + M_A - \sum_i m_i - Q\,,$$

represents the binding energy of a Λ^0 particle to a nucleus containing A nucleons and having a mass M_A. In the above equation M_Λ is the mass of the Λ^0, m_i the masses of the products of the disintegration and Q the sum of their kinetic energies.

TABLE I-21·7. – *Binding energies of* Λ^0 *to nuclei* (*weighted averages on the data of* Tables I, II *and* III-21.2).

Hyperfragment	B_Λ (MeV)	B_n (MeV)	Hyperfragment	B_Λ (MeV)	B_n (MeV)
${}^3_\Lambda$H	0.62 ± 0.27	6.2 (β)	${}^7_\Lambda$Li	4.4 ± 7	7.17
${}^4_\Lambda$H	1.32 ± 0.25	—	${}^8_\Lambda$Li	4.5 ± 1	1.99 (β)
${}^4_\Lambda$He	1.68 ± 0.24	20.55	${}^8_\Lambda$Be	5.8 ± 0.5	18.8 (α)
${}^5_\Lambda$He	2.58 ± 0.21	-0.8 (α)	${}^9_\Lambda$Be	6.3 ± 0.3	1.67
			${}^{11}_\Lambda$C	14.6 ± 6	13.77 (β)

Code: B_Λ = binding energy of Λ^0 to nuclei;
B_n = binding energy of the last neutron to the corresponding nucleus (in brackets the type of radioactivity for those nuclei which are instable).

Average values, calculated for several masses and charges are given in Table I and Fig. 1. In light hyperfragments such as ${}^3_\Lambda$H and ${}^4_\Lambda$He the binding of the Λ^0 is much smaller than that of the corresponding last neutron in a stable ³H or ⁴He nucleus. The failure to observe hyperdeuterons [47] and ${}^3_\Lambda$He can be explained attributing their absence to the weakness of the binding of the Λ with nucleons (*).

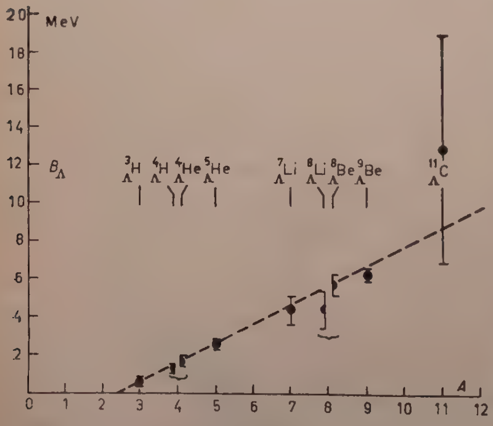

Fig. 1-21·7.

The existence of ${}^4_\Lambda$H and ${}^5_\Lambda$He shows that, despite the weakness of the Λ^0-nucleon binding, the Λ^0 may be favoured in some cases with respect to neutrons in forming aggregates since it is able to elude the exclusion prin-

(*) It cannot be explained as due to detection inefficiency: we can see no reason why the mesonic decay of a ${}^2_\Lambda$H or a ${}^3_\Lambda$He should not be as clearly visible as that of a ${}^3_\Lambda$H.

ciple. For instance, the ^{5}He is not stable because the last neutron is forbidden by Pauli's principle to go in an s state and has to go into a p state. This implies an increase of kinetic energy larger than the (negative) potential energy, i.e. no bound p state exists for the last neutron in this system. On the other hand the Λ^0 can move into an s state which is stable. For the same reasons one may expect to find aggregates such as $^6_\Lambda$He or $^8_\Lambda$Be though no stable ^{6}He nor ^{8}Be exists.

Coming back to the lighter hyperfragments ($Z \leqslant 2$), a few other remarks may be appropriate [48].

a) If charge independence holds good, the (p-Λ^0) and (n-Λ^0) interactions in the same spatial and spin state must be the same. The closeness of the experimental values of B_Λ in $^4_\Lambda$He and $^4_\Lambda$H, which differ from one another only for the replacement of one proton with one neutron, proves that this is so.

b) The esistence of an $^3_\Lambda$H and the failure to observe $^3_\Lambda$He are not in disagreement with charge independence. Assuming charge independence, a system of two nucleons and a Λ^0 may have either $T=0$ or $T=1$. The non existence of $^3_\Lambda$He indicates that for $^3_\Lambda$H the first choice is the correct one, which is very reasonable since the two nucleons are then in a ^3S state of relative motion.

c) The equivalence between Λ^0-p and Λ^0-n forces would not correspond to the truth if the interaction between the Λ^0 and the nucleons could take place through a virtual step such as

(i) $$\Lambda^0 \to \Lambda^0 + \pi^0 ,$$

because the interaction of a π^0 with a neutron has equal magnitude and opposite sign to that with a proton. The experimental fact that the binding energies of $^4_\Lambda$He and $^4_\Lambda$H are almost equal indicates that process (i) does not intervene appreciably. This again is in agreement with the assignment $T=0$ to the Λ and with the conservation of T (⁺). Of course process (i) would not violate the charge independence if a neutral π^0 with $T=0$ existed.

21·8. – The Λ^0 nucleon interaction.

Several attempts have been made (*) to calculate the binding energies of the hyperfragments on the basis of some potential between Λ^0 and nucleon. The range of this potential is generally assumed to be short with respect to $\hbar/m_\pi c$ because the Λ^0 cannot interact with a nucleon through the exchange

(⁺) Notice that the process (i) *does not* violate the conservation of strangeness (i.e. of T_3) but only that of T.

(*) Only a few papers written on the subject will be mentioned in the following; a more complete bibliography may be found in [52].

of a single pion as we have seen in the past section; so the exchange of either two pions or of a K is needed which produces a range of $\hbar/2m_\pi c$ or respectively $\hbar/m_K c$.

Field theory calculations to determine the potential Λ^0-N in the 2π exchange case have been made by DALLAPORTA and FERRARI [49] and by LICHTENBERG and ROSS [50]; in the K exchange case by WENTZEL [51] and by [49]. Field theoretical potentials have, over those phenomenologically assumed, the advantage of containing only one parameter, the coupling constant of the interaction chosen, since the range and the parameters which characterize the spin dependence and the tensor/central force ratio are predicted by the theory. However the theory, in its present state, is not capable of predicting a really reliable potential; it is already so for the nucleon-nucleon force and it is so also in this case; in addition to the uncertainties of the weak coupling treatment, here the choice of the interaction to be taken is still even more doubtful than in the nucleon-nucleon case.

Therefore, in our opinion, the most reasonable thing to do for getting an idea of the Λ^0-nucleon force is to proceed as in the nucleon-nucleon case: to assume a phenomenological potential (*the same for* Λ^0-n *and* Λ^0-p - *compare last section*) and try to determine the parameters appearing in such potential so as to reproduce the observed binding energies.

Assuming spin $\frac{1}{2}$ for the Λ^0 (there is presently nothing against this) the number of parameters, which one has to introduce in this potential, is at least three: the range and two depths, one for the singlet Λ^0 nucleon state, the other for the triplet state. It is clear that this does not exhaust the possible characteristics of such a potential (compare the number of parameters which are necessary in the nucleon-nucleon case to fit the data); but at least the above three parameters are necessary for a simplified treatment.

The determination of the above three parameters may be done in principle using the binding energies of ${}_{\Lambda}^{5}$He, ${}_{\Lambda}^{4}$He and ${}_{\Lambda}^{3}$H. This has been attempted by DALITZ and we shall from now on report his arguments.

The idea is to determine from the ${}_{\Lambda}^{5}$He and ${}_{\Lambda}^{4}$He binding energies the volume integrals of the potential in the singlet and triplet state and to determine next the range, for any assumed shape, using ${}_{\Lambda}^{3}$He. We shall see to what extent this program may be carried on.

DALITZ's [53] starting point is that, assuming the range of the Λ^0-nucleon potential to be much smaller than the radii of the nuclei with which we are dealing, it may be permissible in treating ${}_{\Lambda}^{5}$He and ${}_{\Lambda}^{4}$He to approximate such potential by a point interaction. So:

$$V_{\Lambda^0\text{-}i\text{-th Nucleon}} = -\,U\,\delta(\boldsymbol{r}-\boldsymbol{r}_i)\,, \tag{1-21.8}$$

where $\boldsymbol{r}$ is the position of the Λ^0 and $\boldsymbol{r}_i$ the position of the i-th nucleon in the nucleus with which one is dealing.

If one takes into account the spin dependence of the potential one will have to write, instead of (1-**21**·8),

$$(1'\text{-}\mathbf{21}\cdot 8)\qquad V_{\Lambda^0\text{-}i\text{-th Nucleon}} = -\left[U_t\frac{(3+\boldsymbol{\sigma}_{\Lambda^0}\cdot\boldsymbol{\sigma}_i)}{4} + U_s\frac{(1-\boldsymbol{\sigma}_{\Lambda^0}\cdot\boldsymbol{\sigma}_i)}{4}\right]\delta(\boldsymbol{r}-\boldsymbol{r}_i)\,,$$

where $\boldsymbol{\sigma}_{\Lambda^0}$ is the spin Pauli matrix for the Λ^0, and $\boldsymbol{\sigma}_i$ the spin Pauli matrix for the i-th nucleon, U_t is the depth of the triplet potential and U_s the depth of the singlet one.

Consider now first the ${}^5_\Lambda$He; since the Λ^0-nucleon force is certainly much smaller than the nucleon-nucleon one, the ${}^5_\Lambda$He may be considered as a structure consisting of an essentially undeformed α particle plus a Λ^0; the Λ^0 moving in the effective potential (to be called $V_\alpha(r)$) produced by the α particle. This potential is simply the expectation value of the sum of the potentials between the Λ^0 and the four nucleons of the α particle, calculated in the ground state $|\alpha\rangle$ of the same. One has

$$(2\text{-}\mathbf{21}\cdot 8)\qquad V_\alpha(r) = \sum_i \langle\alpha|V_{\Lambda^0\text{-}i\text{-th Nucleon}}|\alpha\rangle = \\ = -4\left(\frac{3}{4}U_t+\frac{1}{4}U_s\right)\int\varrho(\boldsymbol{r}_i)\,\delta(\boldsymbol{r}-\boldsymbol{r}_i)\,d^3r_i = -4\left(\frac{3}{4}U_t+\frac{1}{4}U_s\right)\varrho_\alpha(r)\,,$$

where $\varrho_\alpha(r)$ is the charge density of the α particle normalized to one. The combination $(\frac{3}{4}U_t+\frac{1}{4}U_s)$ appears from averaging over the spins.

We may now write a Schrödinger equation for the Λ^0 in the above potential:

$$(3\text{-}\mathbf{21}\cdot 8)\qquad -\frac{\hbar^2}{2M'_{\Lambda^0}}u'' + (E+V_\alpha(r))u = 0\,,$$

(where of course it has been assumed that the Λ^0 moves in an S state) and where M'_{Λ^0} is the reduced mass of the Λ^0; $M'_{\Lambda^0} = (4M_{\Lambda^0}M)/(M_{\Lambda^0}+4M) \simeq 0.9M$, M being the nucleon mass.

Choosing for $\varrho_\alpha(r)$ in (2) a gaussian giving a root mean square radius $1.61\cdot 10^{-13}$ cm, in agreement with the experiments of HOFSTADTER *et al.*:

$$(4\text{-}\mathbf{21}\cdot 8)\qquad \varrho_\alpha(r) = \frac{1}{a^3\pi^{\frac{3}{2}}}\exp\left[-\frac{r^2}{a^2}\right],\qquad a = 1.33\cdot 10^{-13}\text{ cm}\,,$$

and inserting (2) in (3) one obtains the correct value (*) for the binding energy

(*) The values of the binding energies of ${}^5_\Lambda$He, ${}^4_\Lambda$He, ${}^3_\Lambda$H, assumed for the following discussion are the same as the ones taken by Dalitz respectively, 2.2 ± 0.4, 1.6 ± 0.3, 0.4 ± 0.3 MeV; they do not differ essentially from the ones of the Table I-**21**·7.

of the Λ^0, if:

$$\bar{U} = \tfrac{3}{4}U_t + \tfrac{1}{4}U_s = 165 \text{ MeV}\cdot 10^{-39} \text{ cm}^3 . \tag{5-21·8}$$

Thus the volume integral of the potential between Λ^0 and average nucleon in the ^{4}He is 165 MeV·10^{-39} cm^3.

A similar calculation may be made for ${}^4_\Lambda$He assuming again for the ^{3}He a gaussian charge density with a radius determined from the Coulomb energy difference ^{3}He — ^{3}H. The potential which one obtains in this case, replacing (2-21·8), is:

$$V_{^3\text{He}}(r) = \left[-3\left(\frac{3U_t}{4}+\frac{U_s}{4}\right)-\frac{1}{4}(U_t-U_s)\right]\varrho_{^3\text{He}}(r)\,, \tag{6-21·8}$$

if the Λ^0 combines its spin with the one of ^{3}He to give total spin one, and

$$V_{^3\text{He}}(r) = \left[-3\left(\frac{3U_t}{4}+\frac{U_s}{4}\right)+\frac{3}{4}(U_t-U_s)\right]\varrho_{^3\text{He}}(r)\,, \tag{7-21·8}$$

if the Λ^0 combines its spin with the one of ^{3}He to give total spin zero. In any case writing the potential $V_{^3\text{He}}(r)$ as:

$$V_{^3\text{He}}(r) = C\varrho_{^3\text{He}}(r)\,, \tag{8-21·8}$$

where C is a constant, it results from the observed binding energy of ${}^4_\Lambda$He and solving an equation similar to (3) that C has the value $C \simeq 720$ MeV·10^{-39} cm^3. Now, if U_t is equal to U_s, that is if the Λ^0-nucleon force is spin independent, one should have from (6) or (7) and (5) $C = 510$; the difference from 510 to 720 is attributed by DALITZ to the effect of a strong spin dependence of the force (1′) that is to: $U_s \neq U_t$.

According to DALITZ a further confirmation of this strong spin dependence may be found in the high binding energy of ${}^7_\Lambda$Li; this needs in fact a value of the potential much larger than, almost the double, $6(\frac{3}{4}U_t+\frac{1}{4}U_s)\varrho_{^6\text{Li}}(r)$ which is the value one would expect in the absence of spin dependence of the forces.

Considering finally the case of the ${}^3_\Lambda$H, strong distortion effects are be to expected. Assuming now a potential with a finite range, the previous evaluations determine the volume integrals of this potential in the singlet or triplet case; so one may use ${}^3_\Lambda$H to determine the range of the potential. DALITZ discusses the situation in the two cases in which the spin of the ${}^4_\Lambda$He is one (form. (6)) and zero (5). In the first case one has $U_t > U_s$, precisely $U_t = 380$, $U_s = -480$; then the ${}^3_\Lambda$H has spin $\frac{3}{2}$, only U_t contributes and with the value $U_t = 380$ one may predict, according to DALITZ, a range of the order $\hbar/2m_\pi c$, having

assumed a Yukawa shape; one may then show also that no hyperdeuteron may be expected to be bound.

In the second case one has $U_t < U_s$, precisely $U_s = 380$, $U_t = 100$; in this case the predictions are less definite; all one can say is that if the range is larger than $0.4 \cdot 10^{-13}$ cm the absence of the hyperdeuteron may be explained. Since no hyperdeuteron has been seen the above condition on the range must be satisfied in any case.

We shall close at this point the discussion, simply remarking that the above estimates depend, though not critically, rather strongly on the values of the binding energies; the errors which affect the values of the binding energies given in the Table I-**21**·7 are small only because obtained compounding many measurements each affected by a very large error; in this situation a single measurement with a small error may be of more value than many measurement as the ones in question; it is obvious that if in the future the binding energy of ${}^{5}_{\Lambda}$He will raise and that of ${}^{4}_{\Lambda}$H remain stationary or decrease with respect to the values assumed here, the conclusion of a strong spin dependence should be reconsidered.

21·9. – Anomalous hyperfragments.

To our knowledge only 3 cases have been observed of spontaneous decay of fragments associated with a release of energy higher than that expected for a Λ^0-hyperfragment, one by the Rome group [12] and the other two by the Wisconsin group [54]. So long as they remain unconfirmed by the observation of other similar cases, it is unwise to attribute too much significance to them. We have reported here below their main characters for convenience of other workers.

The Rome event [12]. – A track, emerged from a 26+9n star produced by cosmic rays, stops in the emulsion after a range 1240 μm long. From δ-ray counting and thin down length the charge has been estimated to be 4 ± 1 units. The secondary star, consisting of three branches one of which heavily ionizing, makes it difficult to accept the value $Z=3$. The mass was found to be 9 ± 1 proton masses.

The terminal disintegration produced three ionizing particles two of which are too short to be identified and the third (5120 μm long) appears to have a charge 2e. According to the Authors, if the missing momentum is balanced by the emission of at least two neutral particles, the total energy release is at least 340 MeV.

The following was suggested as the most probable decay scheme

$$ {}^{10}\mathrm{B}^* \to {}^{4}\mathrm{He} + {}^{3}\mathrm{He} + {}^{1}\mathrm{H} + 2\mathrm{n} + Q \qquad Q \simeq 500 \ \mathrm{MeV}; $$

The first Wisconsin event [55] – The primary star is of type 22+9p. The track of the hyperfragment is 192 μm long and its shape indicates that it stops before disintegrating; its charge being certainly smaller than $5e$ and most probably greater than $2e$. The terminal disintegration produces four tracks: one was identified as an α particle of 14 MeV; two, of charge $1e$ could be protons or deuterons or tritons; if protons they had an energy of 2.7 and (22 ± 2) MeV respectively; the fourth could be either a π meson (of $45 \pm \pm 18$ MeV) or a proton (of 300 ± 125 MeV). If the last particle is taken to be a proton the charge of the fragment must be $5e$, which is inconsistent with direct charge measurements. If it is a π^-, the energy release from the secondary star is 110 MeV, to be compared with 37 MeV expected for mesic Λ^0 hyperfragments.

The second Wisconsin event [56]. – A thick track, only 9 μm long, connects a 21+9n disintegration with a small secondary star having 3 branches: (1) due to an unidentified recoil, (2) due to an isotope of hydrogen and (3) to a particle of charge $1e$ whose mass—as deduced from scattering, grain density and residual range in three different ways—appears to be very close to that of a K meson. Since its end in emulsion is not associated with any visible decay or interaction product, it was interpreted as a $K\bar{\rho}$ event. The total energy release was $\geqslant 550$ MeV including the rest mass of the supported K meson.

Since the nature of the interconnecting track is not established, the event could be due to either a hyperfragment or to the capture of a negative particle. In either cases a hyperon heavier than $\sim 2\,910\ m_e$ or a K heavier than $1075\ m_e$ must be postulated.

We notice here that the only new particle which, besides the Λ^0 may generally form semistable aggregates with nucleons is the $K^{0,+}$, at least if there is an attractive potential between K^0 and nuclei. Perhaps events [12] and [55] may be of this kind. PAIS and SERBER [57] have discussed the appearance of the disintegrations of these K^0 excited fragments and in particular the frequencies of 2π, π decays and non-mesonic decays with the conclusion that the non-mesonic mode should be less probable by a factor ~ 100 than the π or 2π (which have comparable probability).

Finally it may be pointed out that [58] other aggregates, which in principle, may be semistable ones are those of a Σ^- or a Ξ^- with any number of neutrons and of a Σ^- or Ξ^0 with any number of protons. There is no clear proof of the effective existence of such aggregates; the Padua group has [37] one example which may be interpreted as such.

REFERENCES (CH. 21)

The references marked with a (*) are not quoted in the text; they have been used in the construction of the graphs of Sect. **21**·3.

[1] M. DANYSZ and J. PNIEWSKY: *Phil. Mag.*, **44**, 348 (1953).
[2] C. F. POWELL: *Nature*, **173**, 469 (1954).
[3] J. CRUSSARD and D. MORELLET: *Compt. Rend. Ac. Sci.*, **236**, 64 (1953).
[4] A. BONETTI, R. LEVI SETTI, M. PANETTI, L. SCARSI and G. TOMASINI: *Nuovo Cimento*, **11**, 210 (1954).
[5] A. BONETTI, R. LEVI SETTI, M. PANETTI, L. SCARSI and G. TOMASINI: *Nuovo Cimento*, **11**, 330 (1954).
[6] W. F. FRY and G. R. WHITE: *Nuovo Cimento*, **11**, 551 (1954).
[7] S. O. SÖRENSEN: *Meddel.* **186**, University Fys. Inst., Oslo.
[8] R. H. BROWN, U. CAMERINI, P. H. FOWLER, H. HEITLER, D. T. KING and C. F. POWELL: *Phil. Mag.*, **40**, 862 (1949).
[9] M. GRILLI and R. LEVI SETTI: *Suppl. Nuovo Cimento*, **12**, 466 (1955).
[10] R. LEVI SETTI: *Nuovo Cimento*, **2**, 263 (1955).
[11] M. BALDO, G. BELLIBONI, M. CECCARELLI, M. GRILLI, B. SECHI, B. VITALE and G. T. ZORN: *Nuovo Cimento*, **1**, 1180 (1955).
[12] C. CASTAGNOLI, G. CORTINI and C. FRANZINETTI: *Nuovo Cimento*, **2**, 550 (1955).
[13] A. DEBENEDETTI, C. M. GARELLI, T. TALLONE and M. VIGONE: *Nuovo Cimento*, **12**, 466 (1954).
[14] W. F. FRY, J. SCHNEPS and M. S. SWAMI: *Phys. Rev.*, **101**, 1526 (1956).
[15] O. SKJEGGESTAD and S. O. SÖRENSEN: *Nuovo Cimento*, **3**, 652 (1956).
[16] D. M. HASKIN, T. BOWEN, R. G. GLASSER and M. SCHEIN: *Phys. Rev.*, **102**, 244 (1956).
[17] J. CRUSSARD: *The VI Roches er Conference*, mimeogr. rep., V-39.
[18] J. CRUSSARD, V. FOUCHÉ, G. KAYAS, L. LEPRINCE-RINGUET, D. MORELLET, F. RENARD and J. TREMBLEY: *Suppl. Nuovo Cimento*, **4**, 616 (1956).
[19] J. H. DAVIES, D. EVANS, P. H. FOWLER, P. R. FRANCOIS, M. W. FRIEDLANDER, R. HILLER, P. IREDALE, D. KEEFE, M. G. K. MENON, D. H. PERKINS and C. F. POWELL: *Suppl. Nuovo Cimento*, **4**, 628 (1956).
[20] M. W. FRIEDLANDER, D. KEEFE and M. G. K. MENON: *Nuovo Cimento*, **2**, 663 (1955).
[21] M. W. FRIEDLANDER, D. KEEFE and M. G. K. MENON: *Nuovo Cimento*, **1**, 482 (1955).
[22] O. HANGERUD and S. O. SÖRENSEN: *Phys. Rev.*, **99**, 1046 (1955).
[23] J. HORNBOSTEL and E. O. SALANT: *Phys. Rev.*, **102**, 502 (1956).
[24] M. SCHEIN, D. M. HASKIN and D. LEENOV: *Phys. Rev.*, **100**, 1455 (1955).
[25] E. SILVERSTEIN, W. SLATER, R. LEVI SETTI and V. L. TELEGDI: circulated preprints.
[26] PH. ROSSELET, R. WEILL and M. GAILLARD: *Nuovo Cimento*, **3**, 505 (1956).
[27] N. SEEMAN, M. SHAPIRO and B. STILLER: *Phys. Rev.*, **100**, 1480 (1955).
[28] A. G. BARKOW and B. CHAMANY: *Phys. Rev.*, **105**, 692 (1957).
[29] F. C. GILBERT, C. E. VIOLET and R. S. WHITE: *Phys. Rev.*, **103**, 248 (1956).
[30] P. H. FOWLER and K. H. HANSEN: *Nuovo Cimento*, **4**, 158 (1956).
[31] R. D. HILL, E. O. SALANT, M. WIDGOFF, L. S. OSBORNE, A. PEVSNER, D. M. RITSON, J. CRUSSARD and W. D. WALKER: *Phys. Rev.*, **101**, 1127 (1956).
[32] W. F. FRY, J. SCHNEPS and M. S. SWAMI: *Phys. Rev.*, **99**, 1561 (1955).
[33] G. ALVIAL, A. BONETTI, C. DILWORTH, M. LADU, J. MORGAN and G. OCCHIALINI: *Suppl. Nuovo Cimento*, **4**, 244 (1956).
[34] M. BLAU: *Phys. Rev.*, **102**, 495 (1956).

[35] W. CHESTON and H. PRIMAKOFF: *Phys. Rev.*, **92**, 1537 (1953).
[36] M. RUDERMAN and R. KARPLUS: *Phys. Rev.*, 102, 247 (1956).
[37] W. F. FRY: Lecture held at Rome University (March 1957).
[38] G. ALEXANDER, J. P. ASTBURY, C. BALLARIO, R. BIZZARRI, B. BRUNELLI, A. DE MARCO, A. MICHELINI, G. C. MONETI, E. ZAVATTINI and A. ZICHICHI: *Nuovo Cimento*, **2**, 365 (1955).
[39] F. BRISBOUT, M. W. FRIEDLANDER and P. IREDALE: *Nuovo Cimento*, **4**, 948 (1956).
[40] F. A. BRISBOUT and V. D. HOPPER: *Austr. Journ. of Phys.*, **1**, 352 (1954).
[41] B. WALDESKOG: *Ark. f. Fys.*, **8**, 369 (1954).
[42] P. CIOK, M. DANYSZ, J. GIERULA, E. SLANZYPCZAK and A. WROBLEWSKI: *Suppl. Nuovo Cimento*, **4**, 619 (1956).
[43] W. H. BARKAS, W. F. DUDZIAK, P. C. GILES, H. H. HECKMAN, F. W. INMAN, C. J. MASON, N. A. NICKOLS and F. M. SMITH: UCRL 3627.
[44] E. P. GEORGE, A. J. HERZ, J. H. NOON and N. SOLNTSEFF: *Nuovo Cimento*, **3**, 44 (1956).
[45] D. M. HASKIN, T. BOWEN and M. SCHEIN: *Phys. Rev.*, **103**, 1512 (1956).
[46] A. B. JASTROW: *Phys. Rev.*, **97**, 179 (1955).
[47] H. PRIMAKOFF and W. CHESTON: *Phys. Rev.*, **93**, 908 (1954).
[48] R. DALITZ: *Phys. Rev.*, **99**, 1475 (1955).
[49] N. DALLAPORTA and F. FERRARI: *Nuovo Cimento*, **5**, 3 (1957).
[50] D. LICHTENBERG and M. ROSS: *Phys. Rev.*, **103**, 1131 (1956).
[51] G. WENTZEL: *Phys. Rev.*, **101**, 835 (1956).
[52] J. J. JONES and J. M. KELLER: *Nuovo Cimento*, **4**, 1329 (1956).
[53] R. DALITZ: *Rochester Conference* (1956); also lecture given at Iowa University (March 1957).
[54] W. F. FRY, J. SCHNEPS, G. A. SNOW and M. S. SWAMI: *Phys. Rev.*, **100**, 1448 (1955).
[55] W. F. FRY and M. S. SWAMI: *Phys. Rev.*, **96**, 809 (1954).
[56] W. F. FRY, J. SCHNEPS and M. S. SWAMI: *Phys. Rev.*, **97**, 1189 (1955).
[57] A. PAIS and R. SERBER: *Phys. Rev.*, **99**, 1551 (1955).
[58] D. HOLLADAY: quoted by R. G. SACHS: *Phys. Rev.*, **99**, 15 (1955).
[59] M. GRILLI, B. SECHI and G. T. ZORN: *Suppl. Nuovo Cimento*, **12**, 310 (1954).
[*] F. ANDERSON, G. LAWLOR and T. E. NEVIN: *Nuovo Cimento*, **2**, 605 (1955).
[*] P. H. BARRETT: *Phys. Rev.*, **94**, 1328 (1954).
[*] M. CECCARELLI, N. DALLAPORTA, M. GRILLI, M. MERLIN, G. SALANDIN, B. SECHI and M. LADU: *Nuovo Cimento*, **2**, 542 (1955).
[*] P. CIOK, M. DANYSZ and J. GIERULA: *Nuovo Cimento*, **11**, 436 (1954).
[*] J. CRUSSARD, M. F. KAPLON, J. KLARMAN and J. H. NOON: *Phys. Rev.*, **95**, 584 (1954).
[*] P. S. FREIER, G. W. ANDERSON and J. E. NAUGLE: *Phys. Rev.*, **94**, 677 (1954).
[*] K. GOTTSTEIN: *Suppl. Nuovo Cimento*, **12**, 309 (1954).
[*] M. LADU, R. LEVI SETTI, L. SCARSI and G. TOMASINI: *Suppl. Nuovo Cimento*, **4**, 621 (1956).
[*] G. LOVERA, L. BARBANTI-SILVA, C. BONACINI, C. DE PIETRI, R. PERILLI FEDELI and A. ROVERI: *Nuovo Cimento*, **10**, 986 (1953).
[*] J. E. NAUGLE, E. P. NEY, P. S. FREIER and W. B. CHESTON: *Phys. Rev.*, **96**, 1383 (1954).
[*] M. SCHEIN, D. M. HASKIN and R. G. GLASER: *Nuovo Cimento*, **3**, 131 (1956).
[*] O. SKJEGGESTAD and S. O. SÖRENSEN: *Phys. Rev.*, **104**, 511 (1956).
[*] A. SOLHEIM and S. O. SÖRENSEN: *Phil. Mag.*, **45**, 1284 (1954).
[*] J. O. SORRELS, G. M. TRILLING and R. B. LEIGHTON: *Phys. Rev.*, **100**, 1484 (1955).
[*] H. YAGODA: *Phys. Rev.*, **98**, 153 (1955).

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